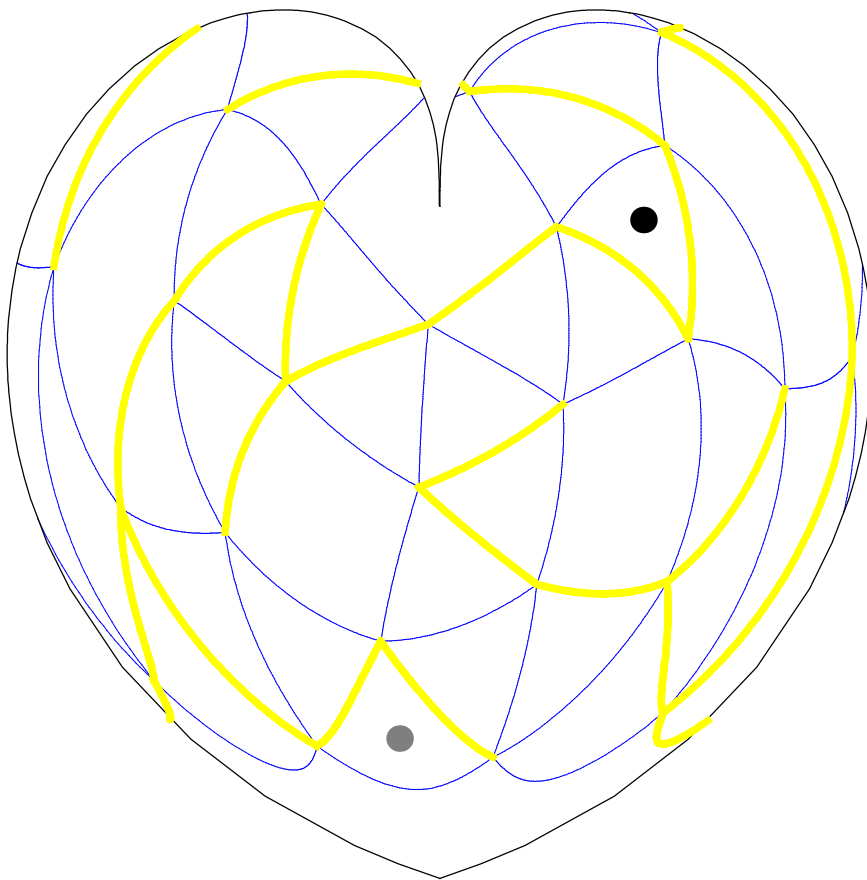
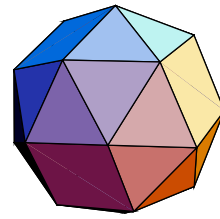
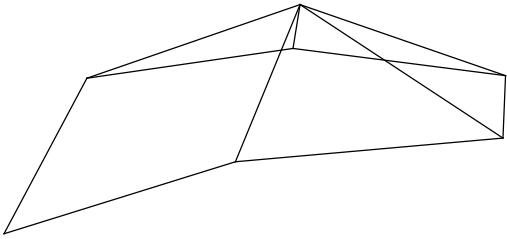


Izidor Hafner

Mazes on Uniform Polyhedra

Bonnet-Werner projection

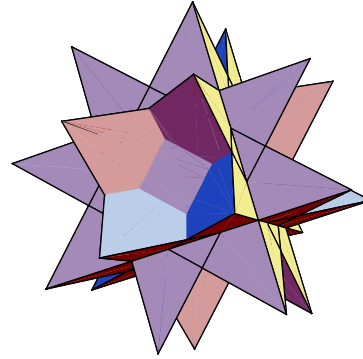
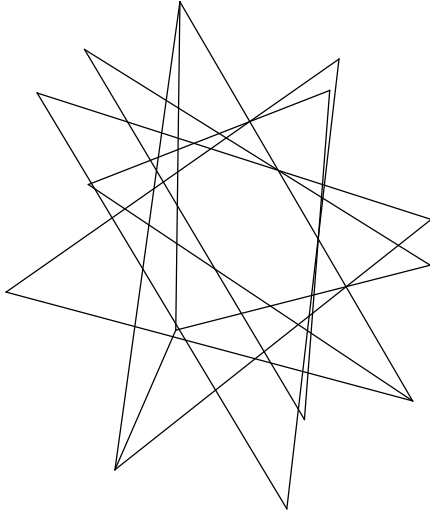


Introduction

Let us take an example. We are given a uniform polyhedron.

stellated truncated hexahedron

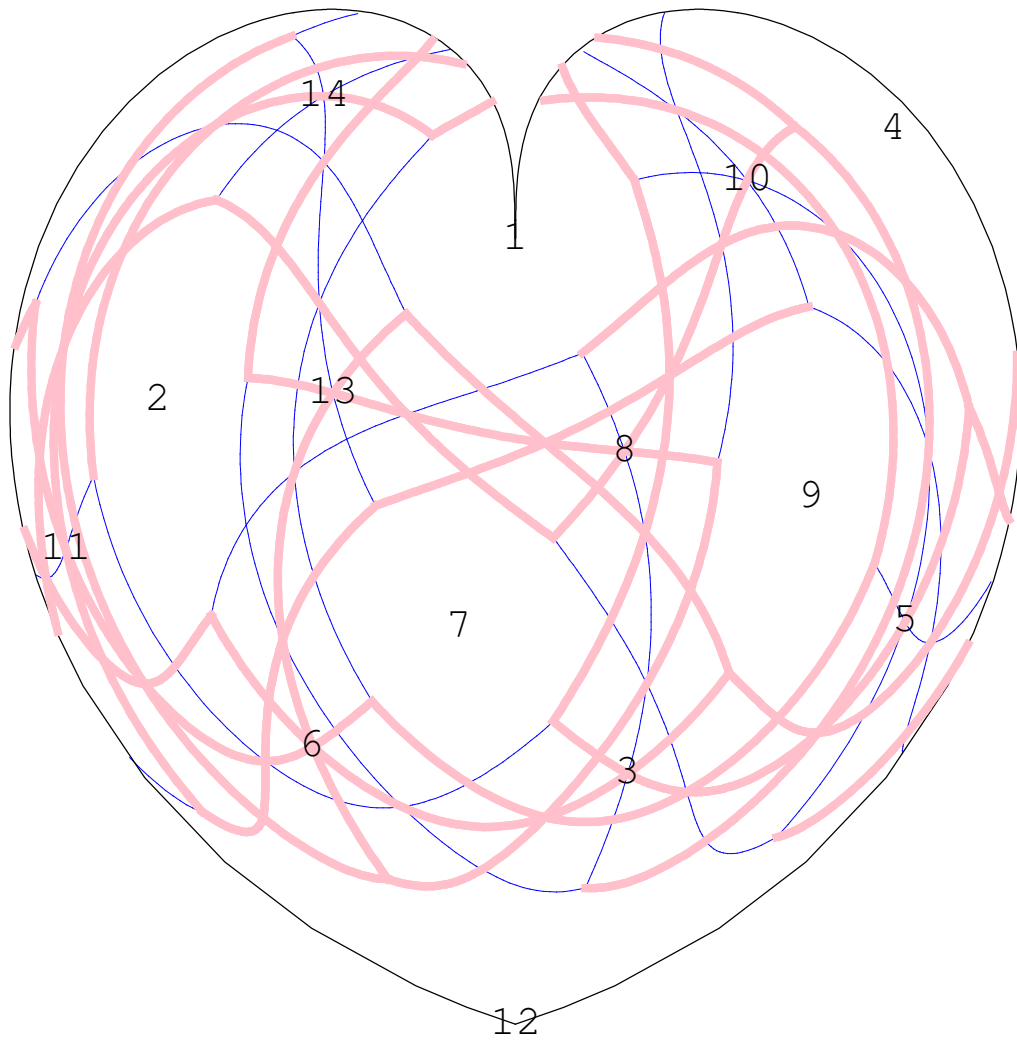
$$\left\{ \frac{8}{3}, \frac{8}{3}, 3 \right\}$$

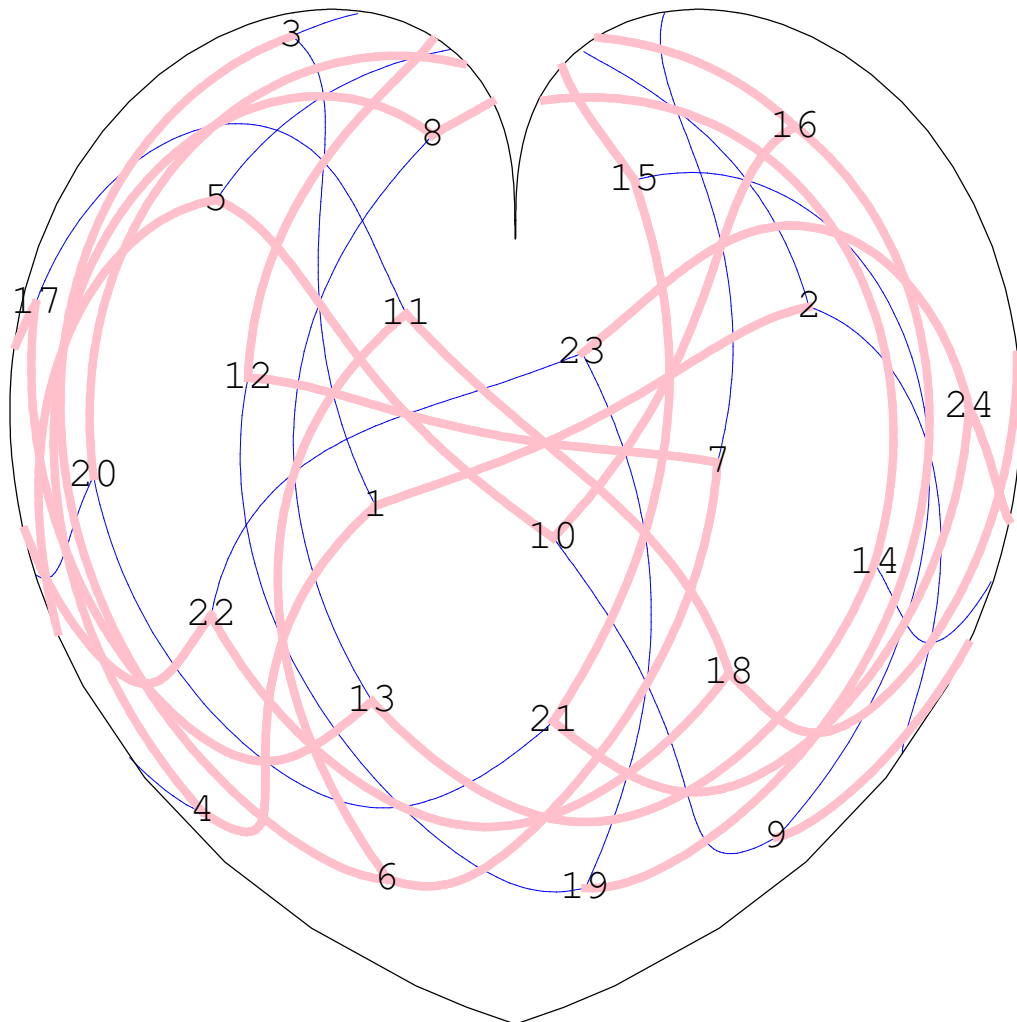


In Mathematica the polyhedron is given by a list of faces and with a list of coordinates of vertices [Roman E. Maeder, *The Mathematica Programmer II*, Academic Press 1996]. The list of faces consists of a list of lists, where a face is represented by a list of vertices, which is given by a matrix. Let us show the first five faces:

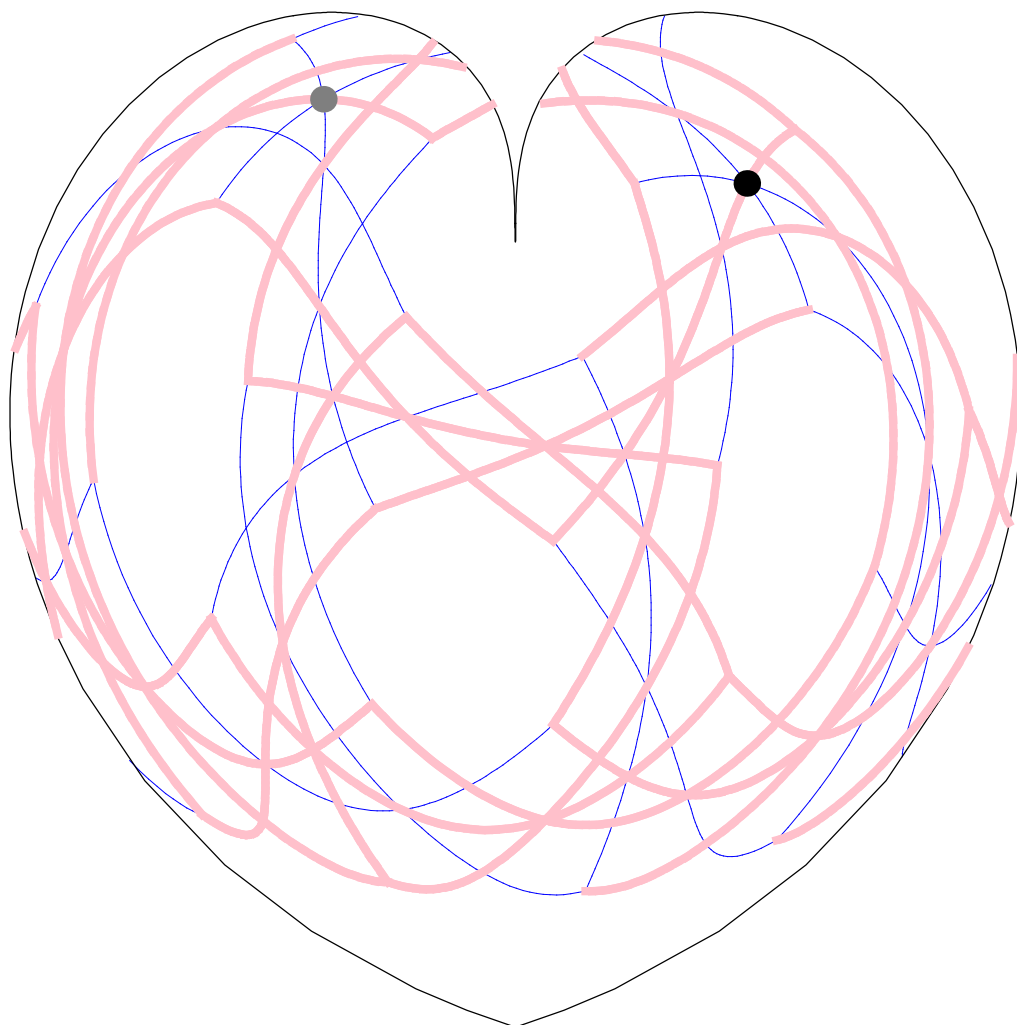
$$\begin{pmatrix} \{1, 2, 5, 10, 16, 12, 7, 3\} \\ \{1, 3, 6, 11, 17, 13, 8, 4\} \\ \{1, 4, 2\} \\ \{2, 4, 8, 14, 20, 15, 9, 5\} \\ \{3, 7, 6\} \end{pmatrix}$$

The next two figures represent faces and vertices. The polyhedron is projected onto a sphere and the sphere is projected by a cartographic projection.

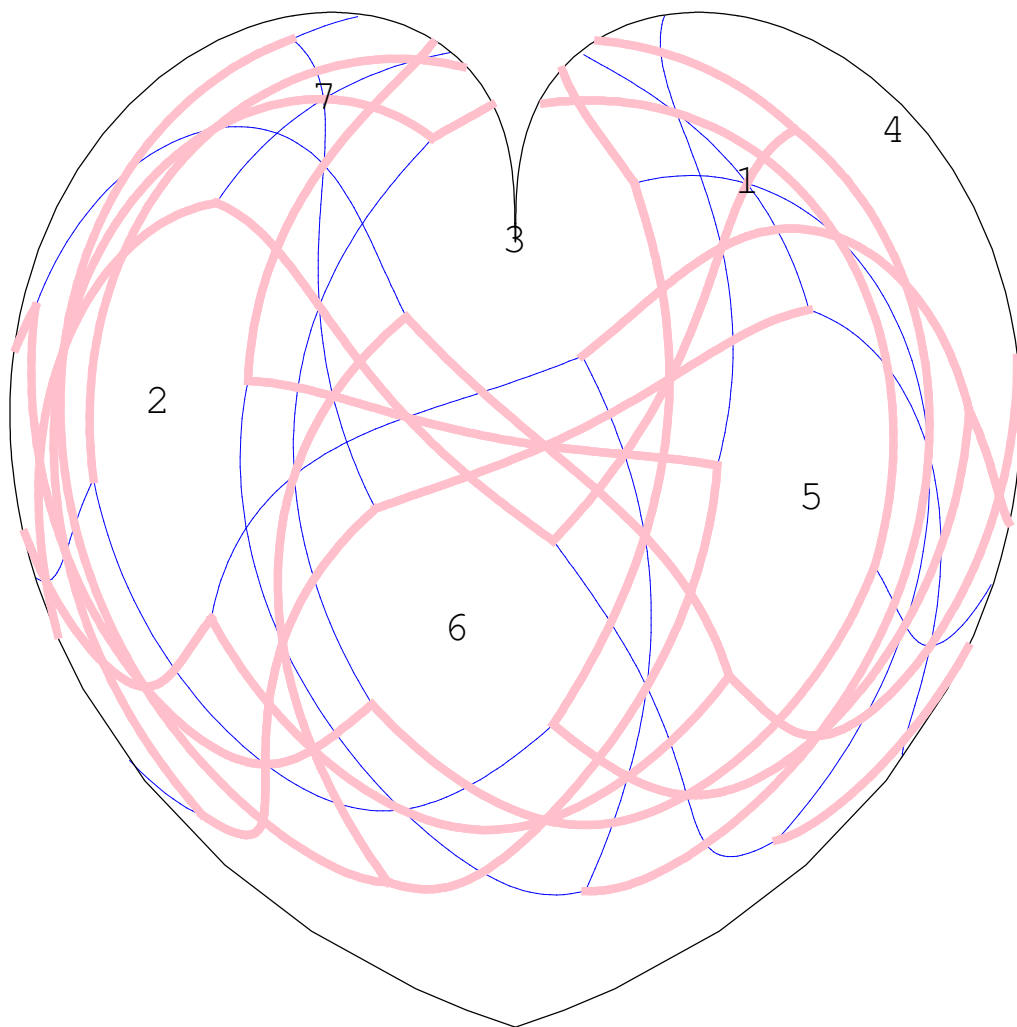




The problem is to find the path from the black dot to gray dot, where thick lines represent walls of a maze.



The solution is given by a list of faces passed from the black to gray dot.

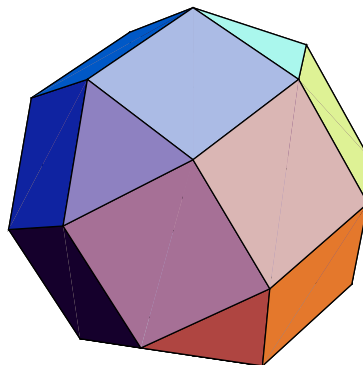
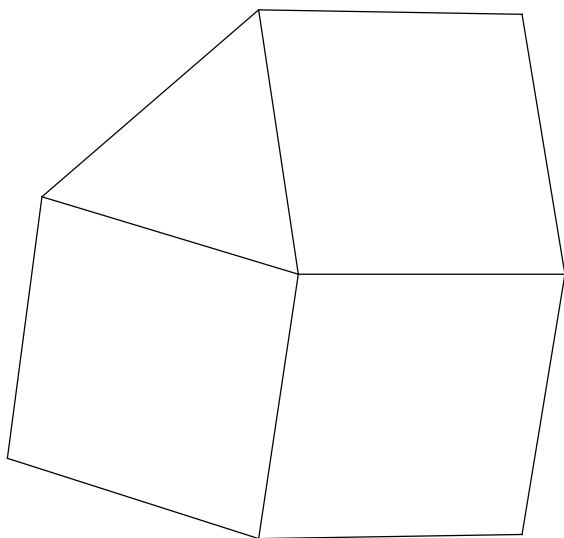


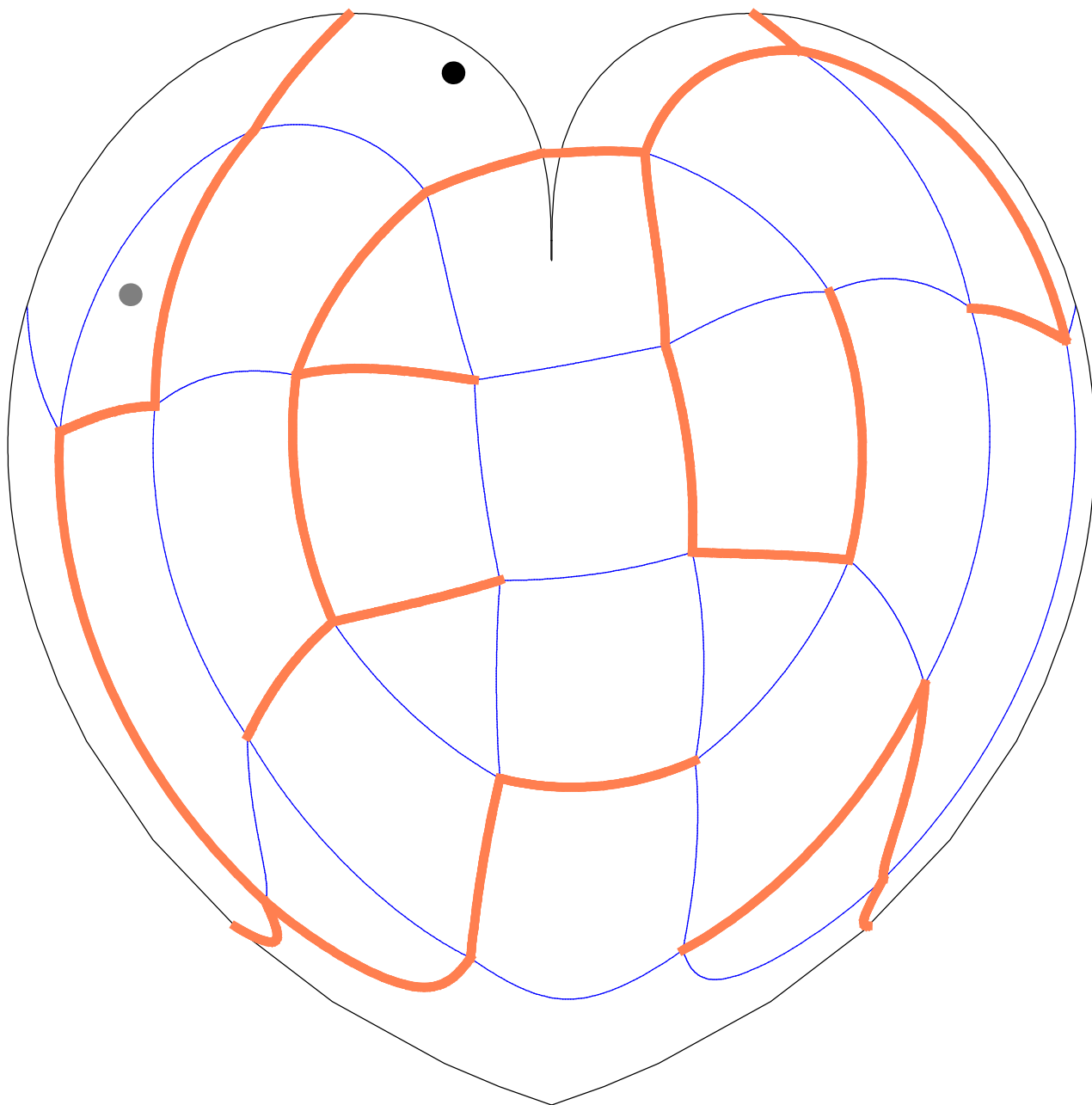
Problems

1.

rhombicuboctahedron

{4, 3, 4, 4}

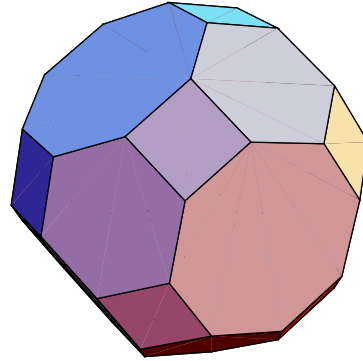
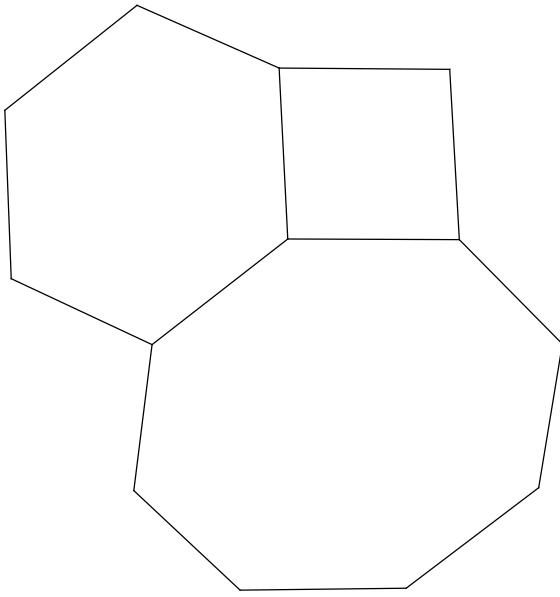


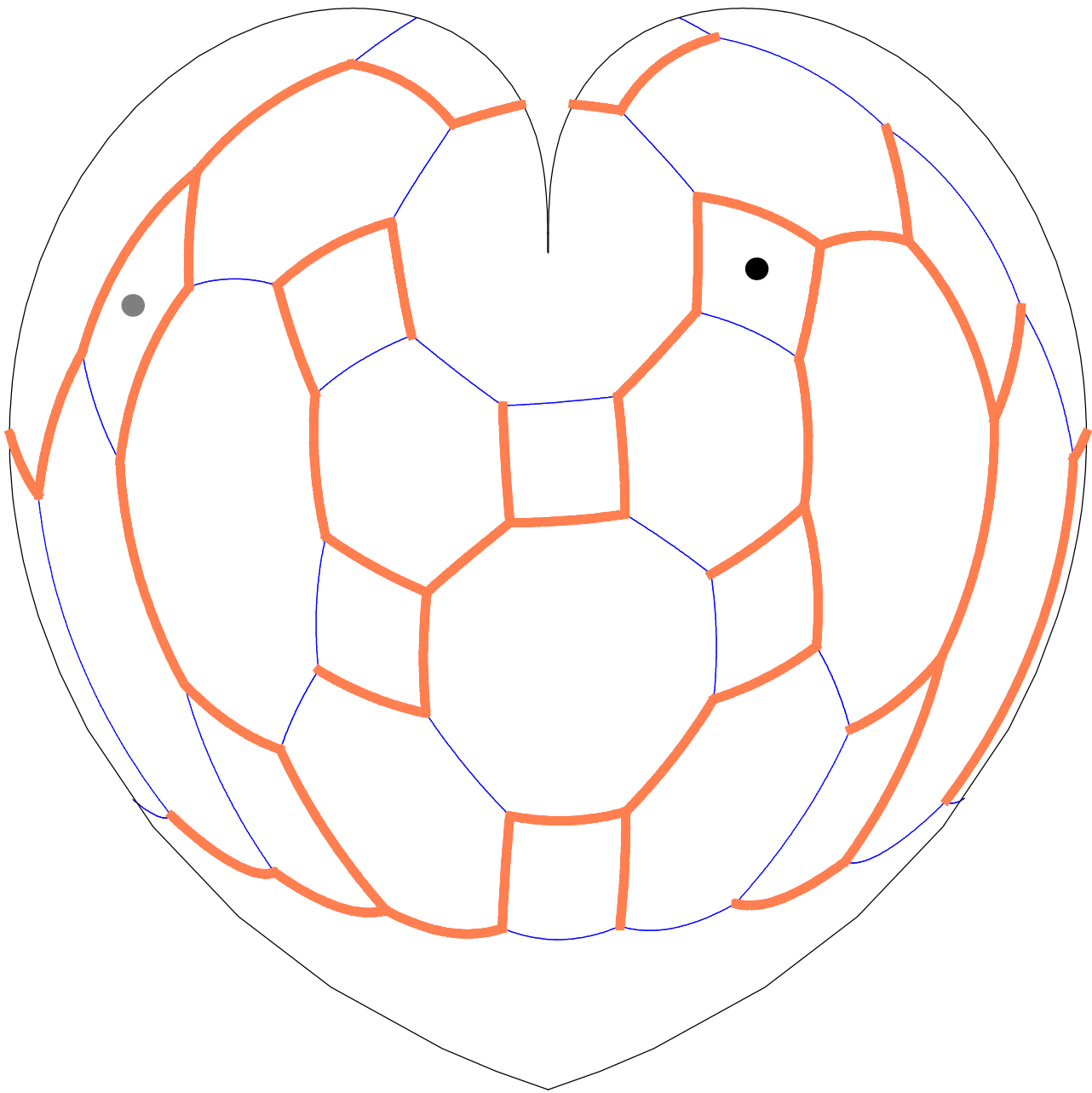


2.

truncated cuboctahedron

{4, 6, 8}

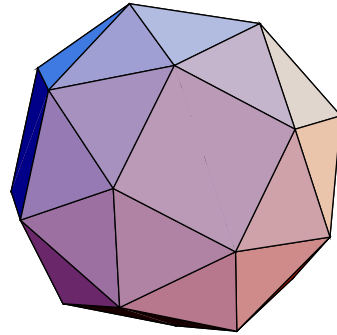
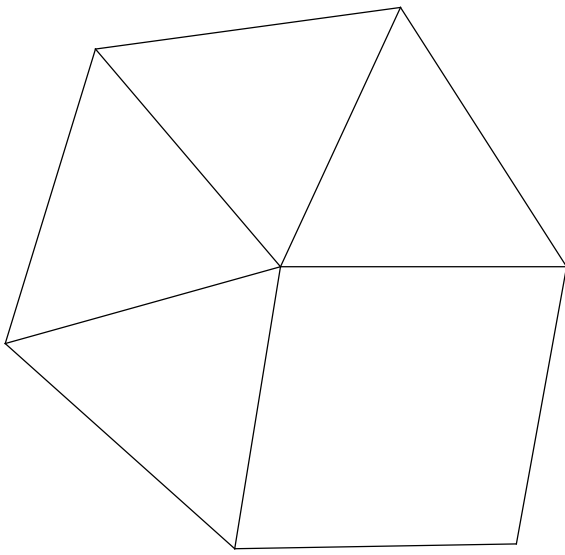


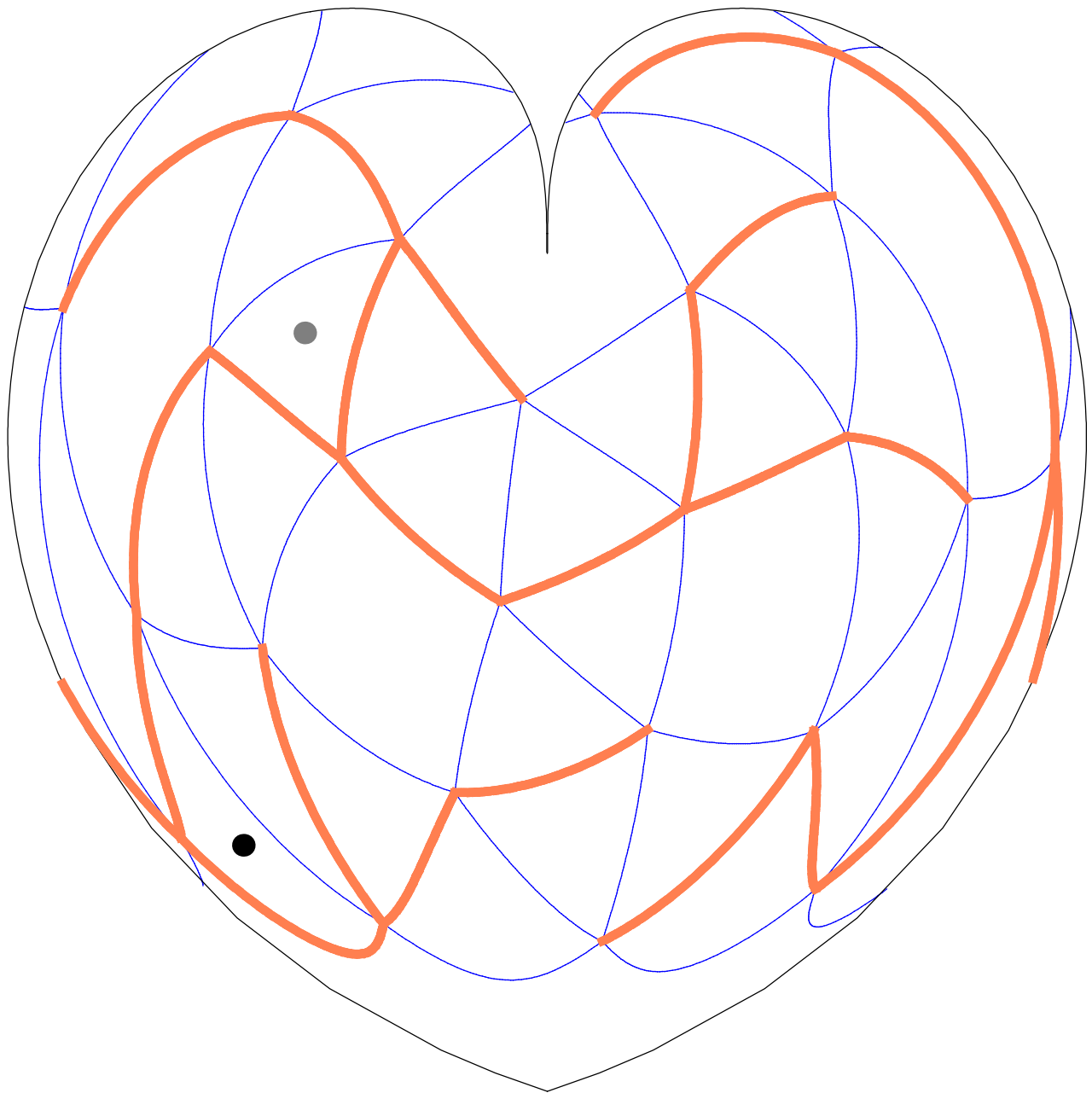


3.

snub cube

{3, 3, 3, 3, 4}

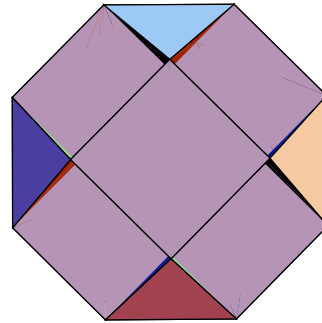
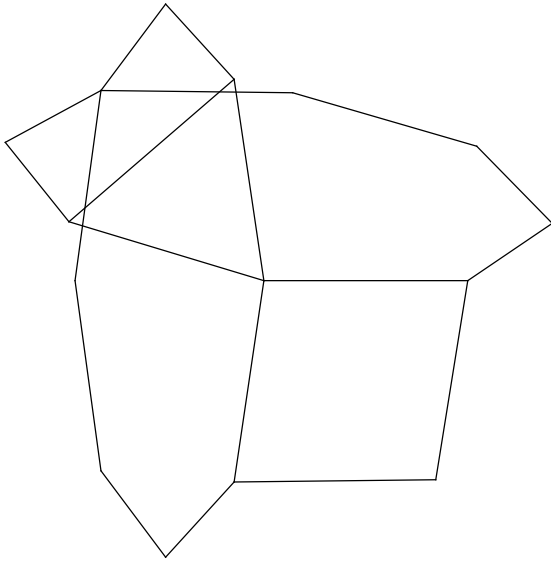


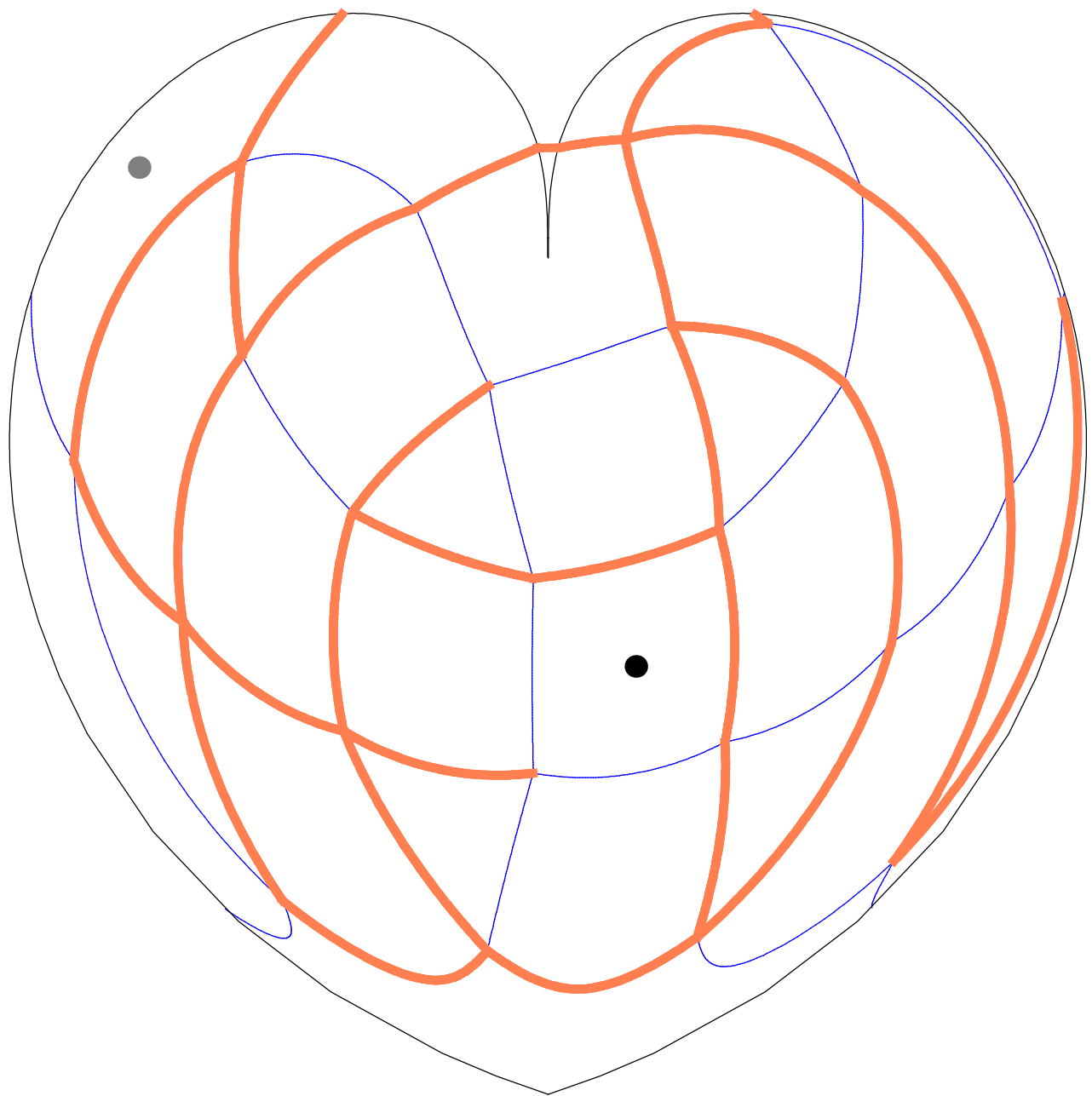


4.

small cubicuboctahedron

$$\left\{8, \frac{3}{2}, 8, 4\right\}$$

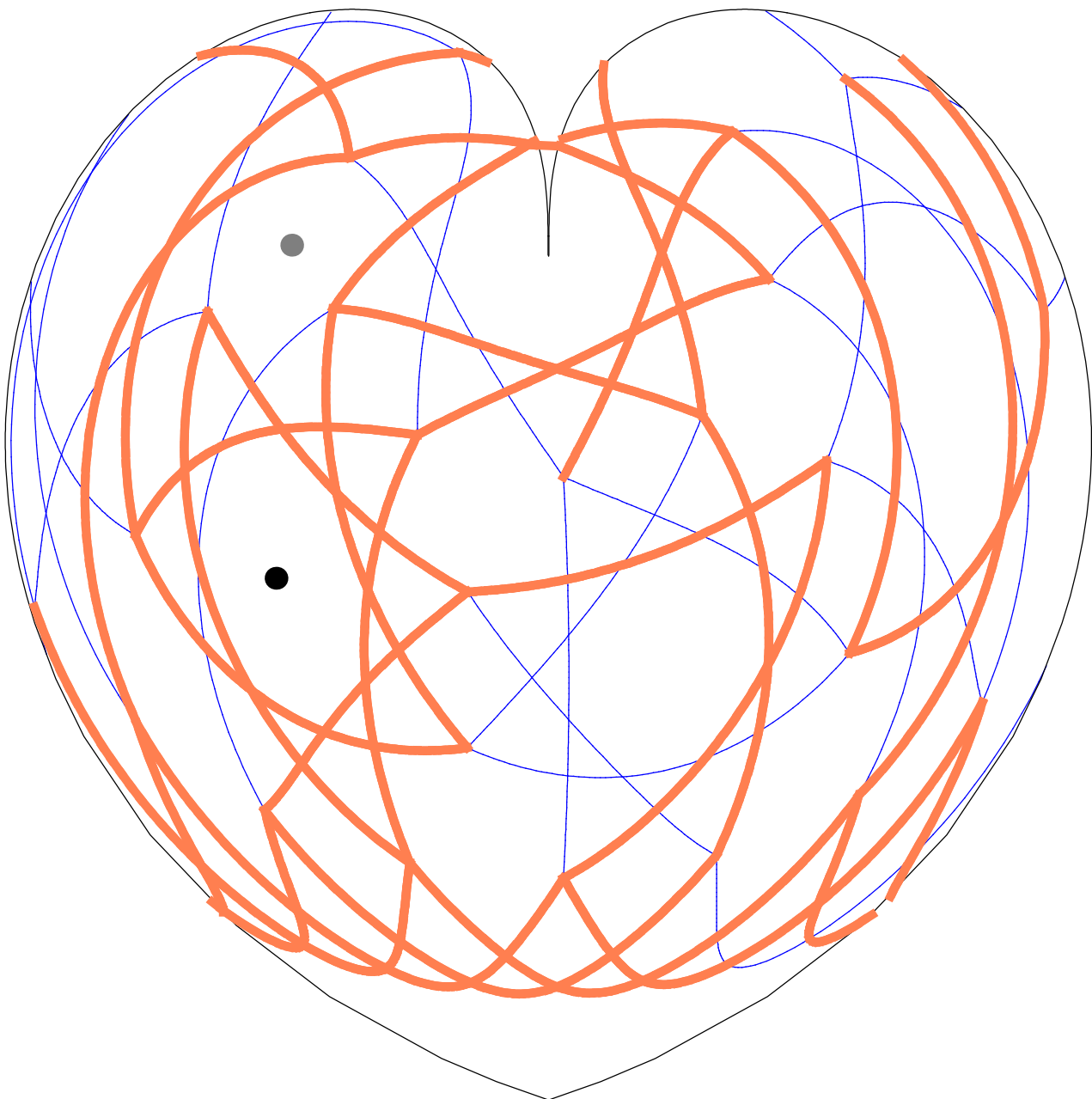
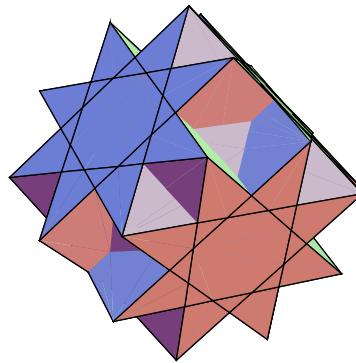
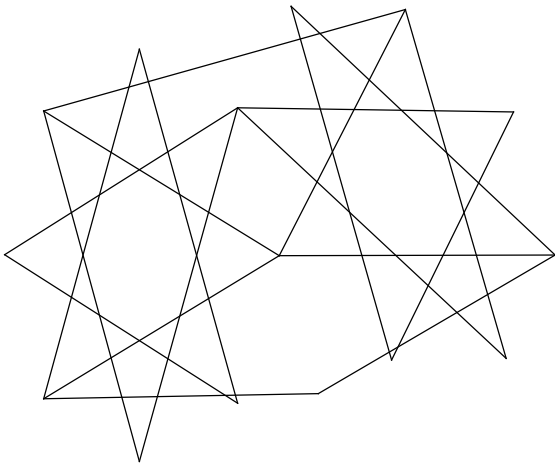




5.

great cubicuboctahedron

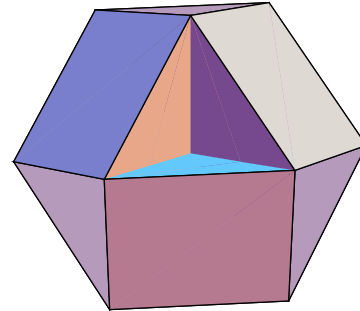
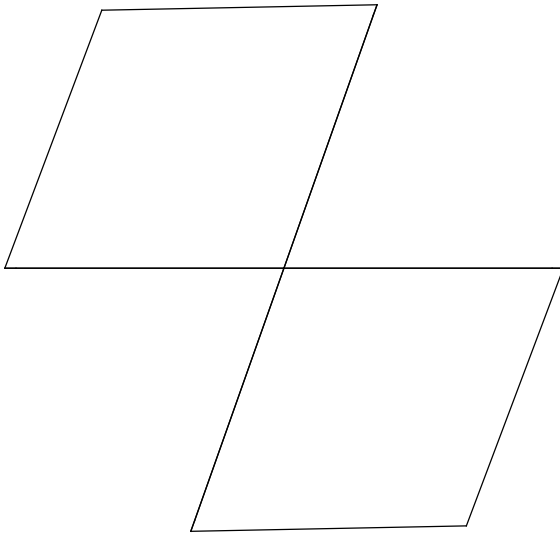
$$\left\{ \frac{8}{3}, 3, \frac{8}{3}, 4 \right\}$$

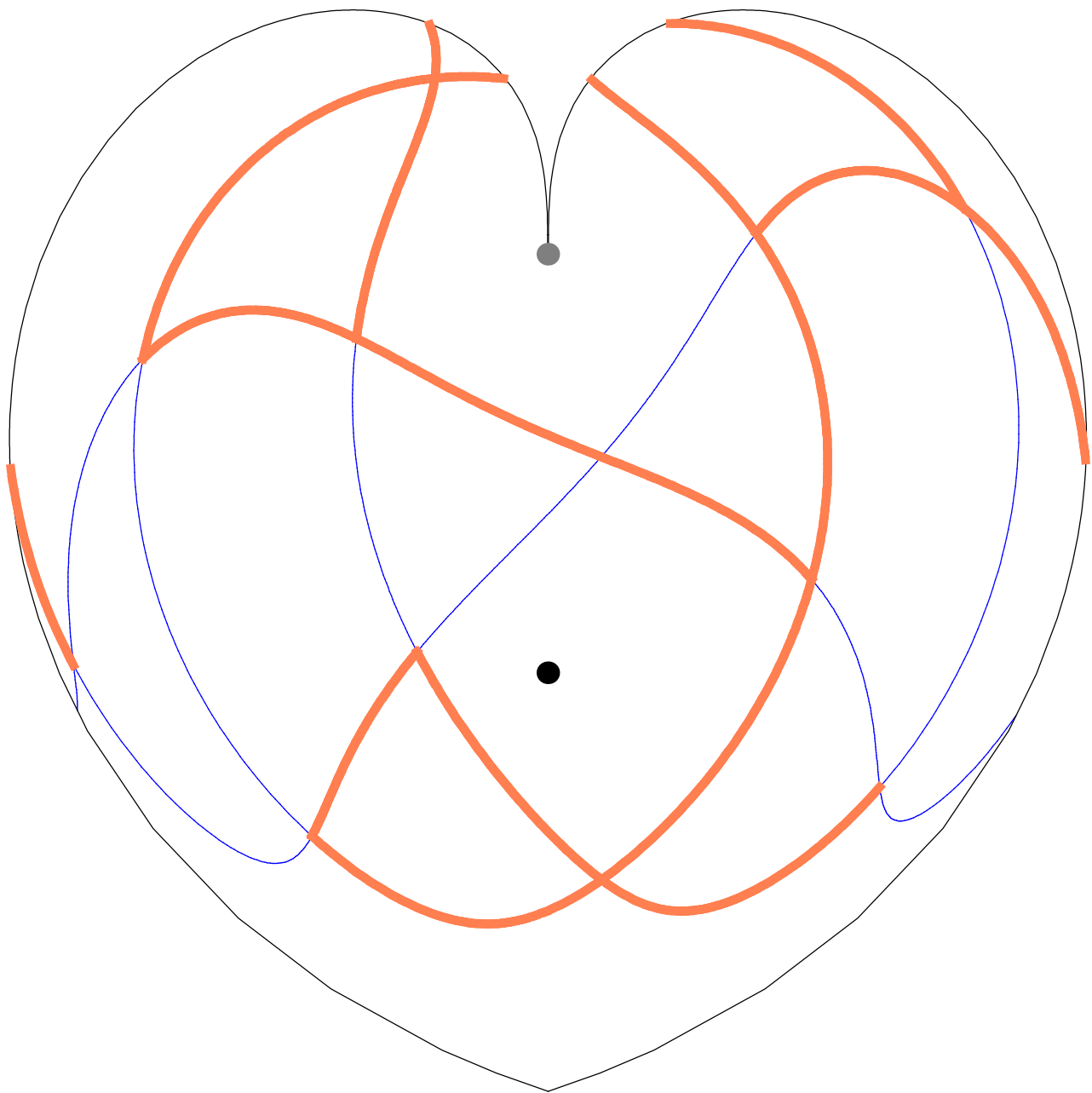


6.

cubohemioctahedron

$$\left\{6, \frac{4}{3}, 6, 4\right\}$$

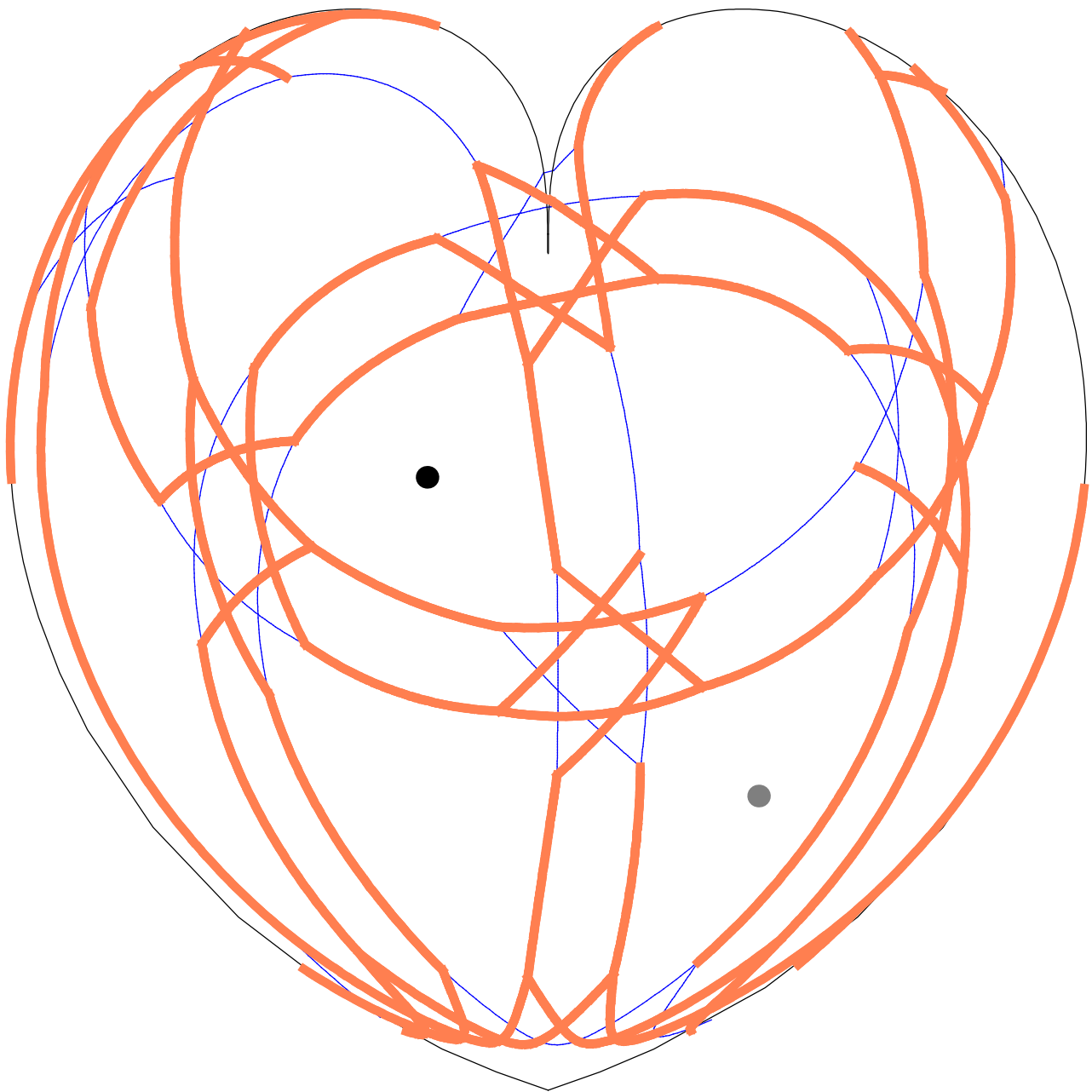
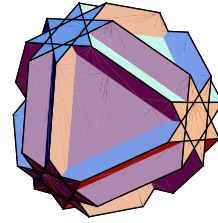
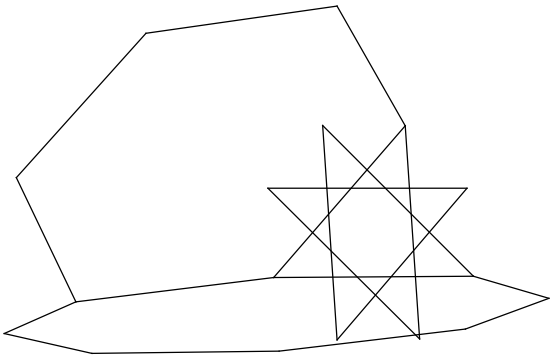




7.

cubitruncated cuboctahedron

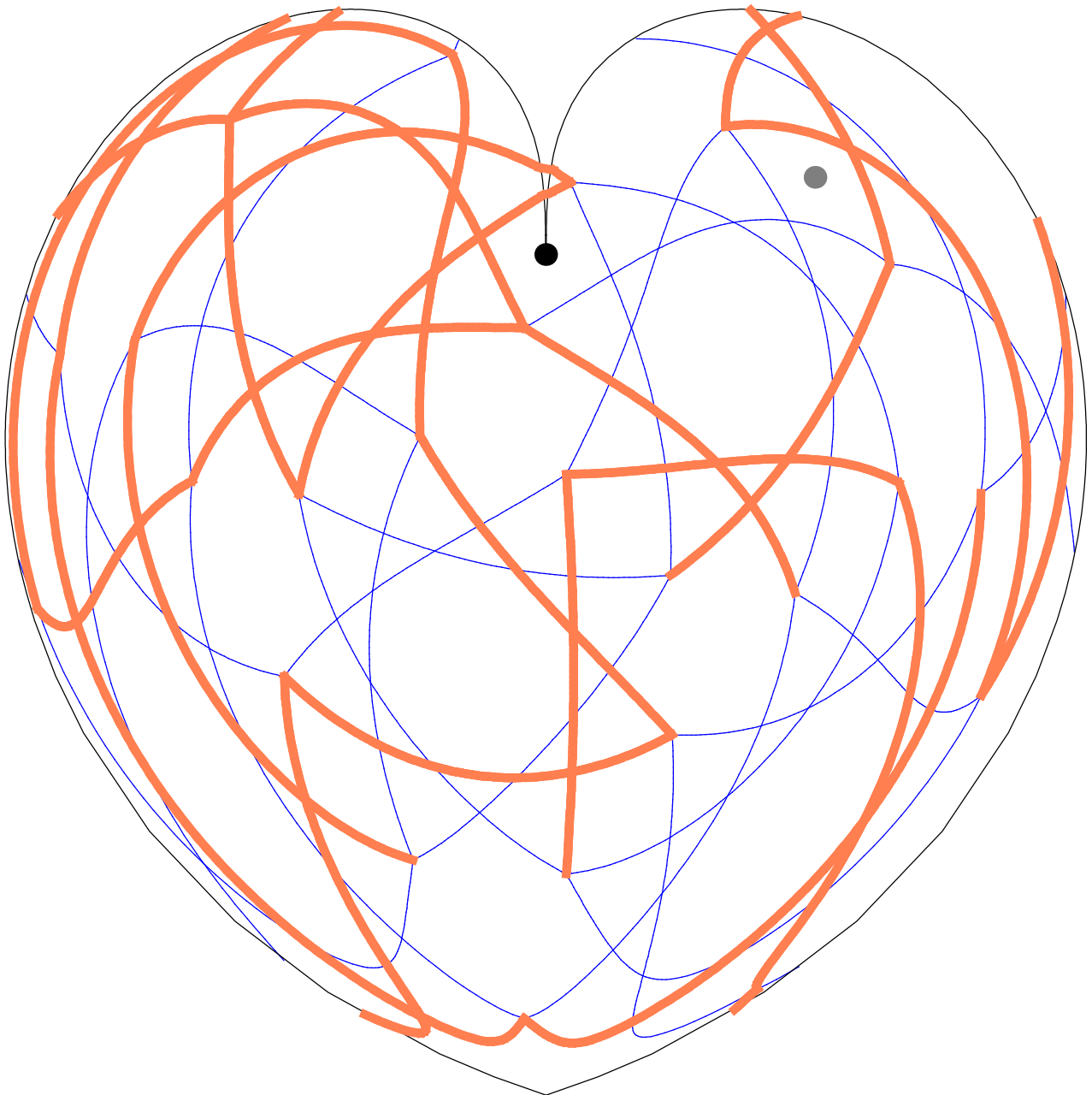
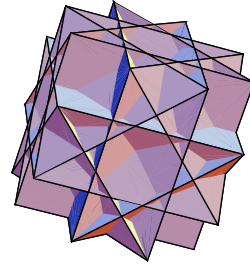
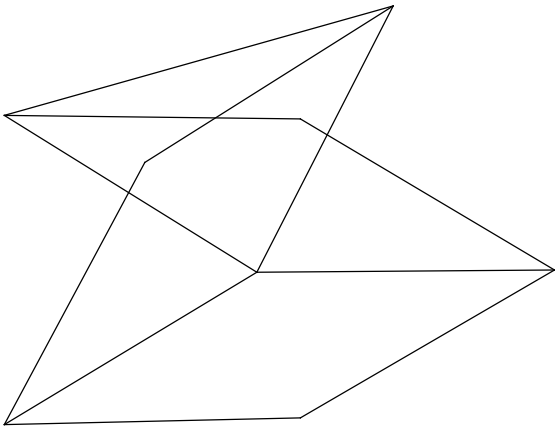
$$\left\{ \frac{8}{3}, 6, 8 \right\}$$



8.

great rhombicuboctahedron

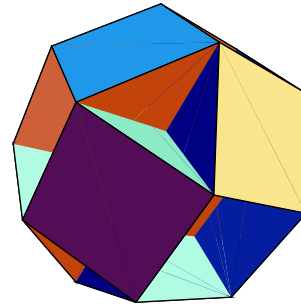
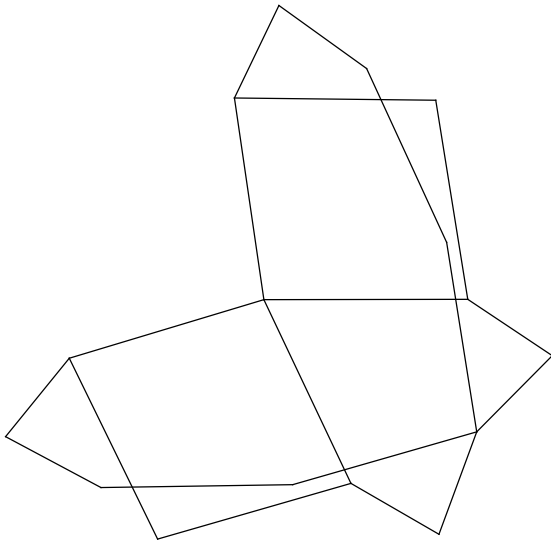
$$\left\{4, \frac{3}{2}, 4, 4\right\}$$

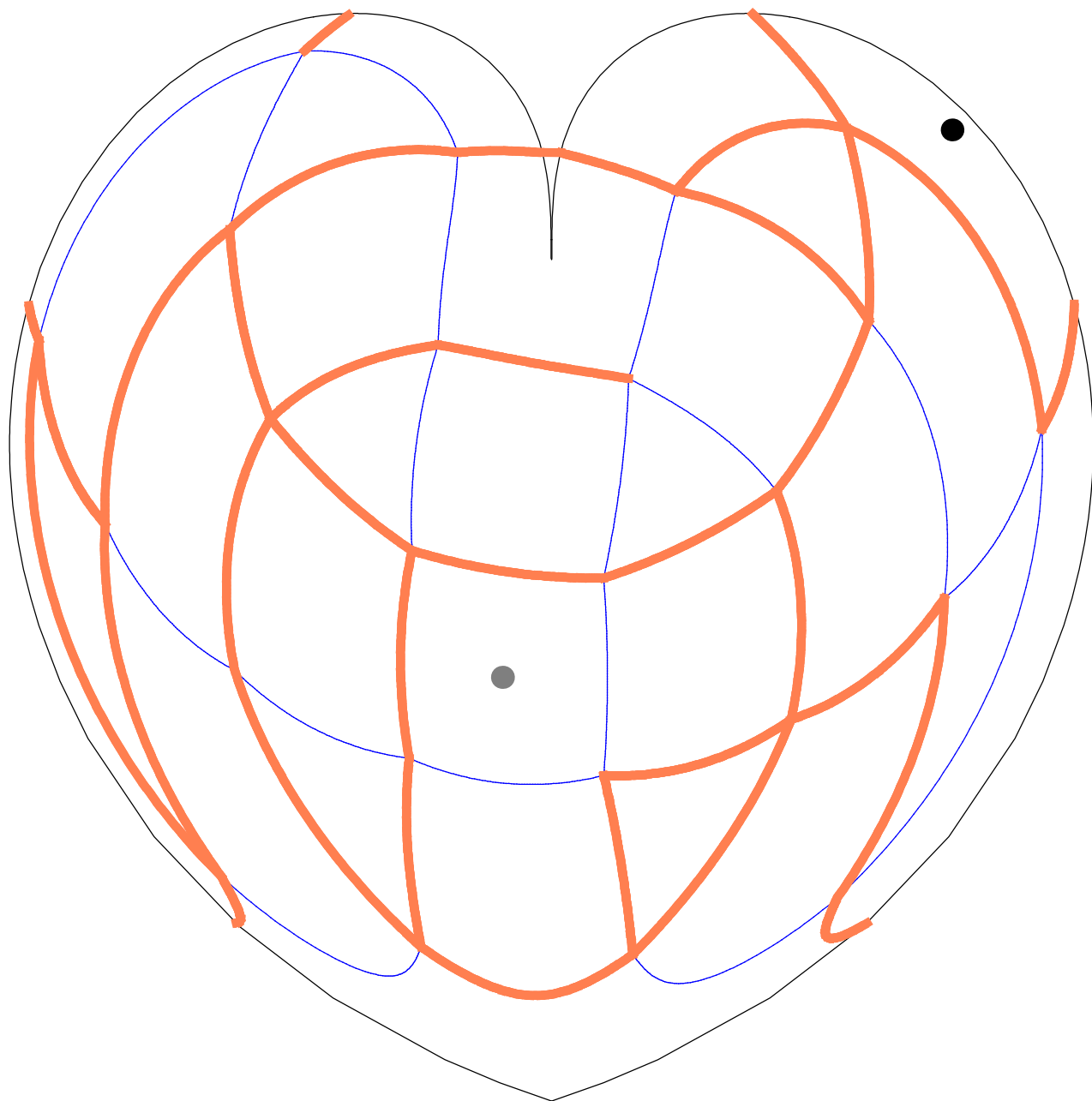


9.

small rhombihexahedron

$$\left\{8, 4, \frac{8}{7}, \frac{4}{3}\right\}$$

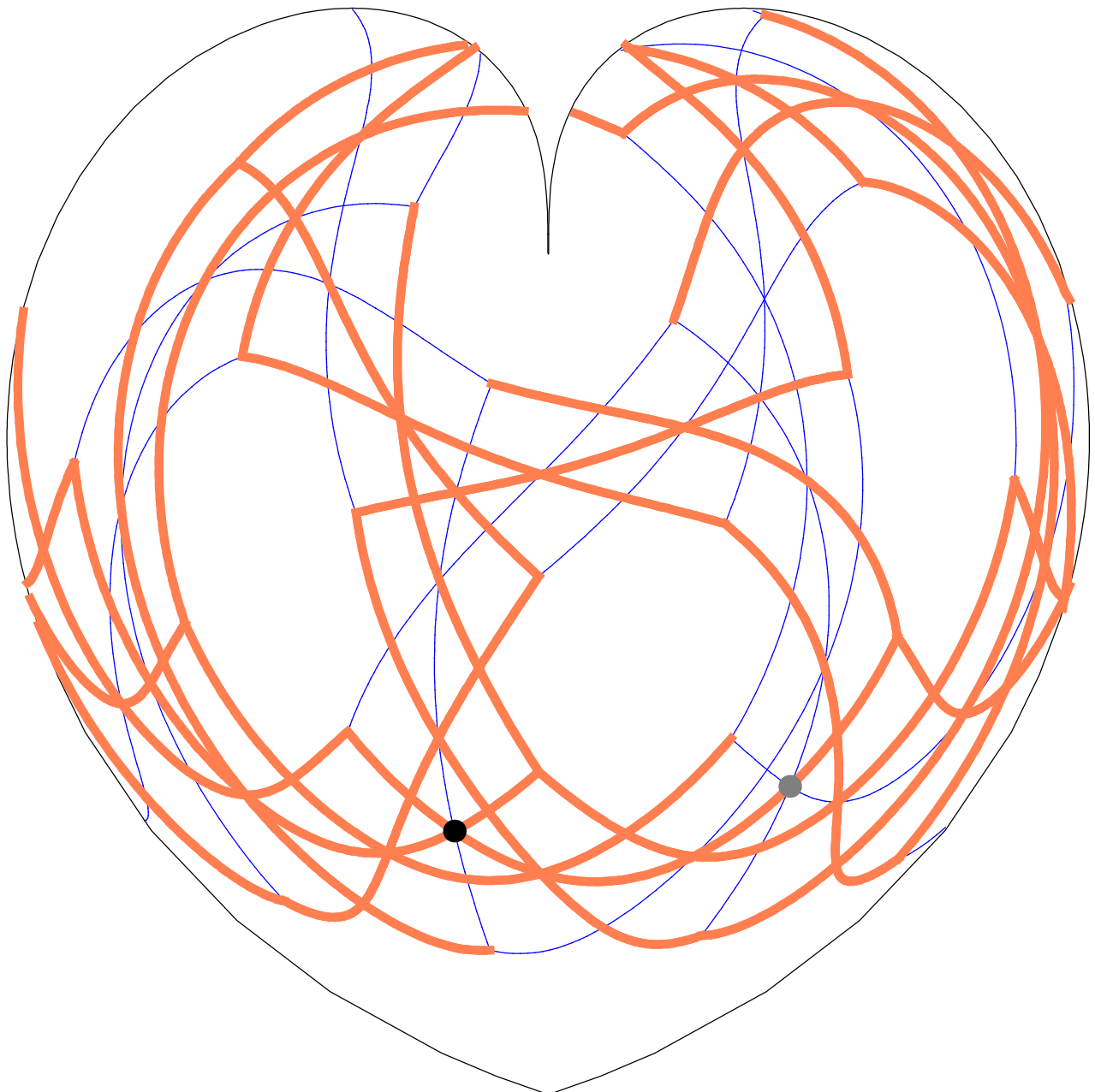
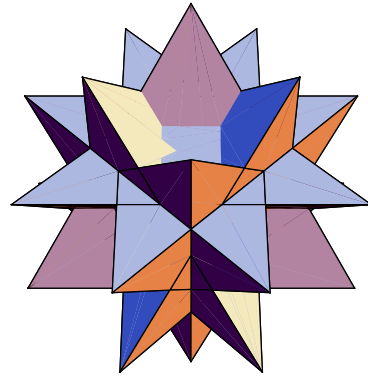
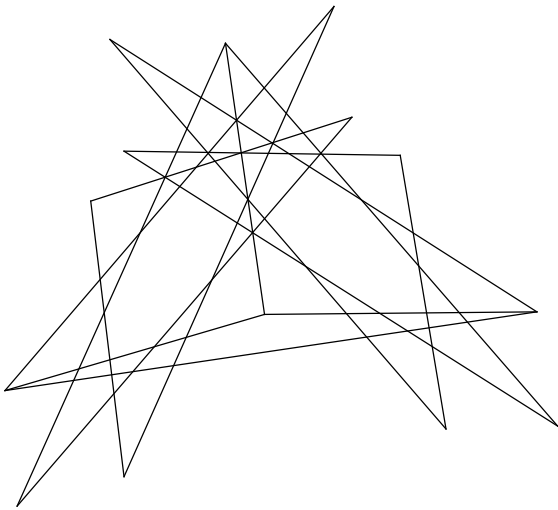




10.

stellated truncated hexahedron

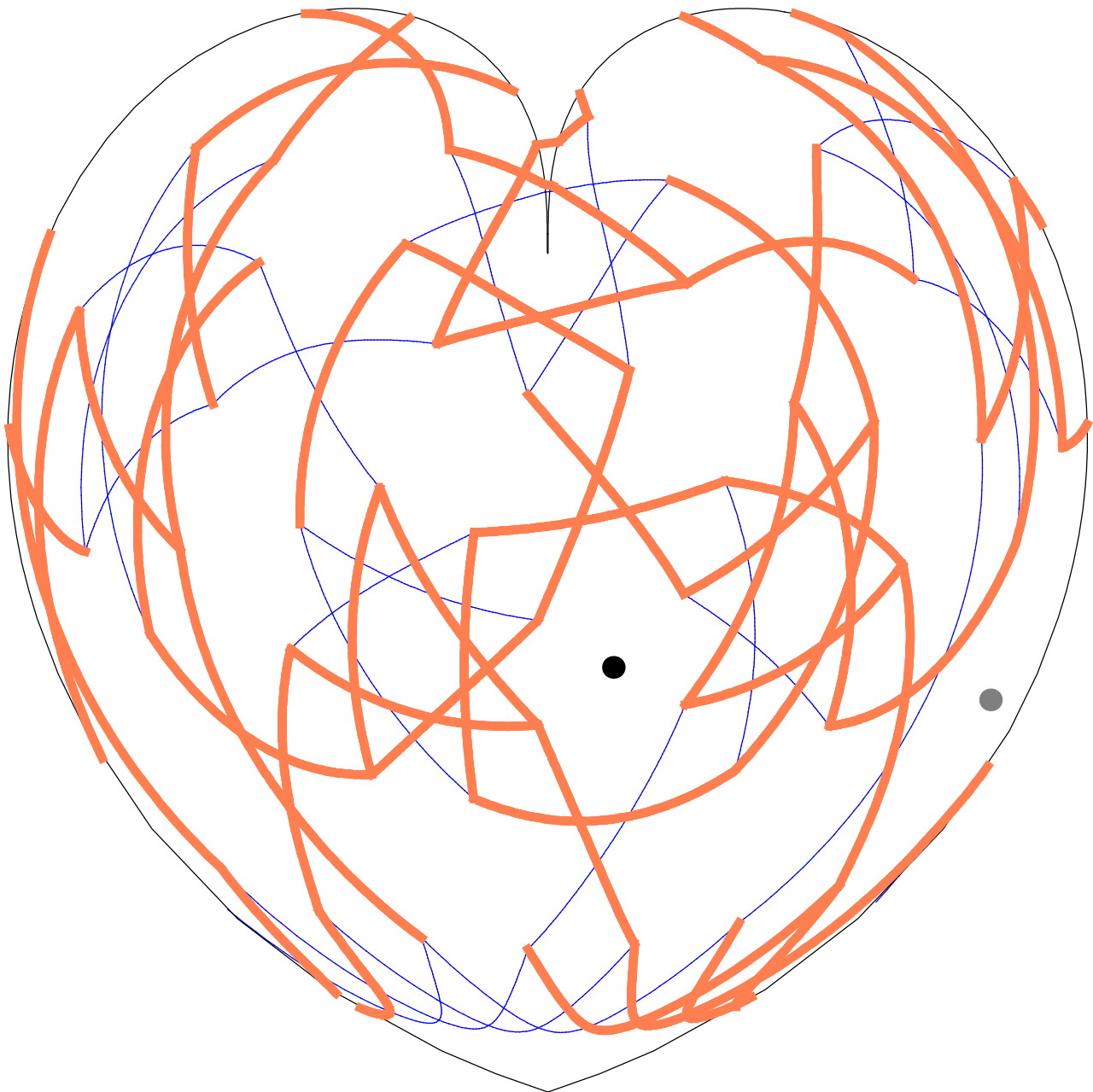
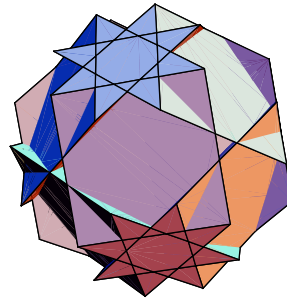
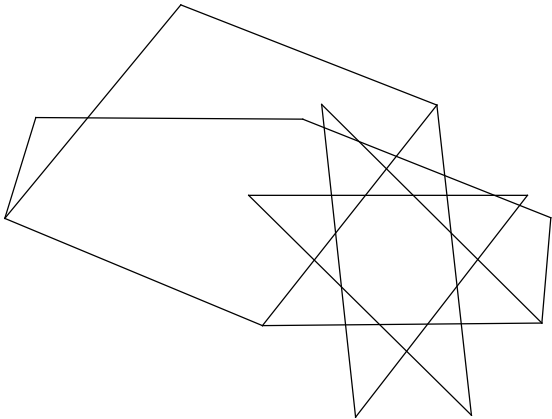
$$\left\{ \frac{8}{3}, \frac{8}{3}, 3 \right\}$$



11.

great truncated cuboctahedron

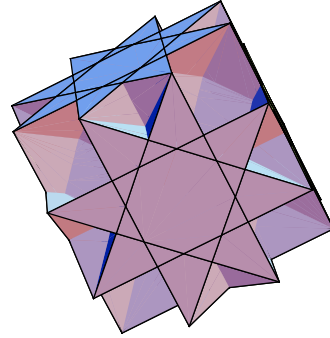
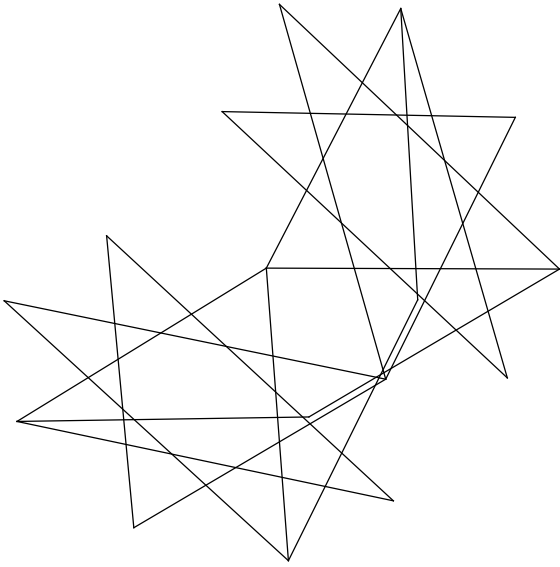
$$\left\{ \frac{8}{3}, 4, 6 \right\}$$

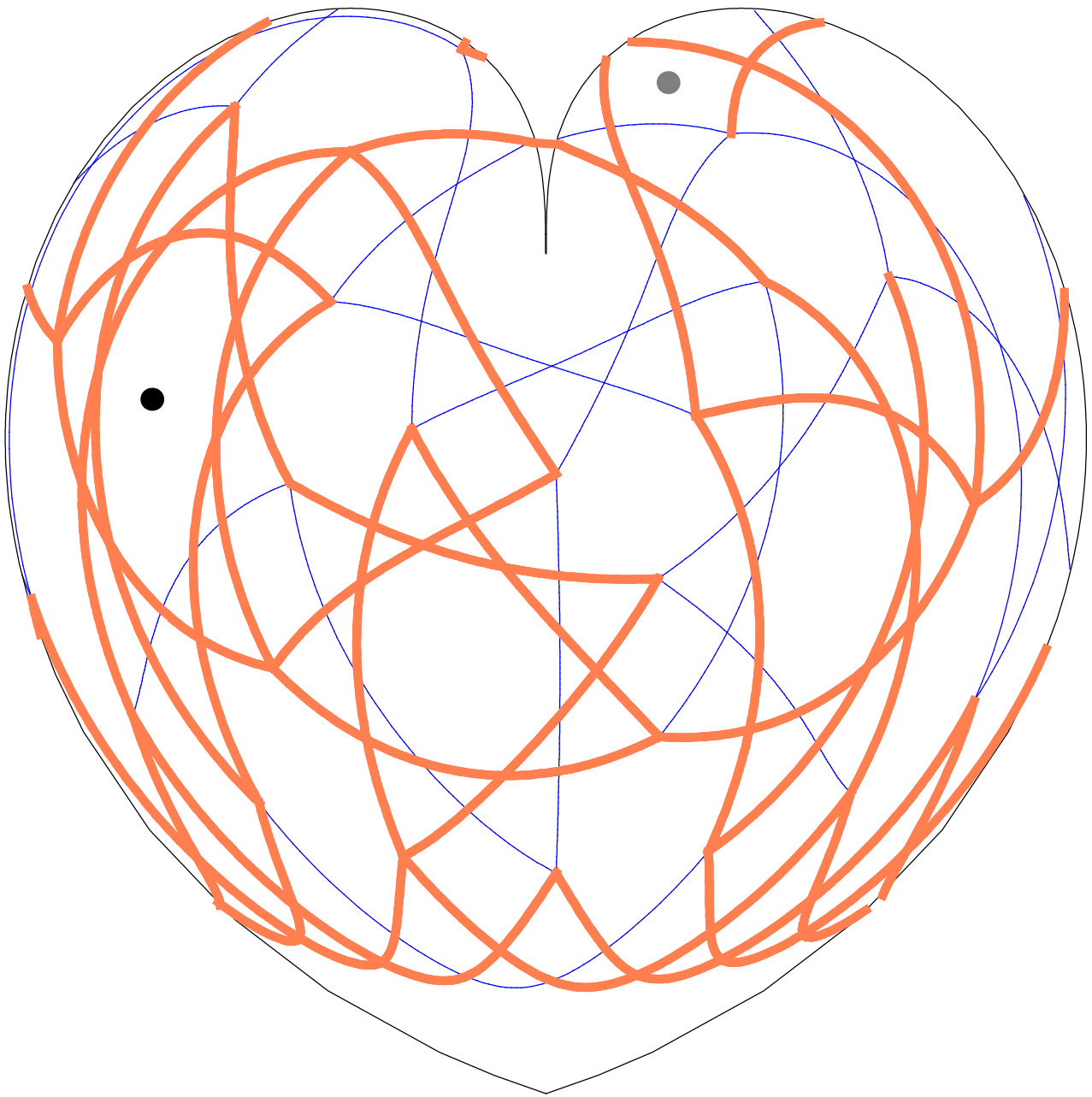


12.

great rhombihexahedron

$$\left\{4, \frac{8}{3}, \frac{4}{3}, \frac{8}{5}\right\}$$

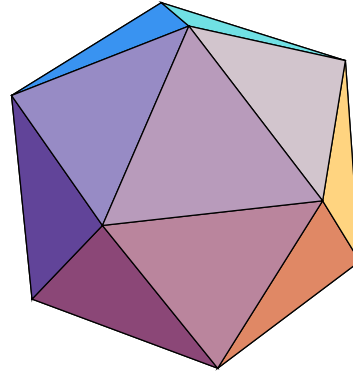
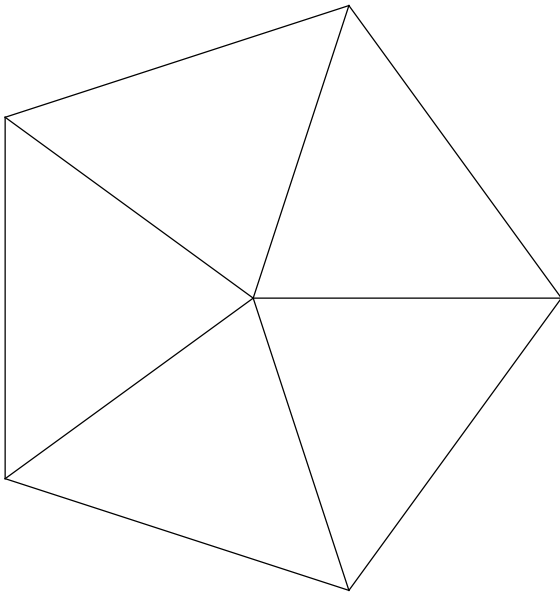


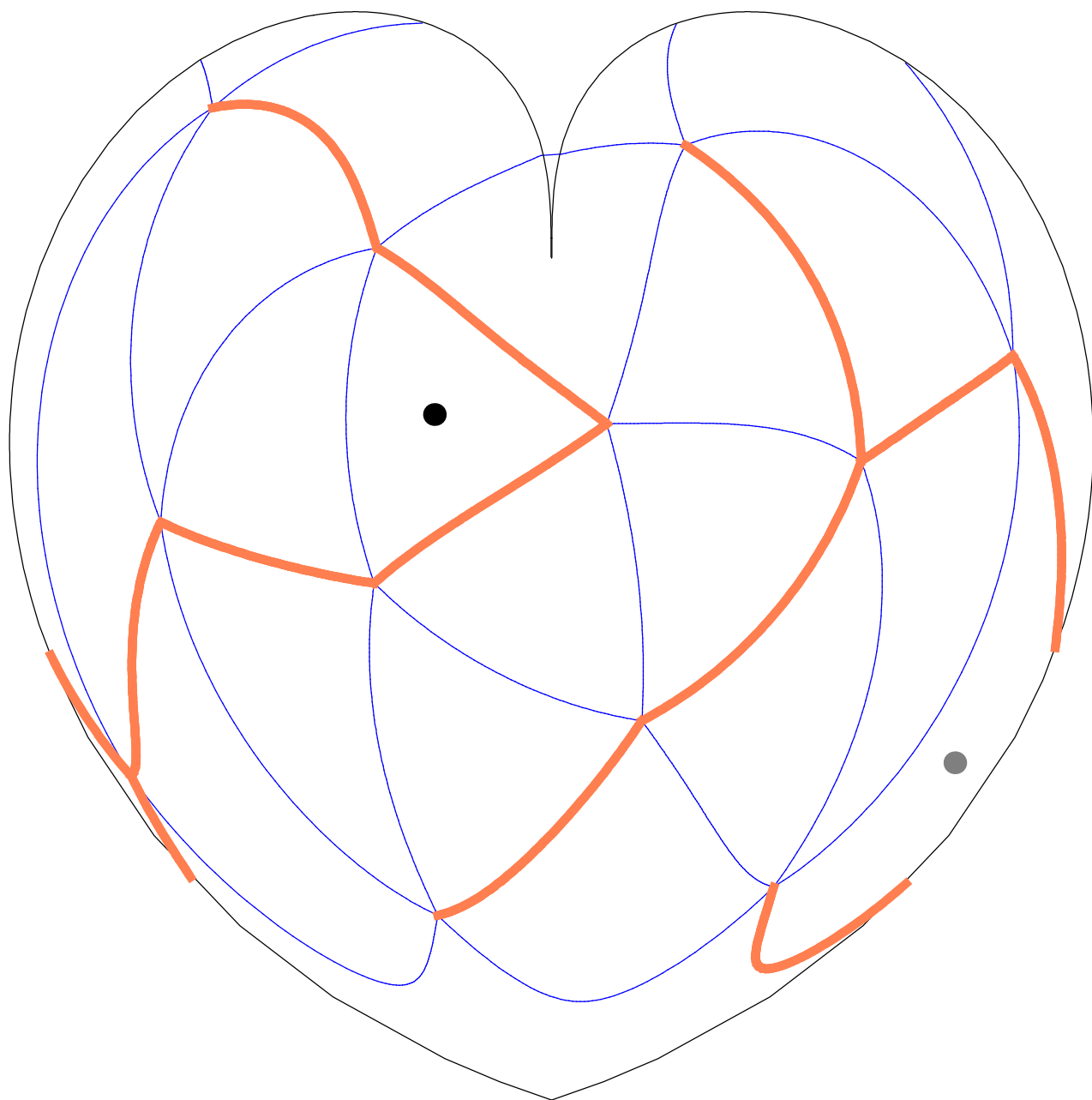


13.

icosahedron

{3, 3, 3, 3, 3}

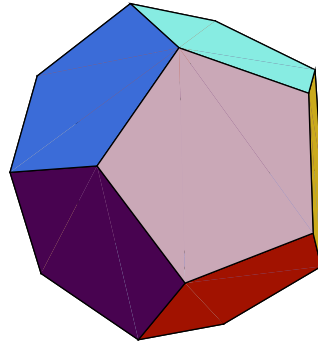
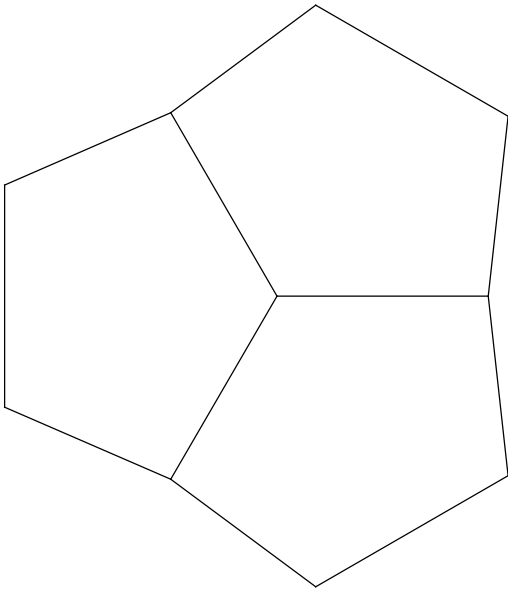


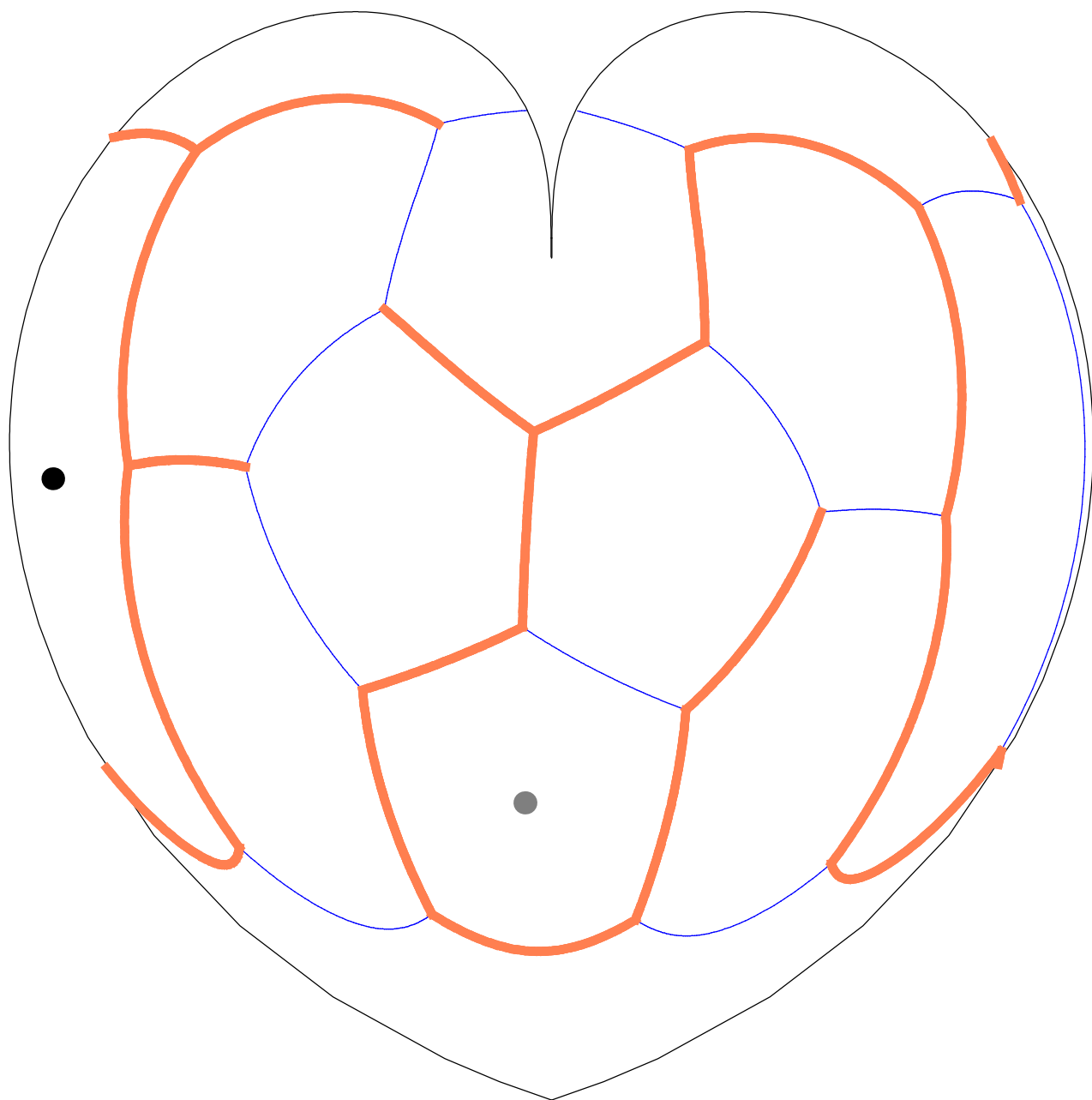


14.

dodecahedron

{5, 5, 5}

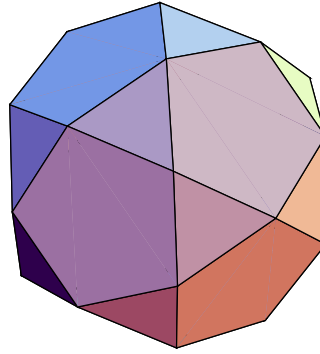
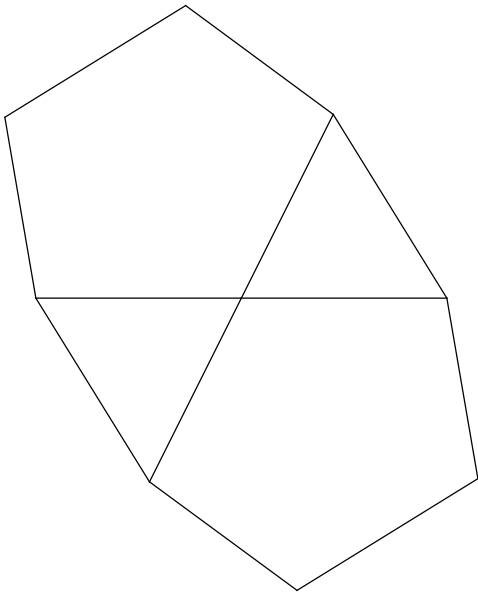


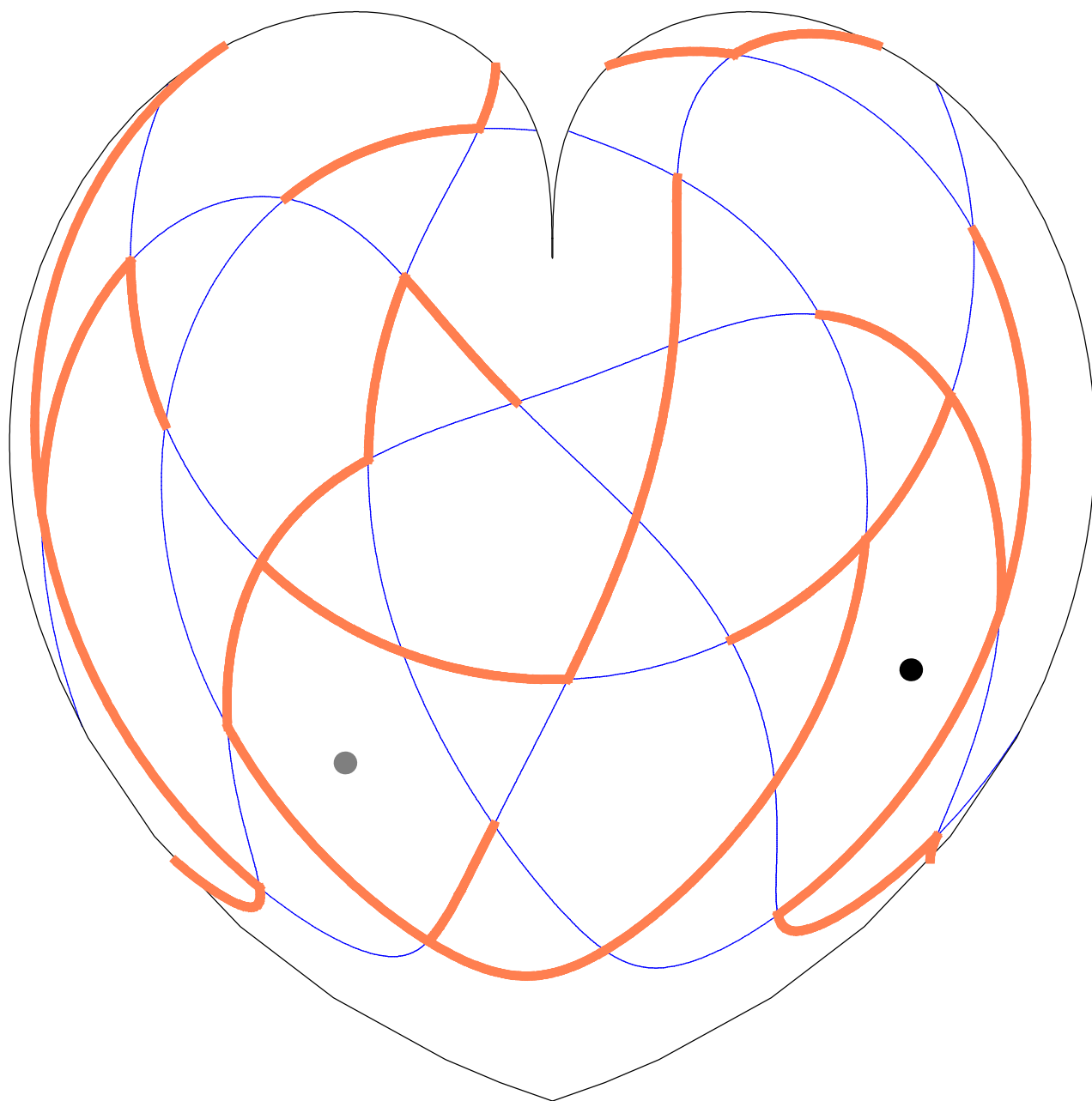


15.

icosidodecahedron

{3, 5, 3, 5}

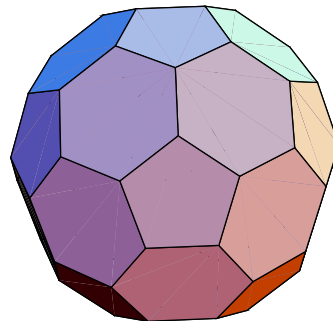
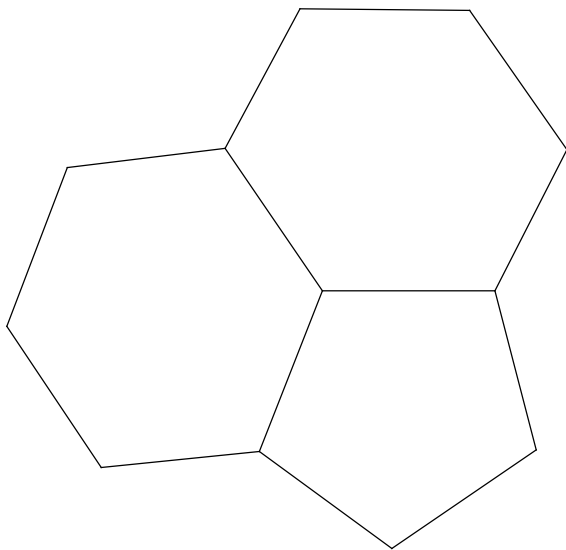


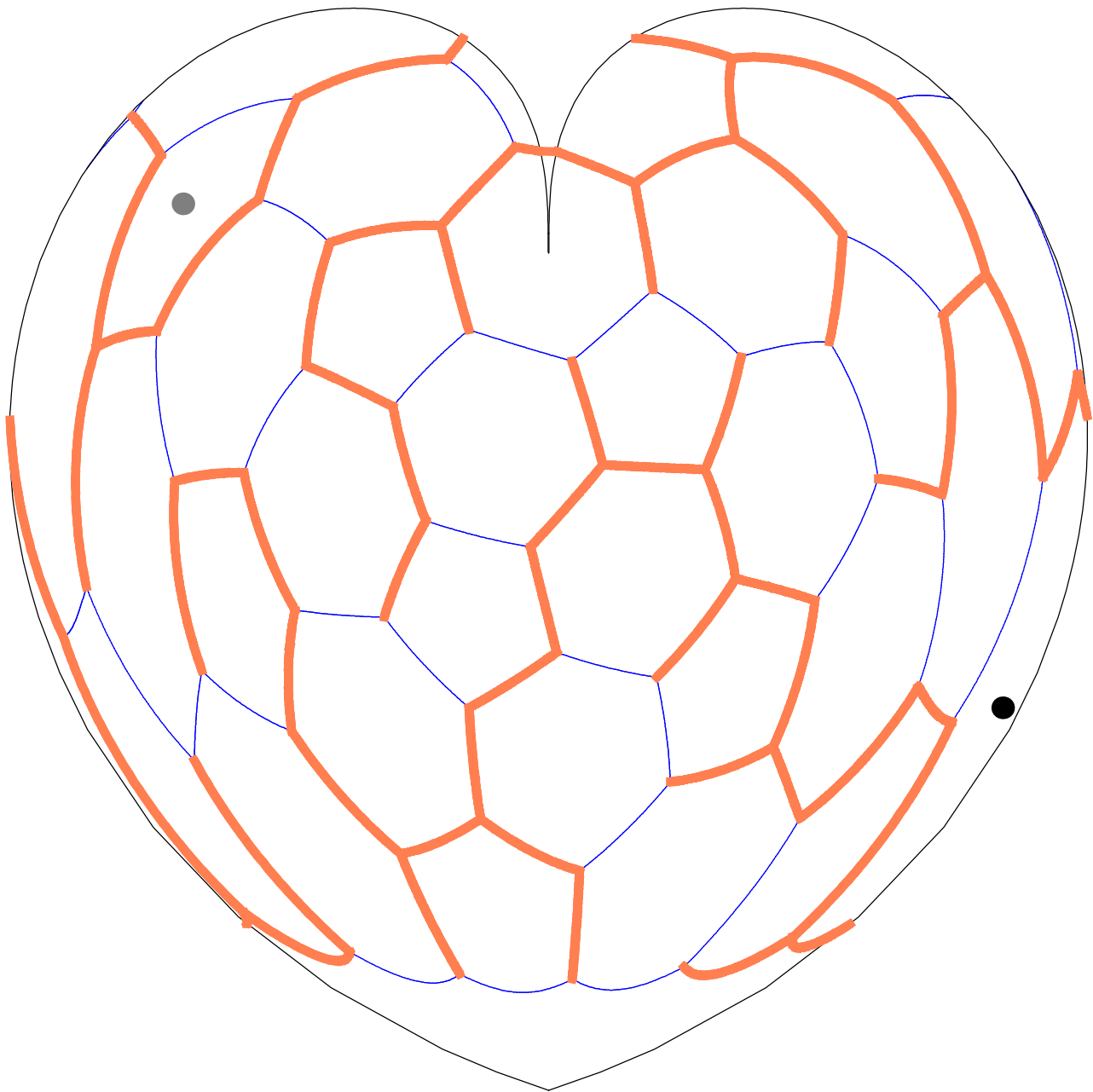


16.

truncated icosahedron

{6, 6, 5}

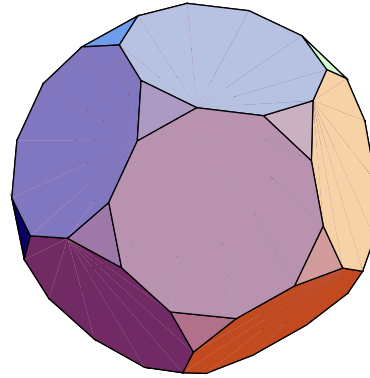
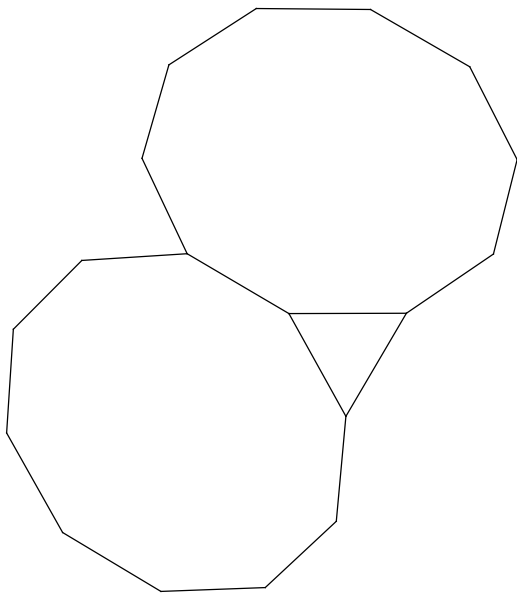


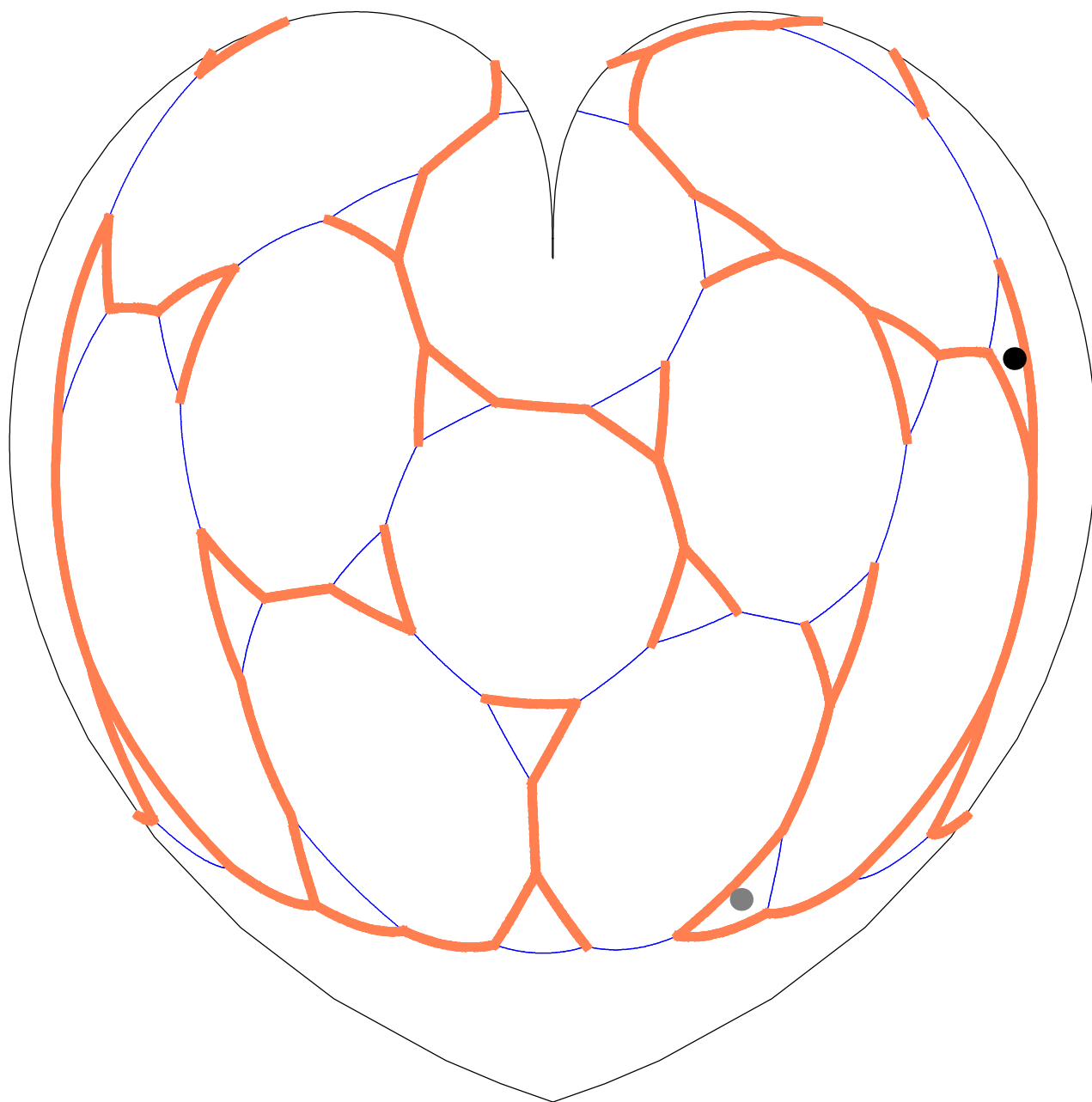


17.

truncated dodecahedron

{10, 10, 3}

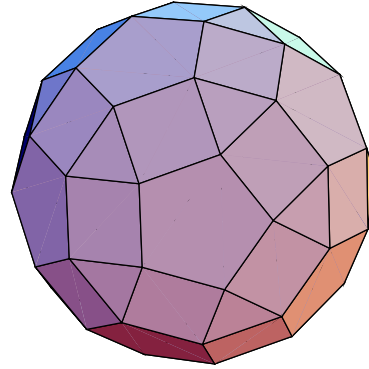
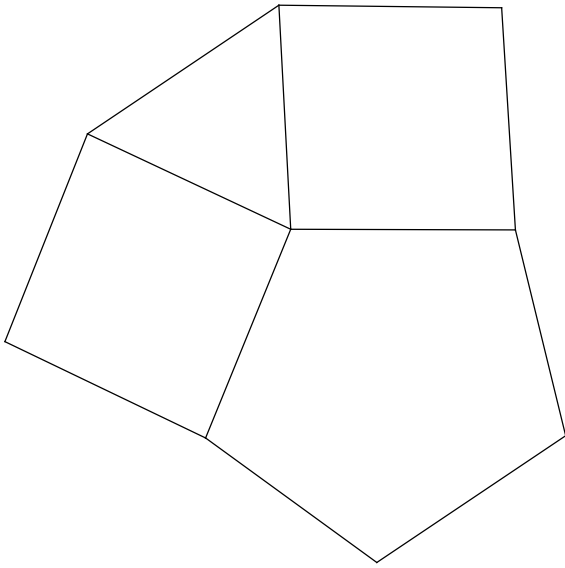


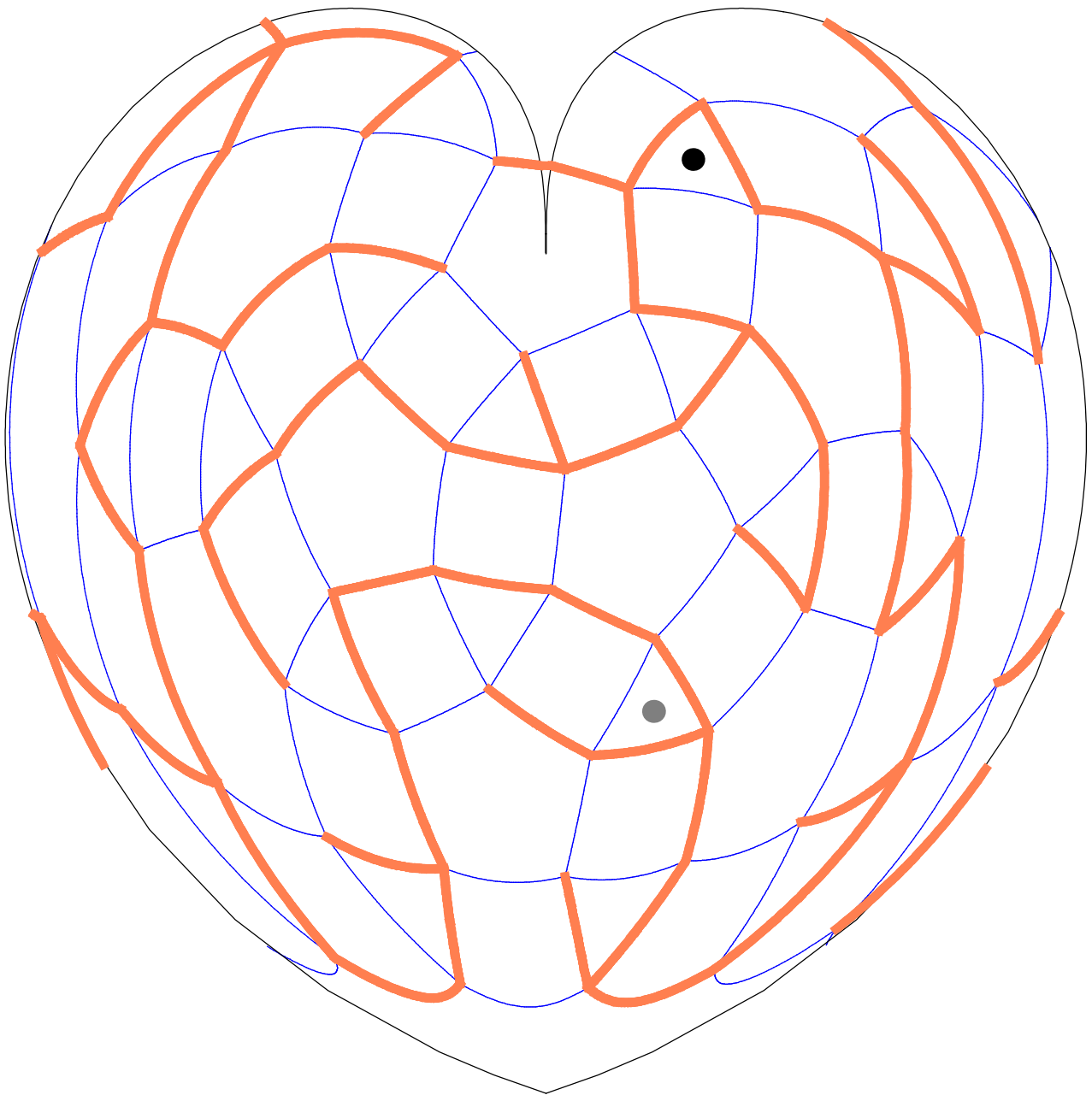


18.

rhombicosidodecahedron

{4, 3, 4, 5}

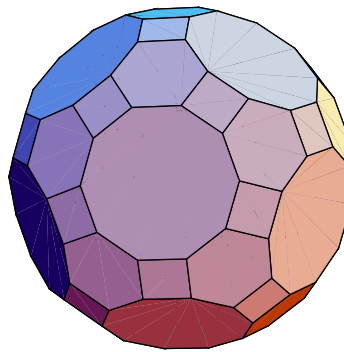
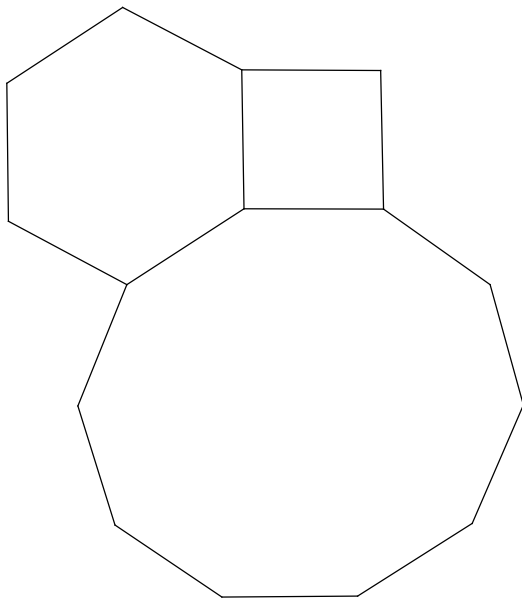


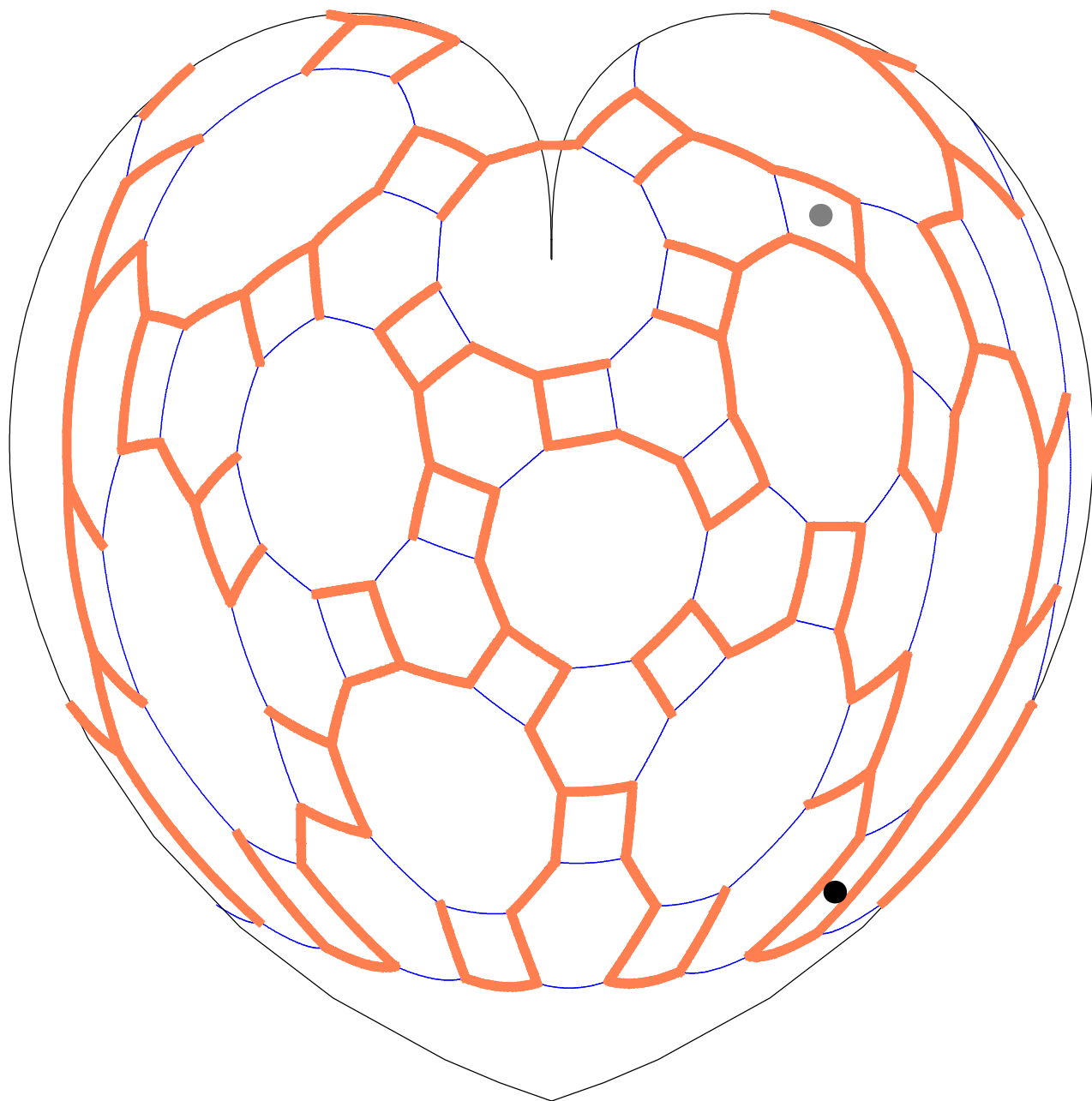


19.

truncated icosidodecahedron

{4, 6, 10}

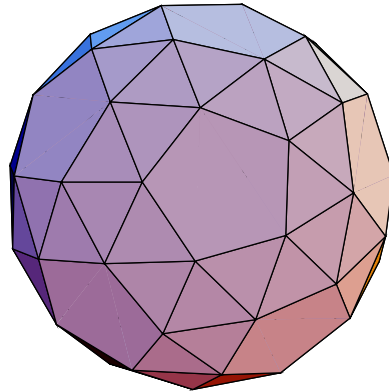
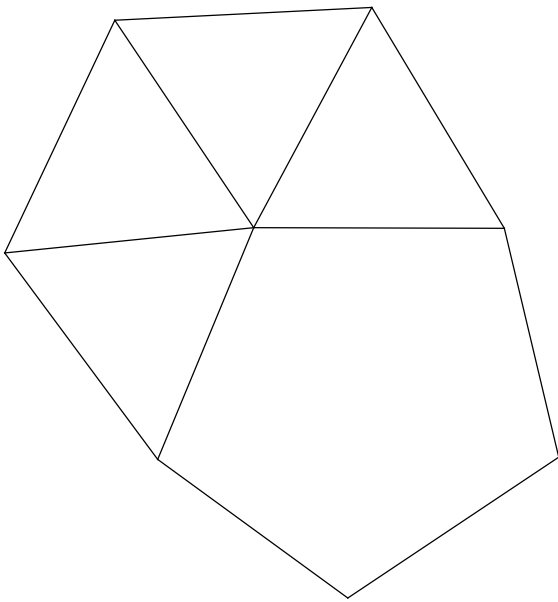


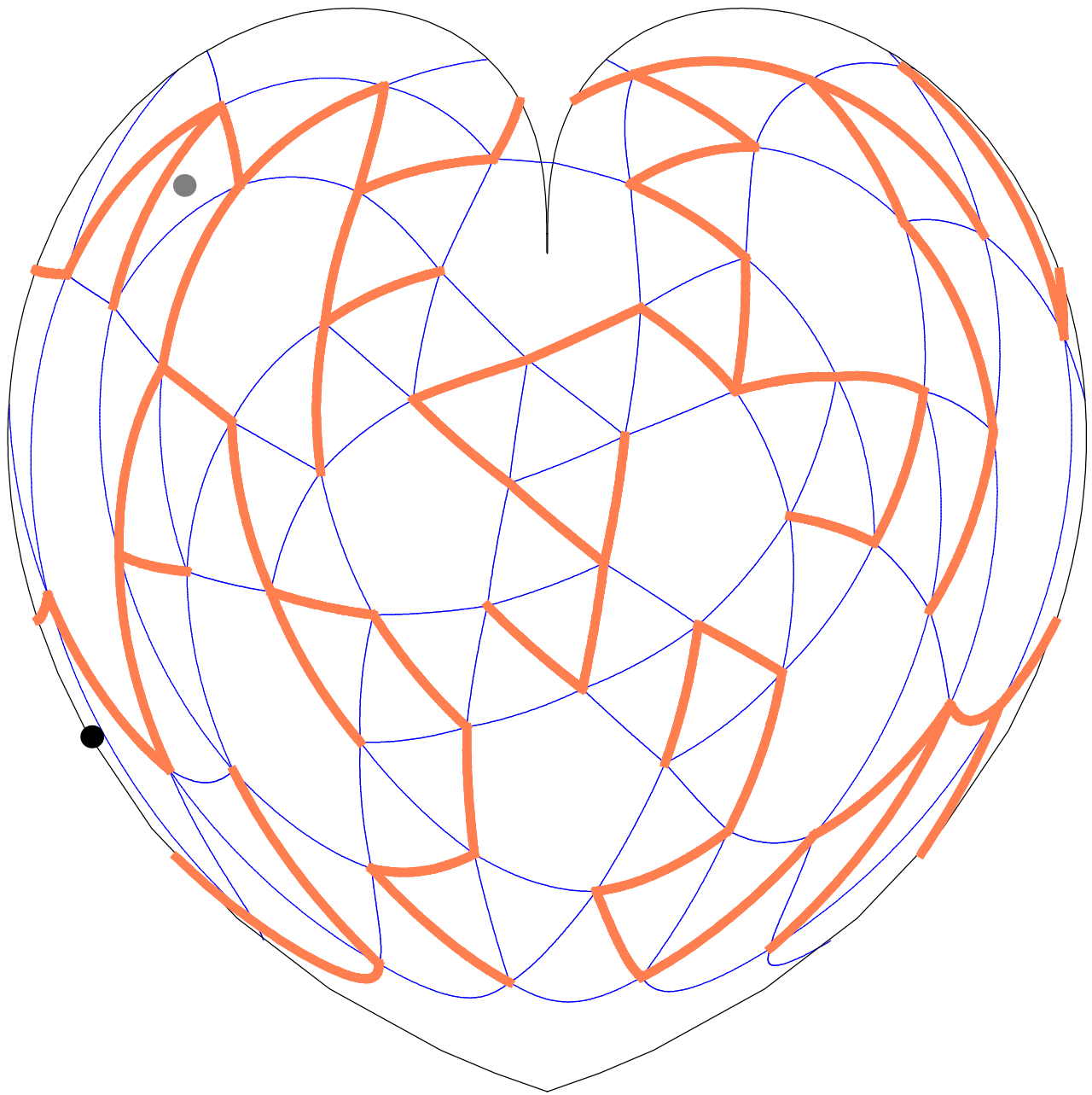


20.

snub dodecahedron

$\{3, 3, 3, 3, 5\}$

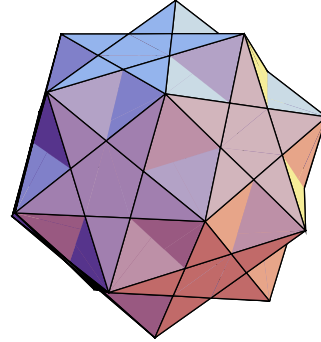
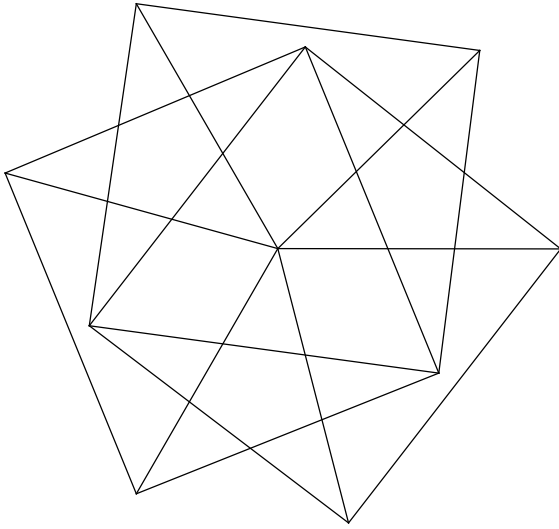


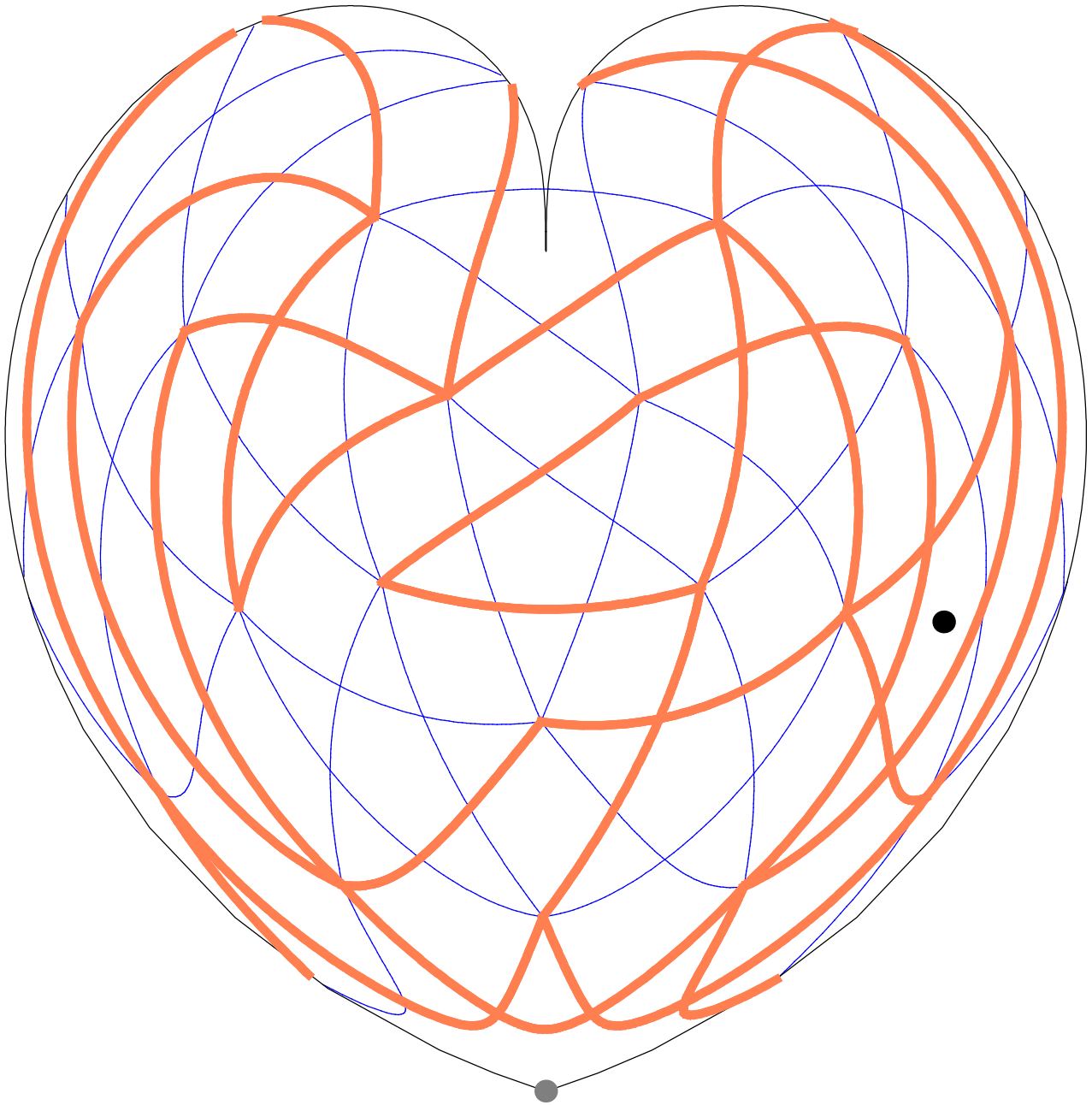


21.

small ditrigonal icosidodecahedron

$$\left\{ \frac{5}{2}, 3, \frac{5}{2}, 3, \frac{5}{2}, 3 \right\}$$

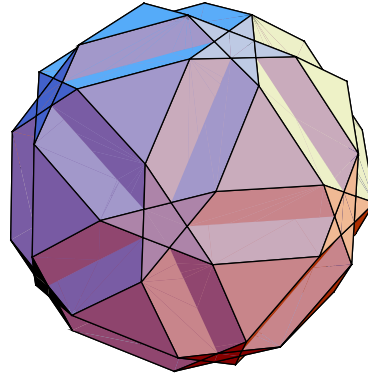
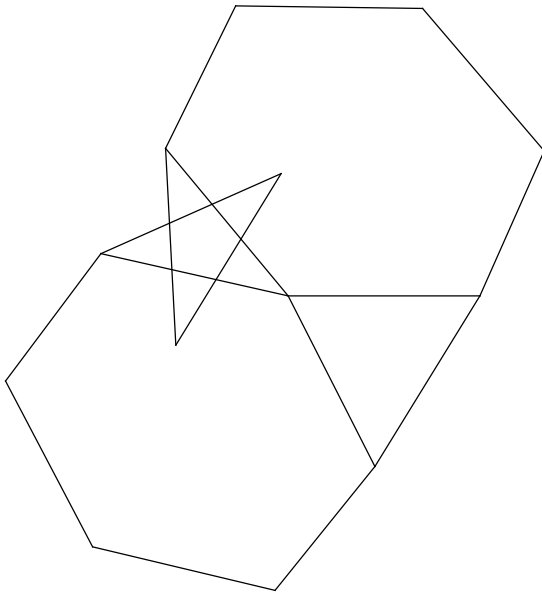


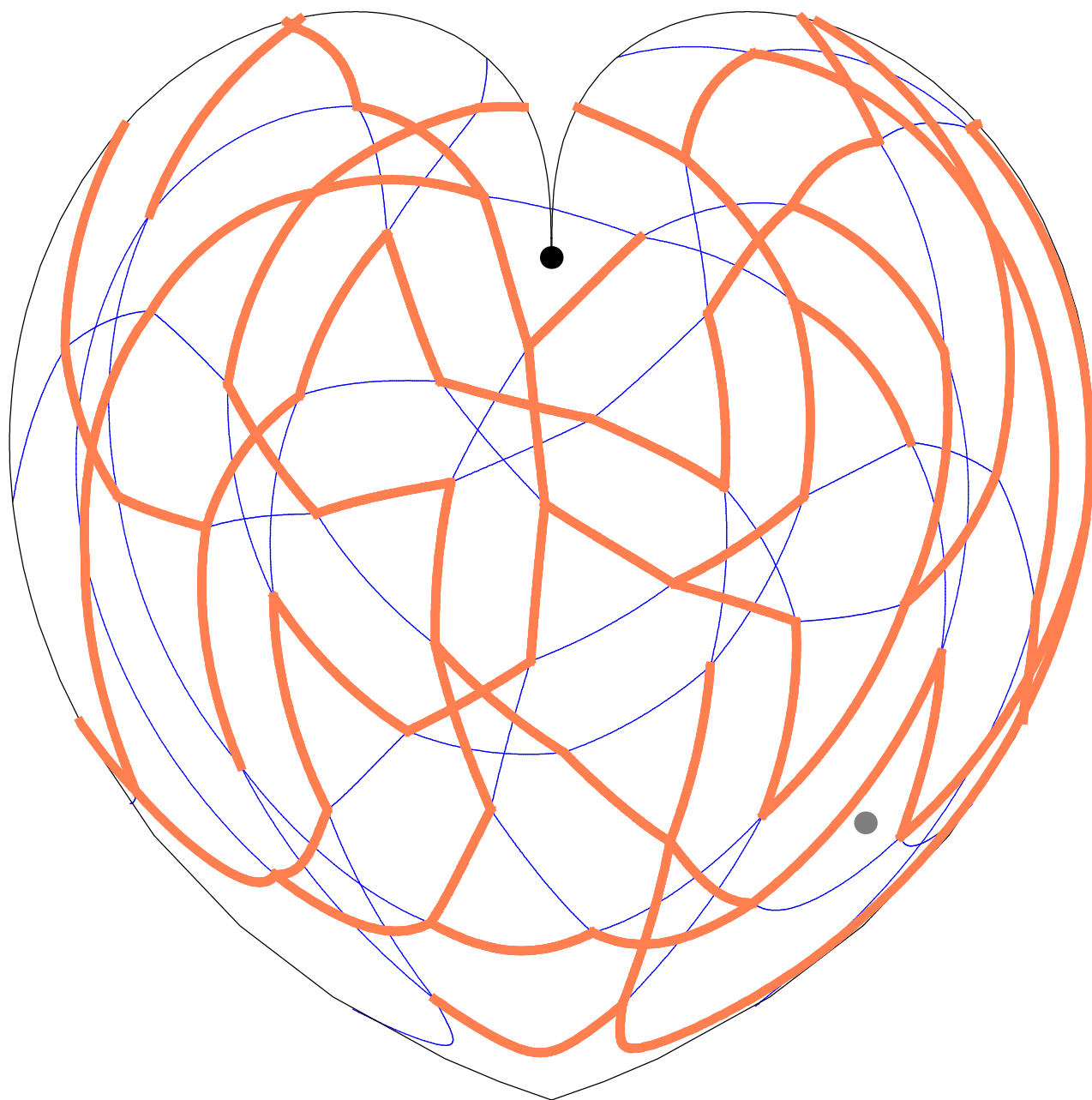


22.

small icosicosidodecahedron

$$\left\{6, \frac{5}{2}, 6, 3\right\}$$

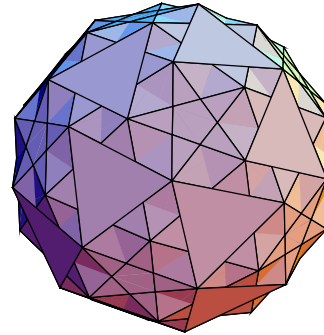
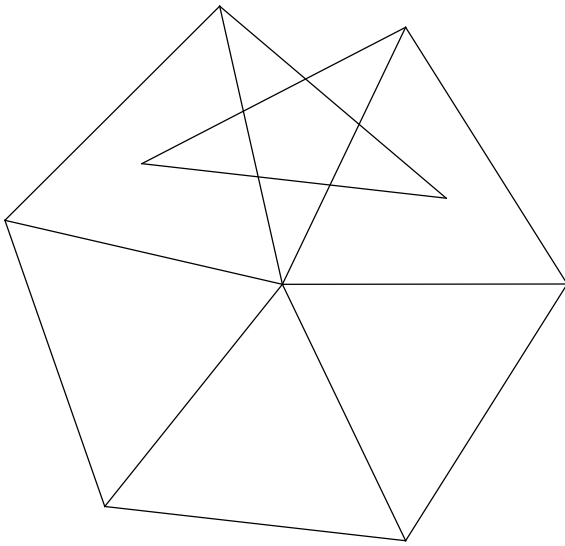


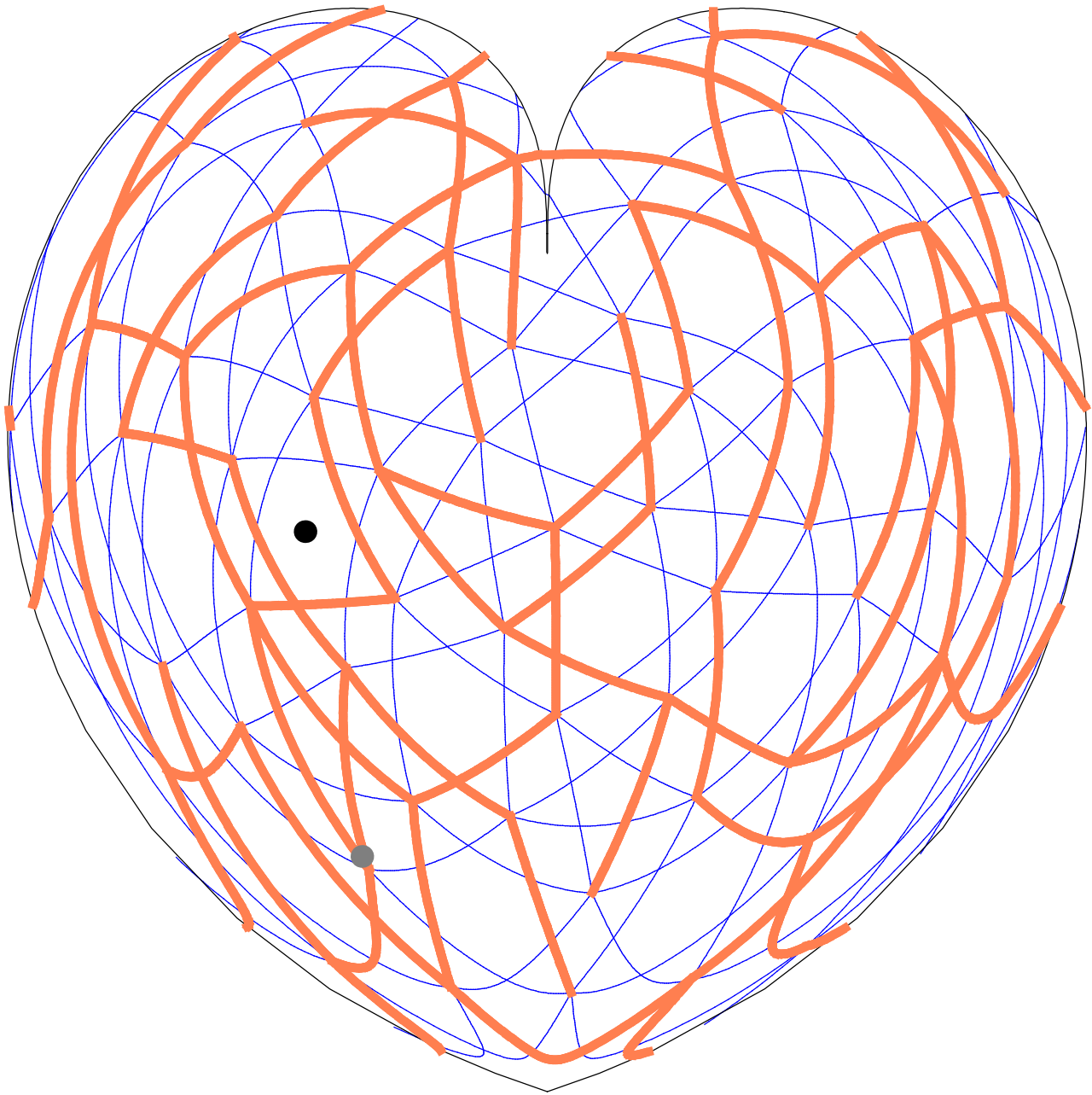


23.

small snub icosicosidodecahedron

$$\left\{3, \frac{5}{2}, 3, 3, 3, 3\right\}$$

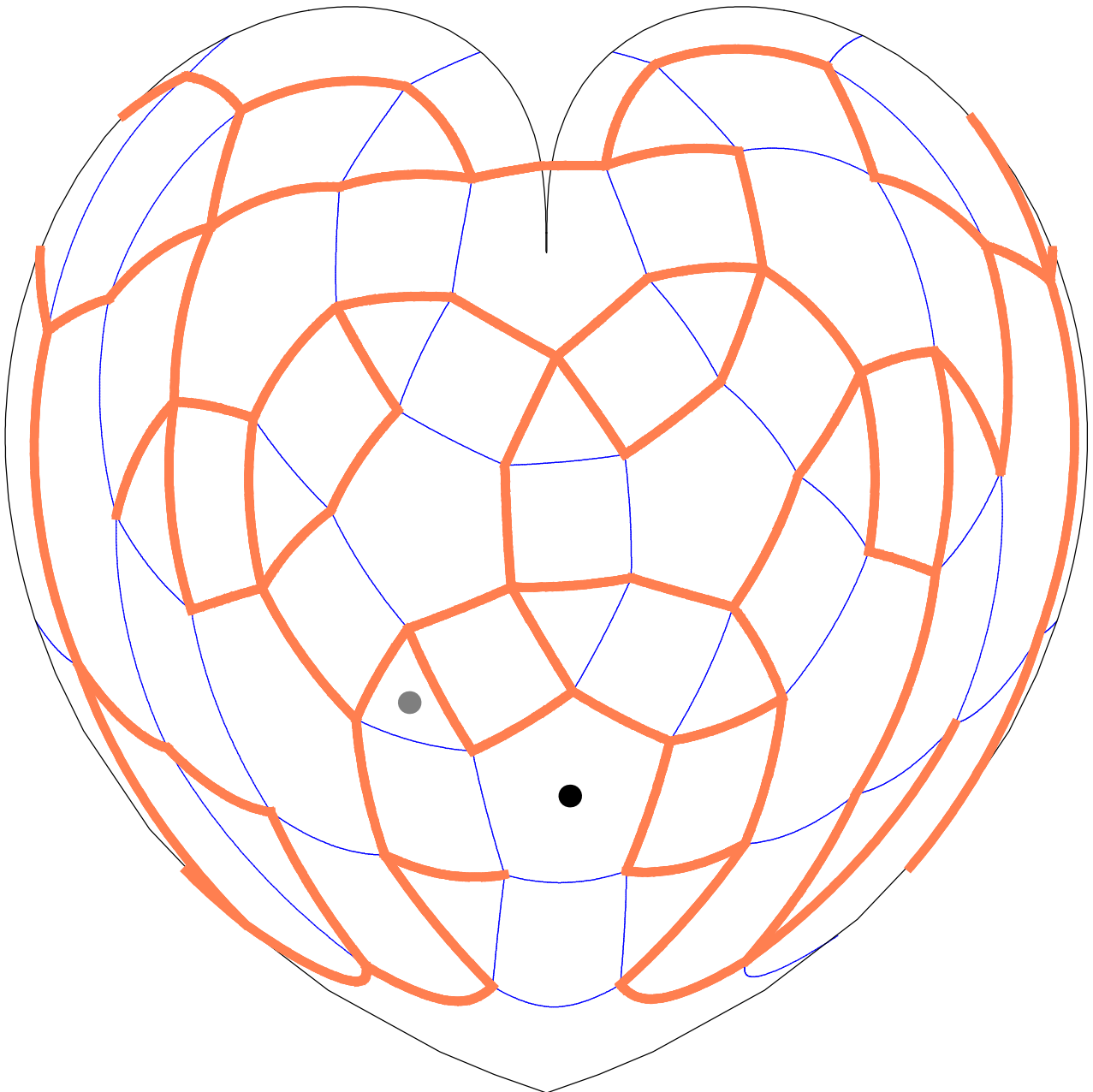
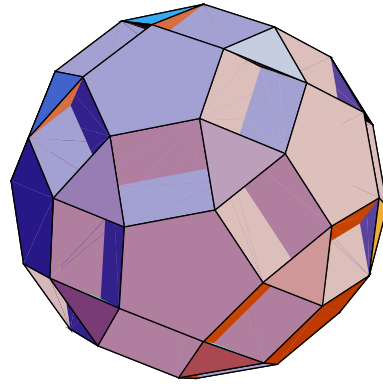
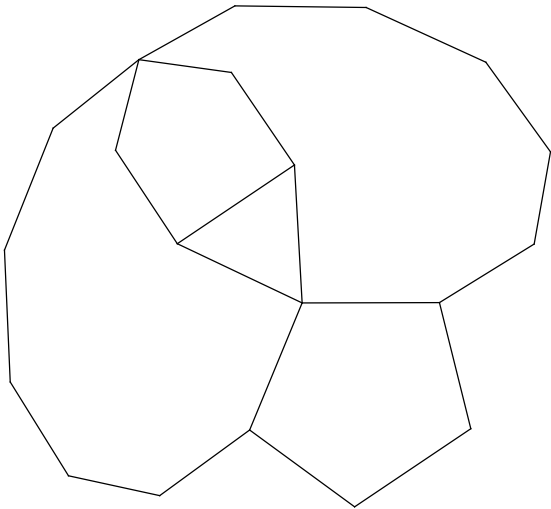




24.

small dodecicosidodecahedron

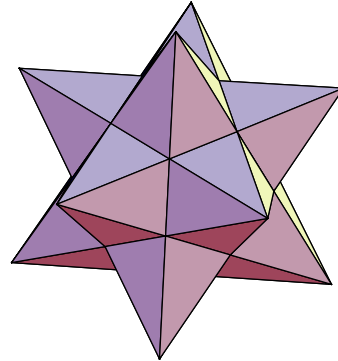
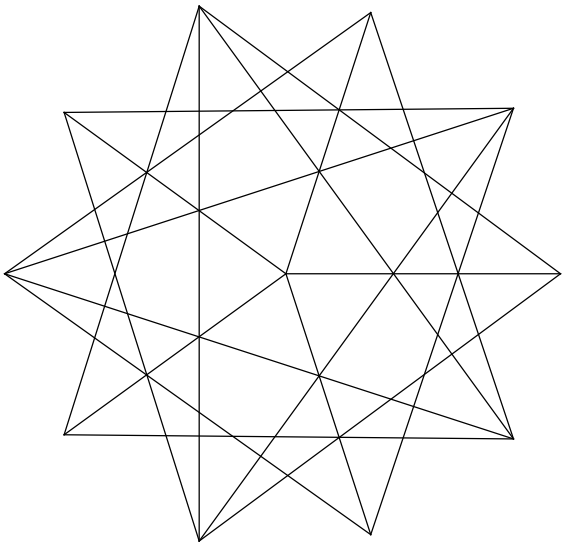
$$\left\{10, \frac{3}{2}, 10, 5\right\}$$

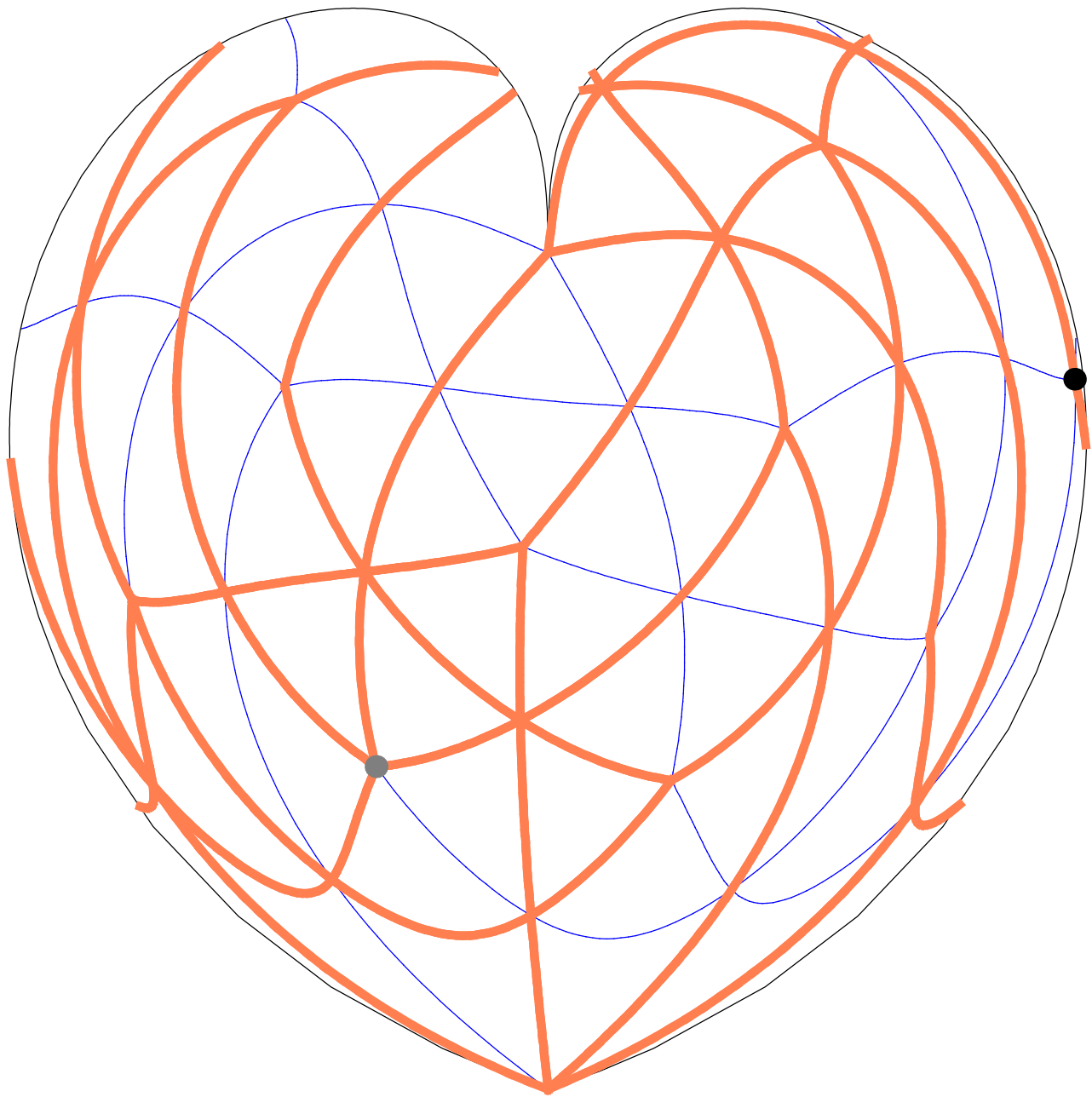


25.

small stellated dodecahedron

$$\left\{ \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2} \right\}$$

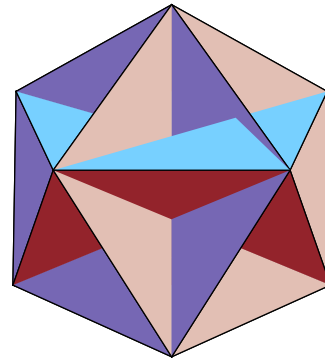
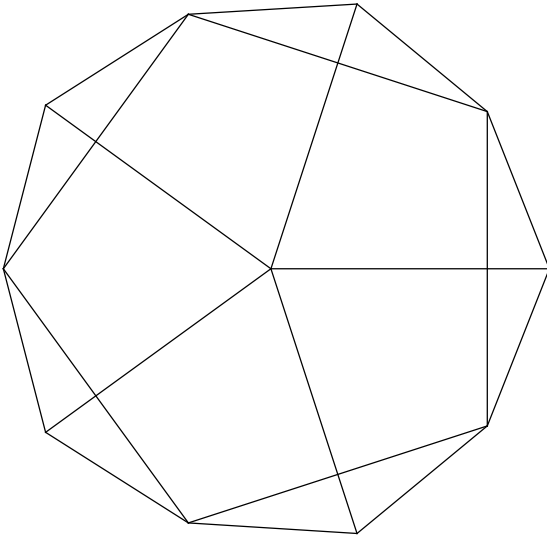


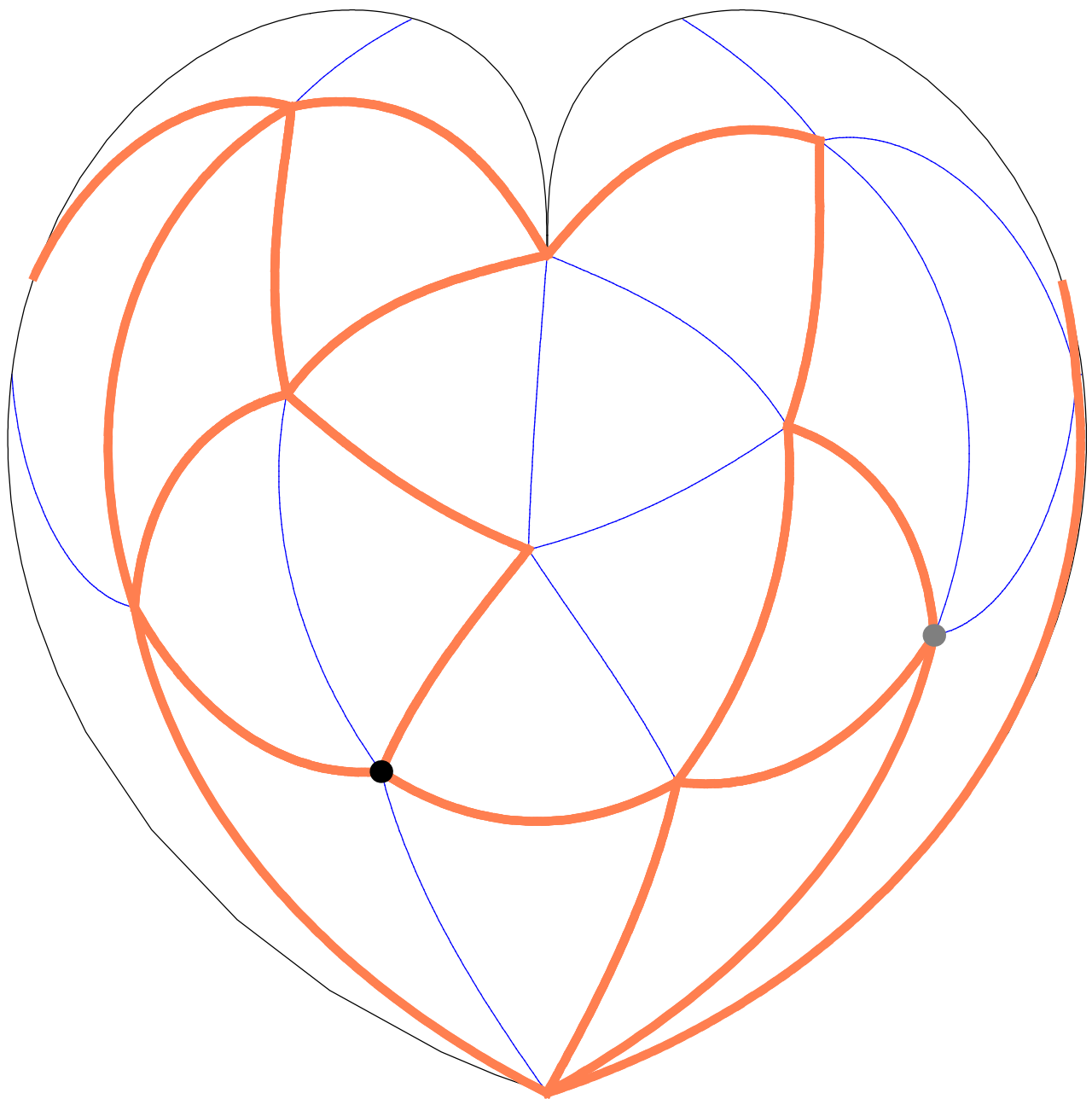


26.

great dodecahedron

$$\frac{1}{2} \{5, 5, 5, 5, 5\}$$

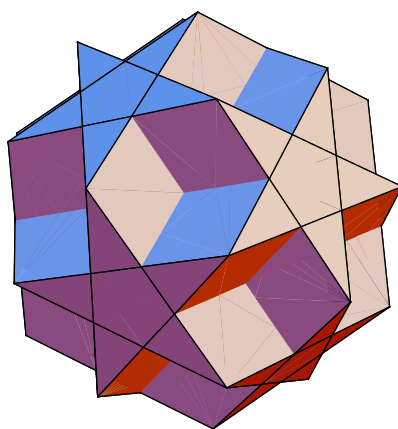
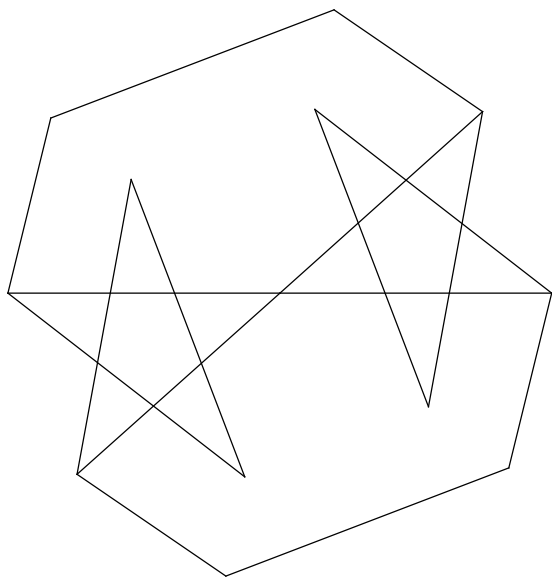


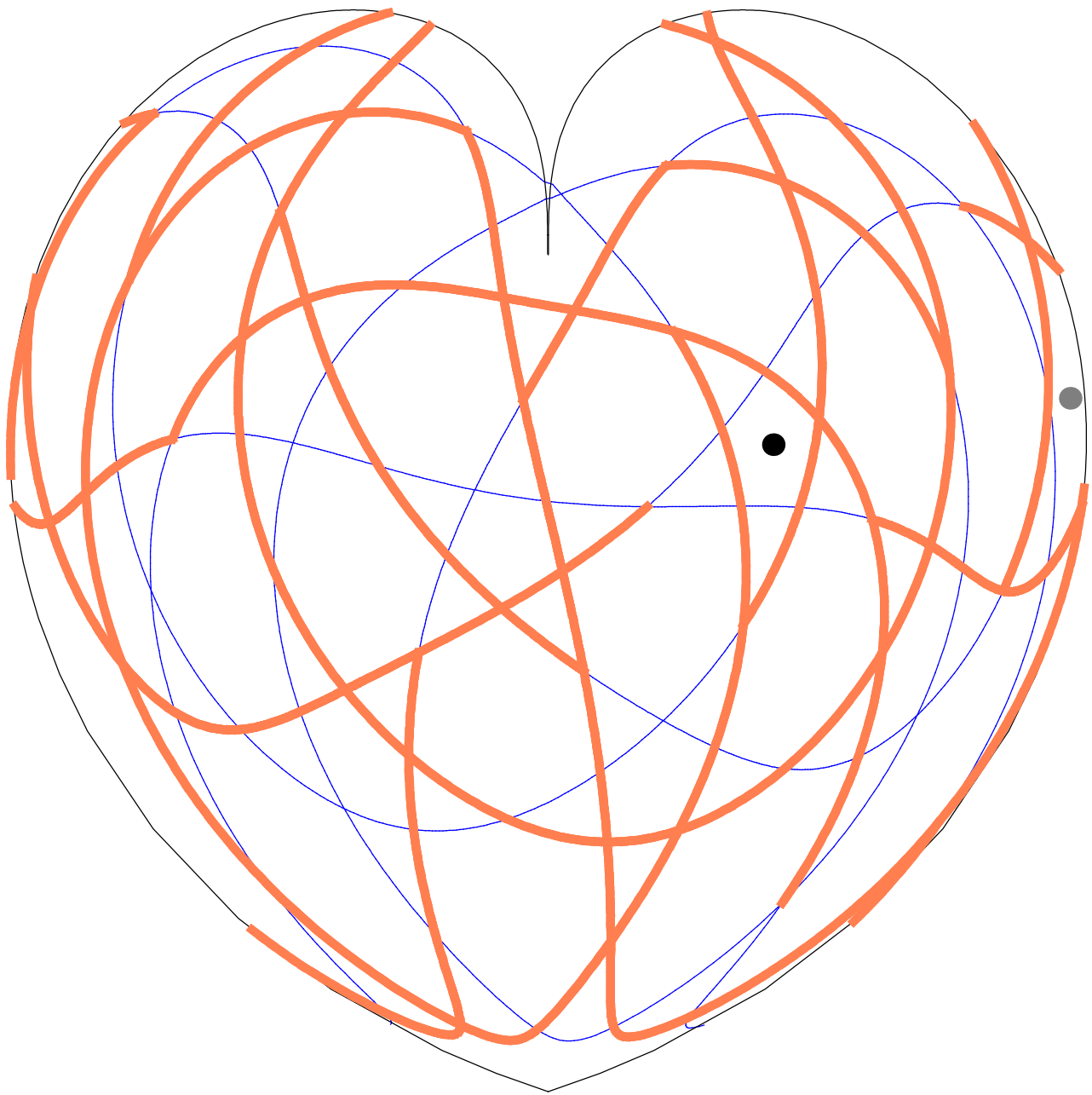


27.

dodecadodecahedron

$$\left\{\frac{5}{2}, 5, \frac{5}{2}, 5\right\}$$

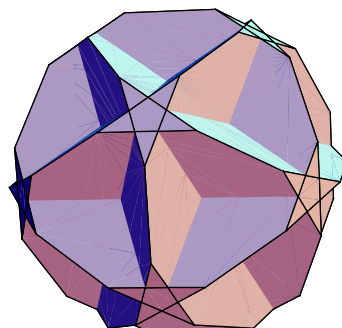
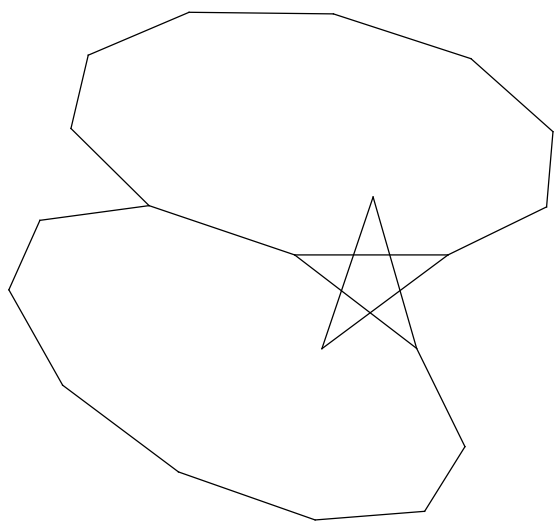


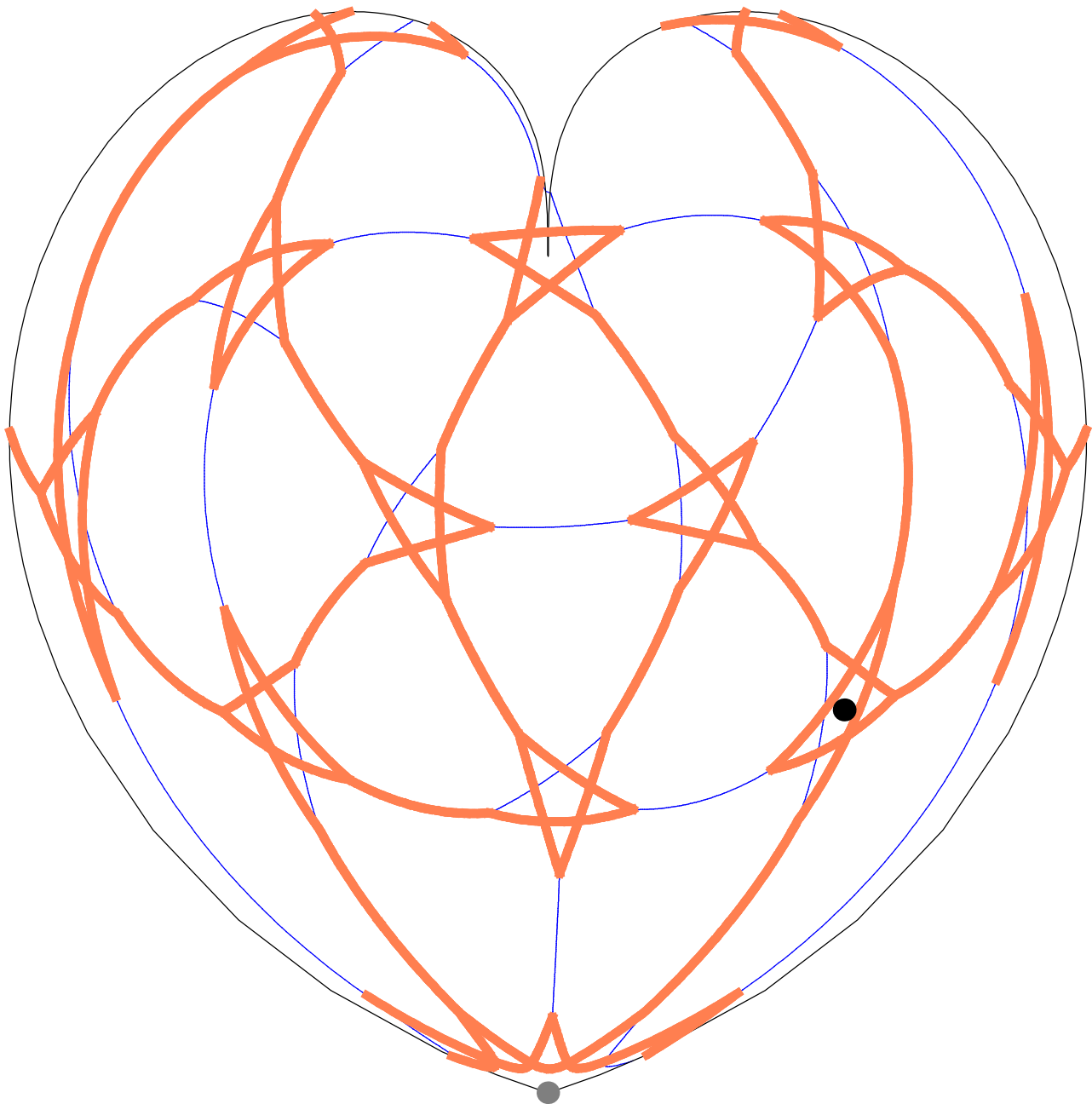


28.

truncated great dodecahedron

$$\{10, 10, \frac{5}{2}\}$$

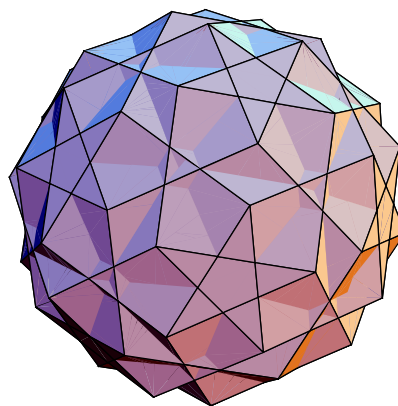
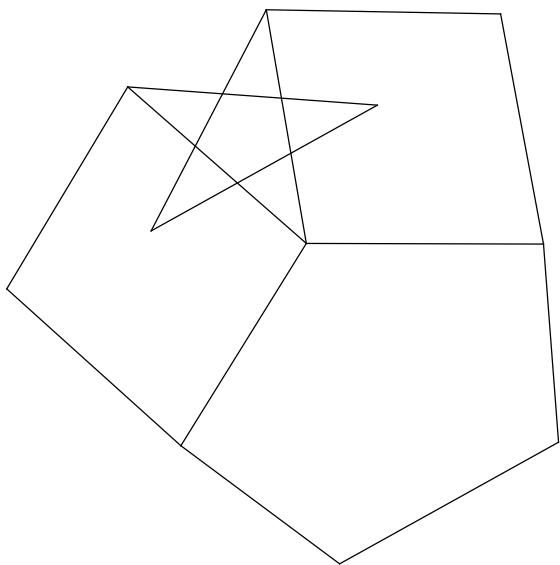


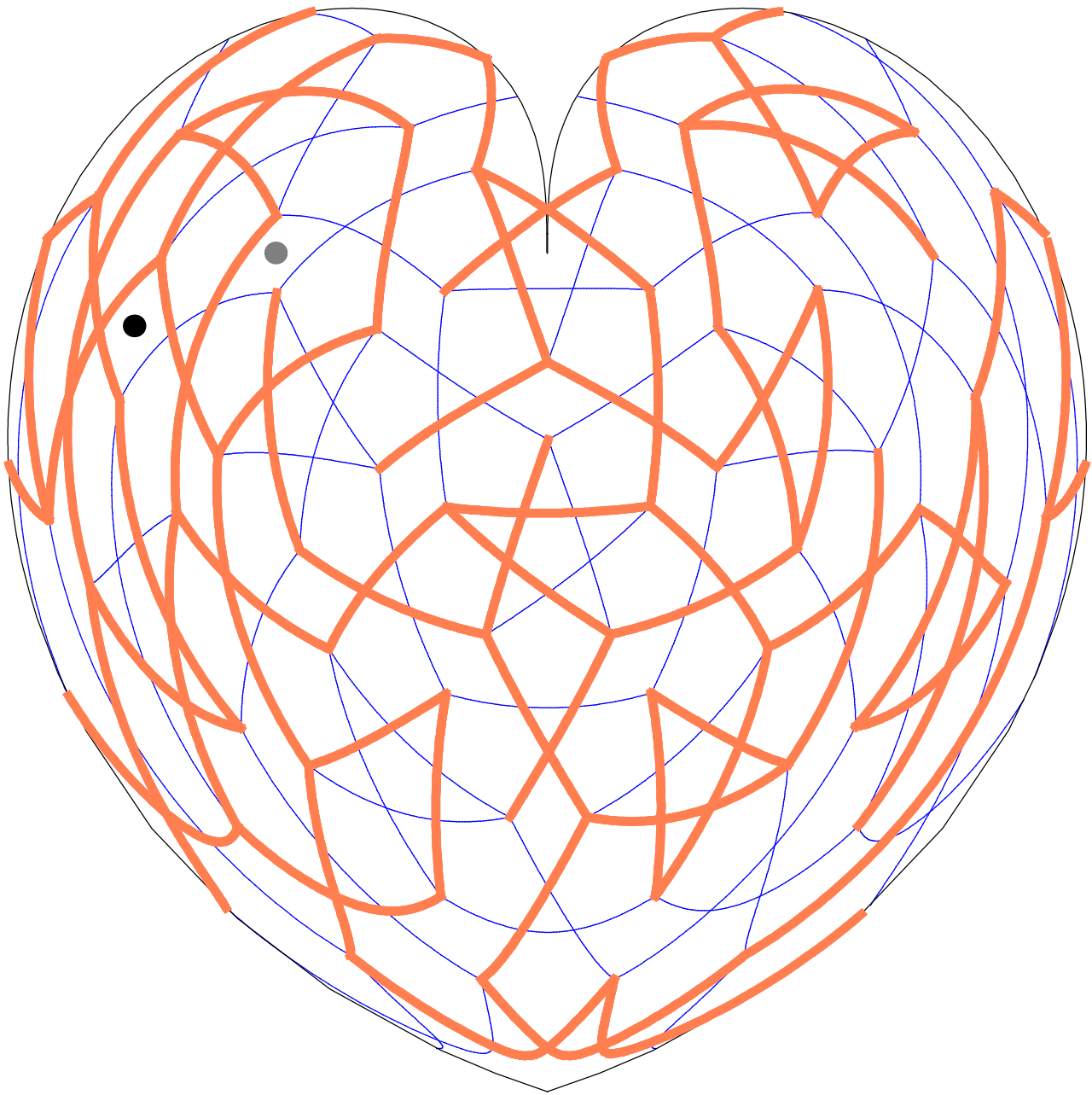


29.

rhombidodecadodecahedron

$$\{4, \frac{5}{2}, 4, 5\}$$

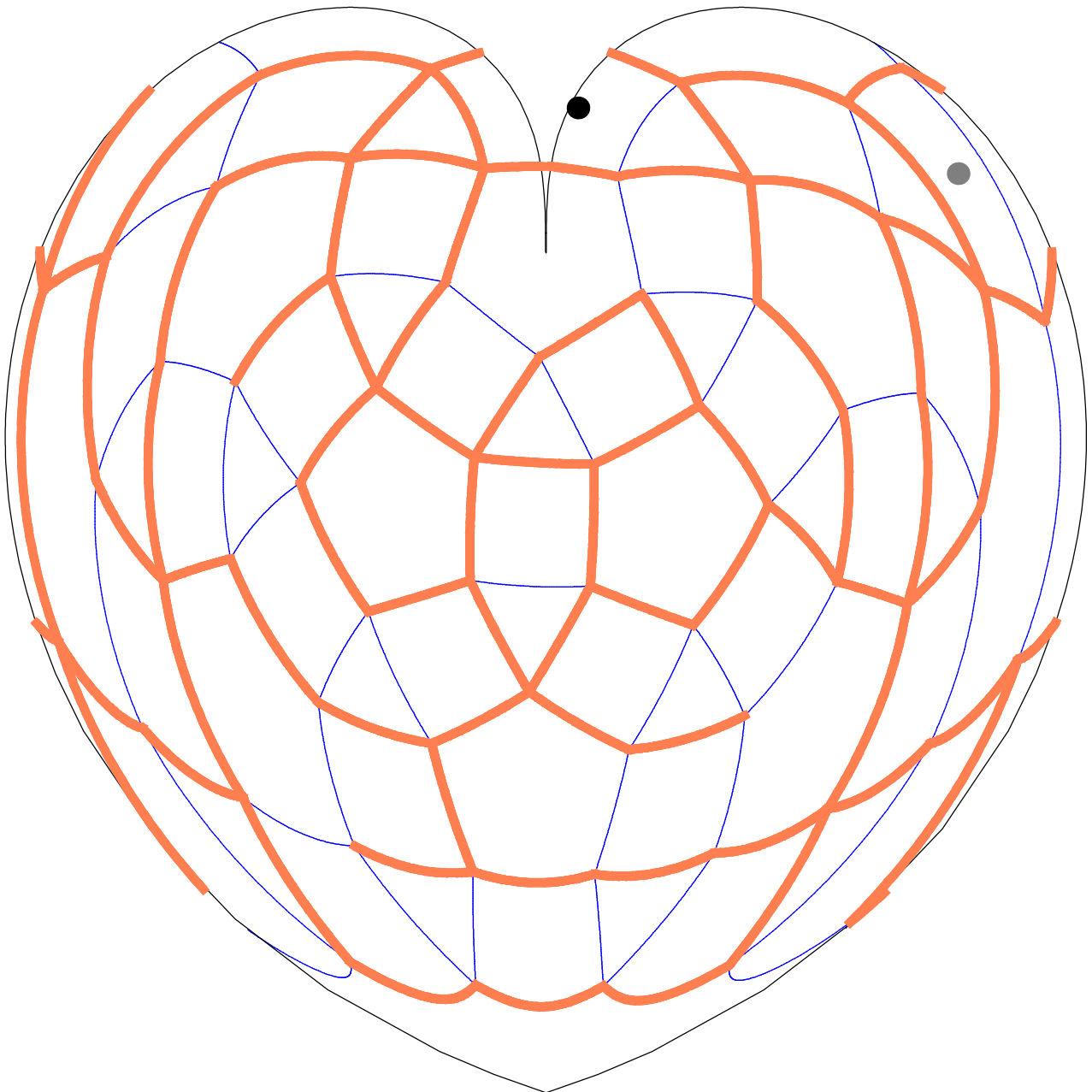
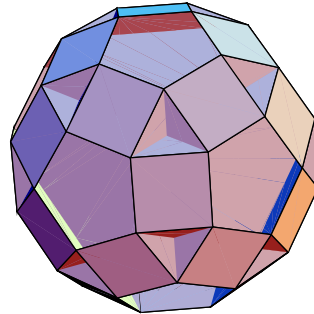
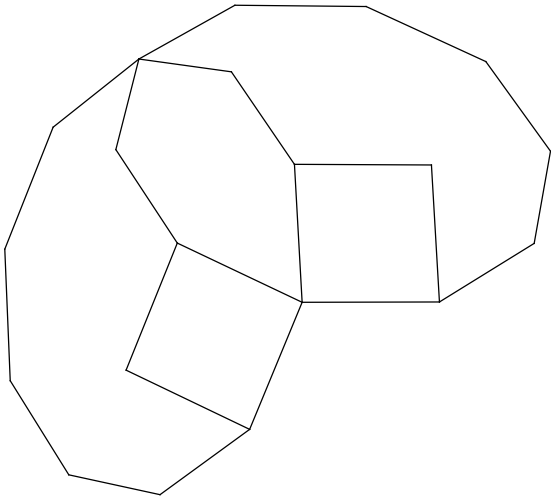




30.

small rhombidodecahedron

$$\left\{10, 4, \frac{10}{9}, \frac{4}{3}\right\}$$

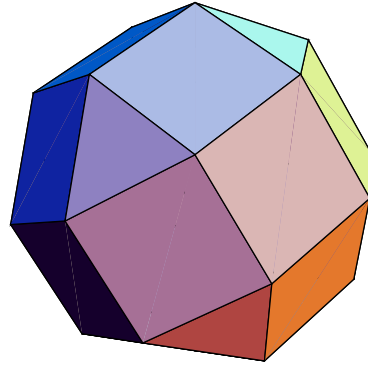
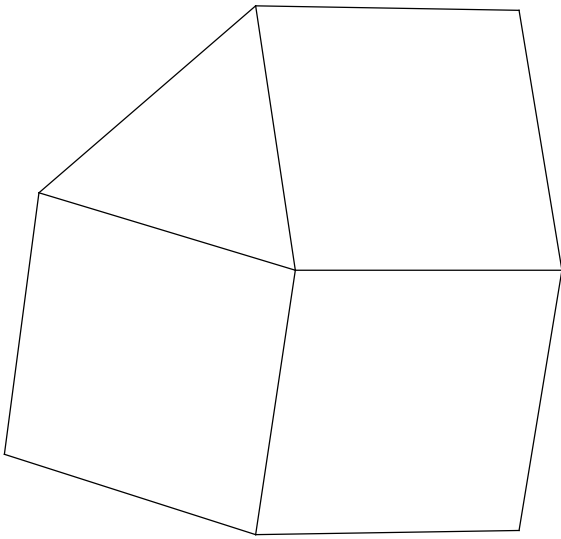


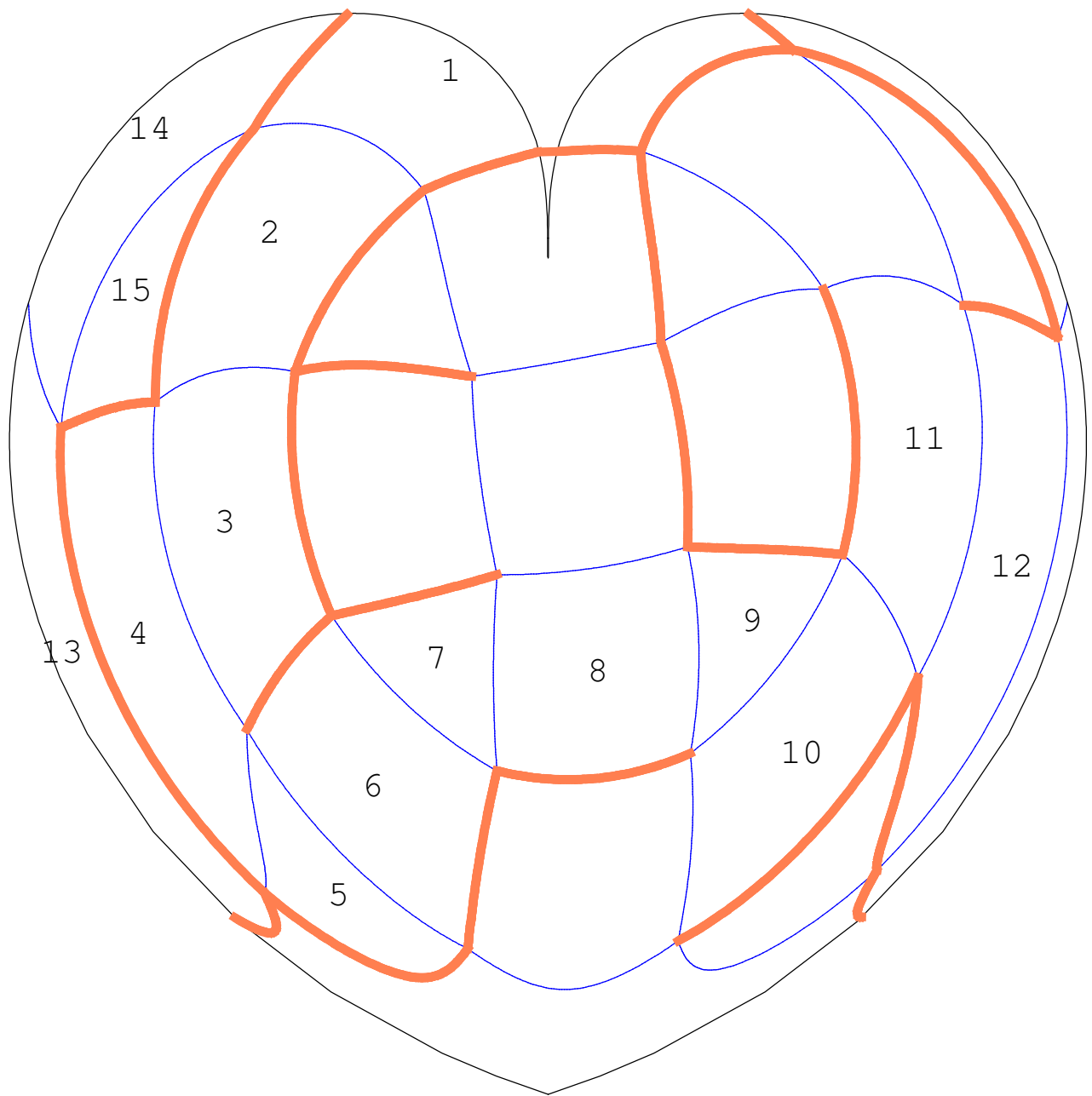
Solutions

1.

rhombicuboctahedron

{4, 3, 4, 4}

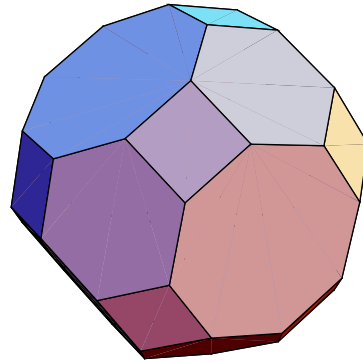
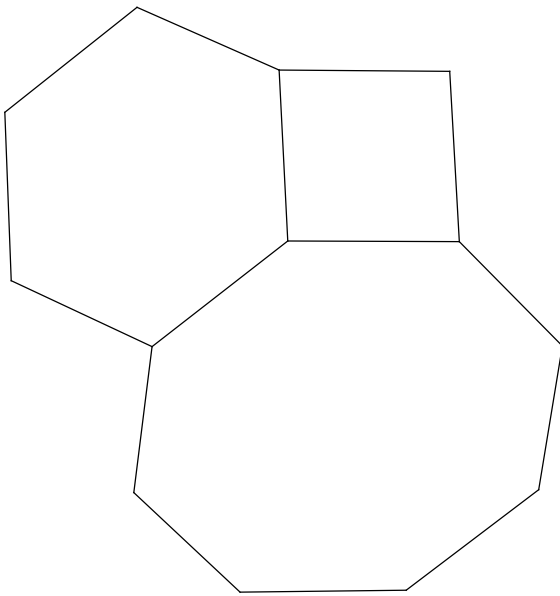


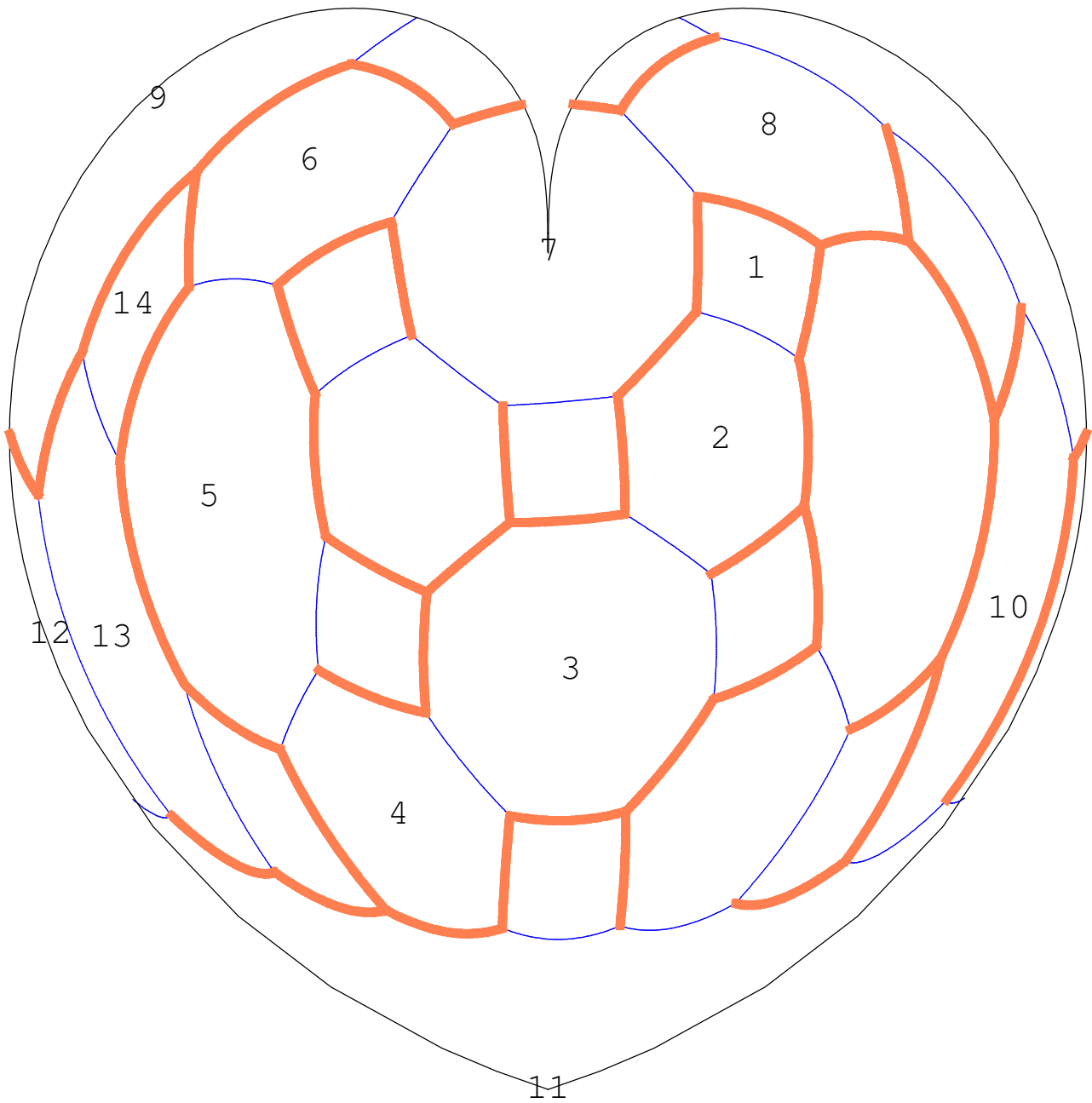


2.

truncated cuboctahedron

{4, 6, 8}

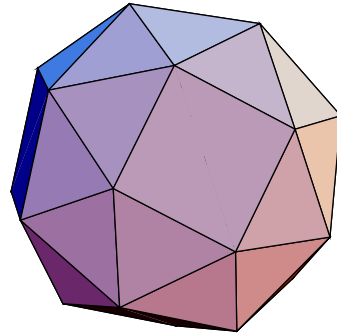
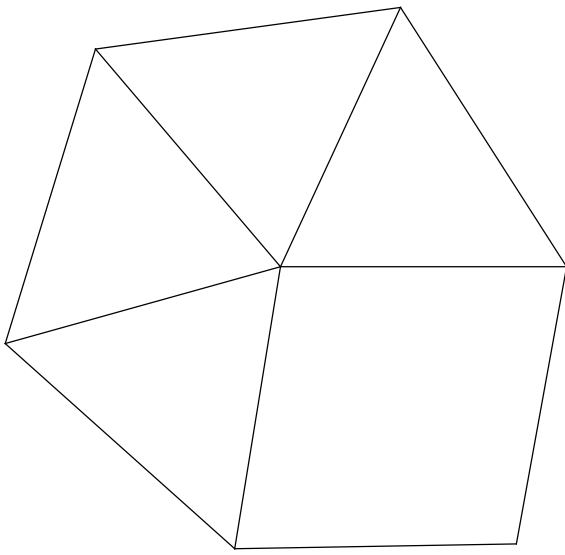


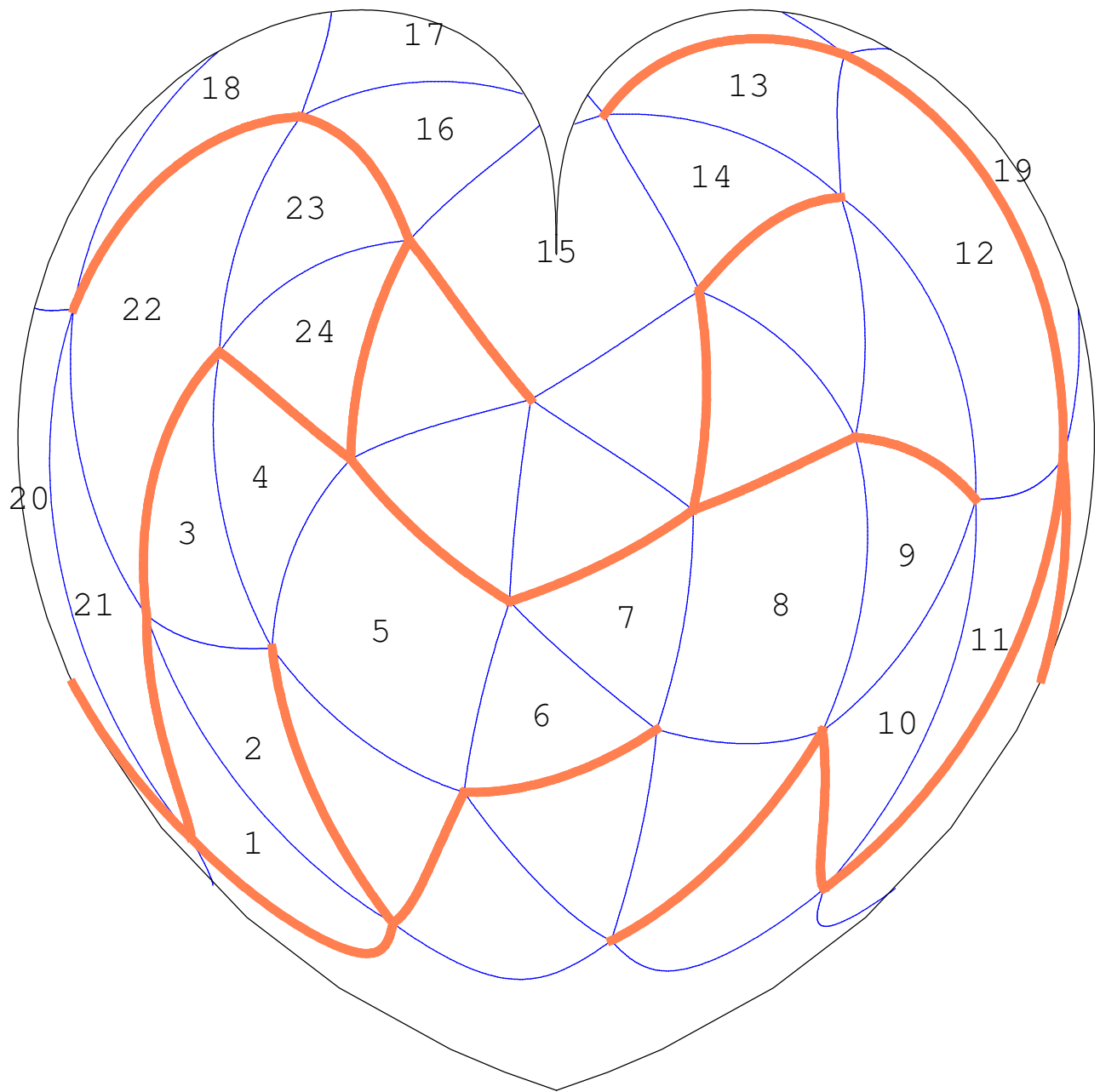


3.

snub cube

{3, 3, 3, 3, 4}

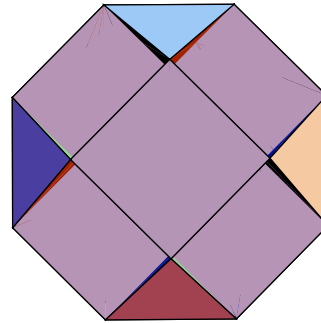
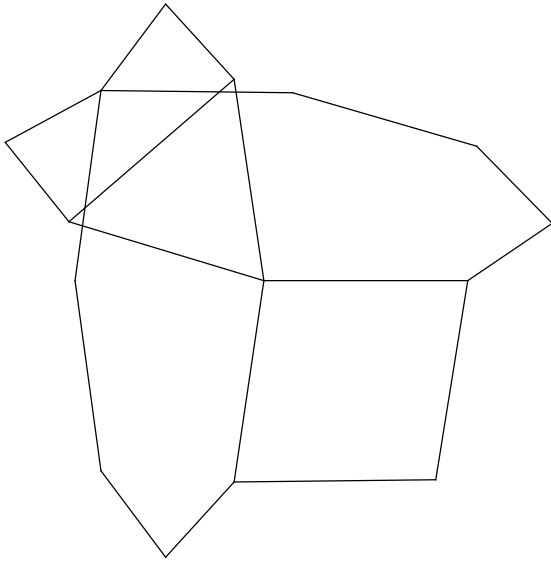


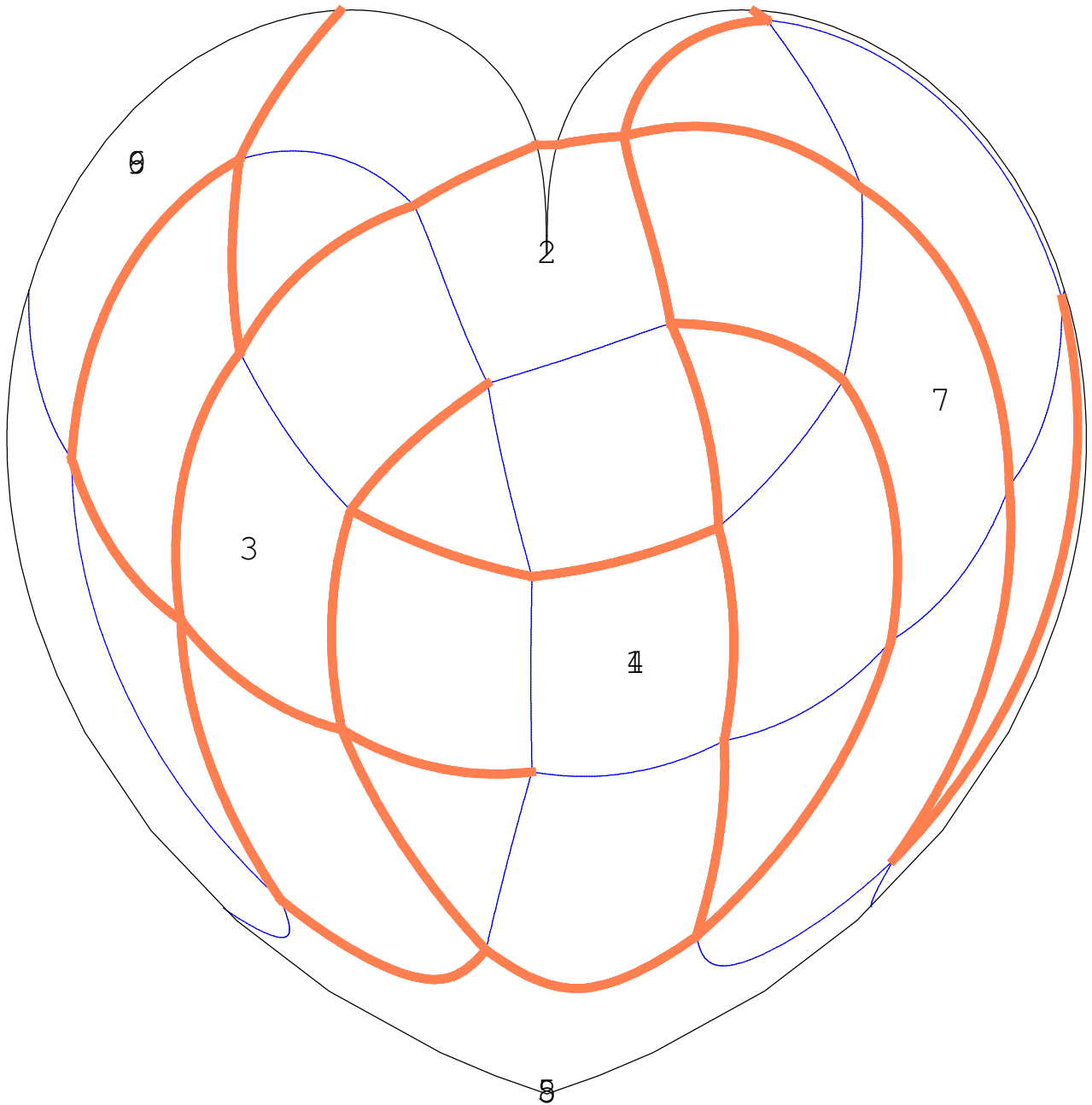


4.

small cubicuboctahedron

$$\left\{8, \frac{3}{2}, 8, 4\right\}$$

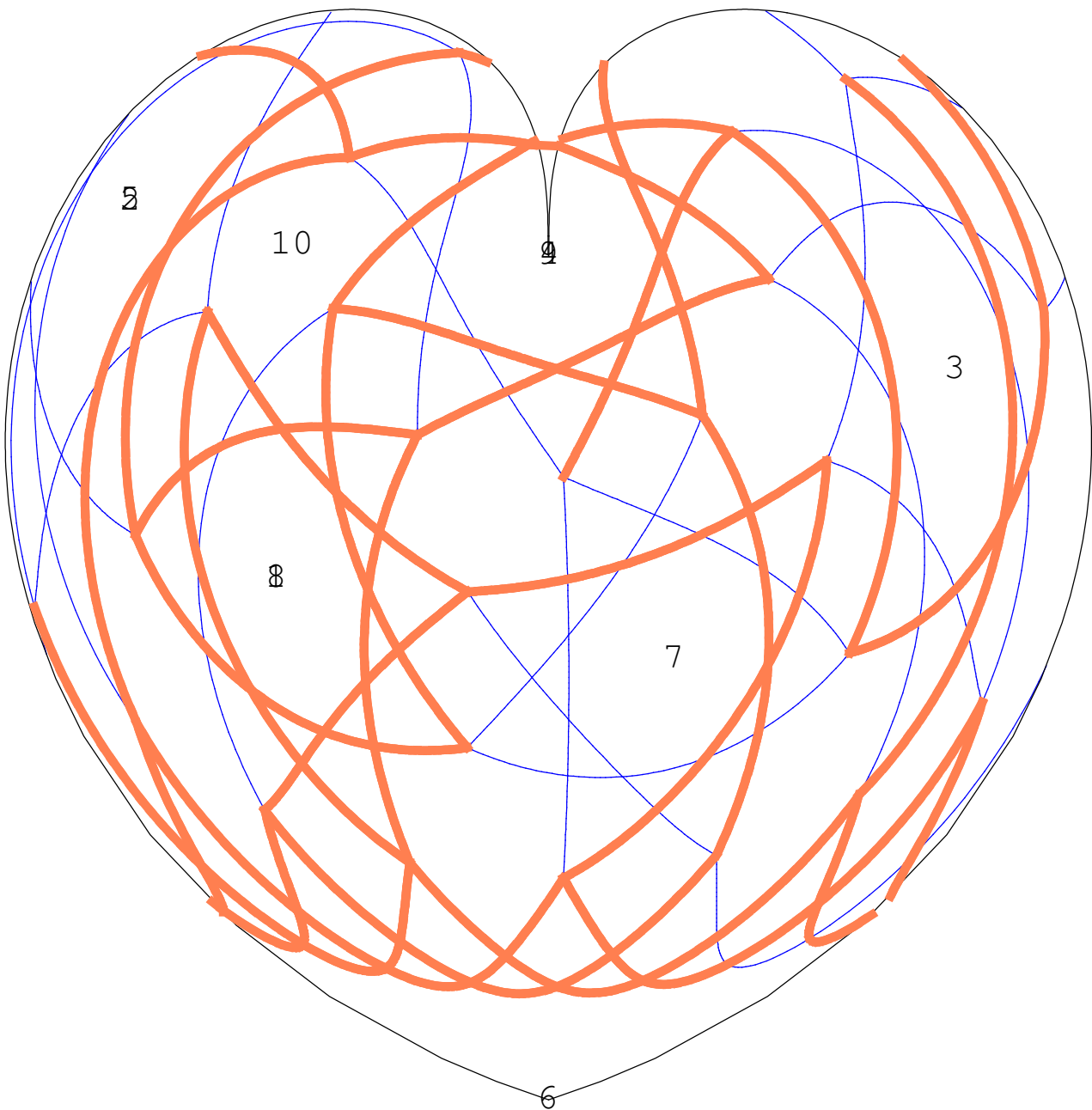
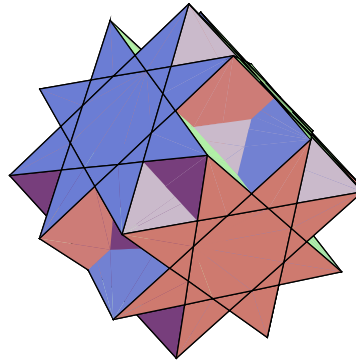
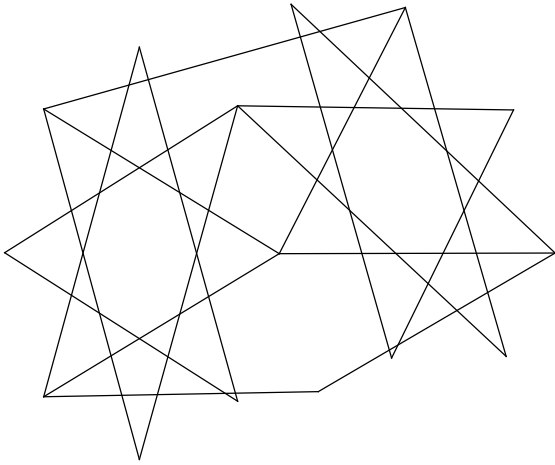




5.

great cubicuboctahedron

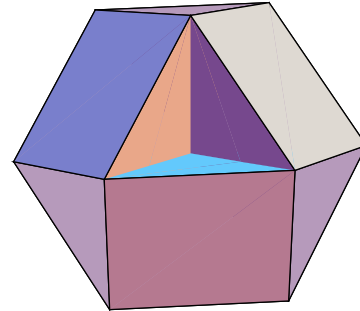
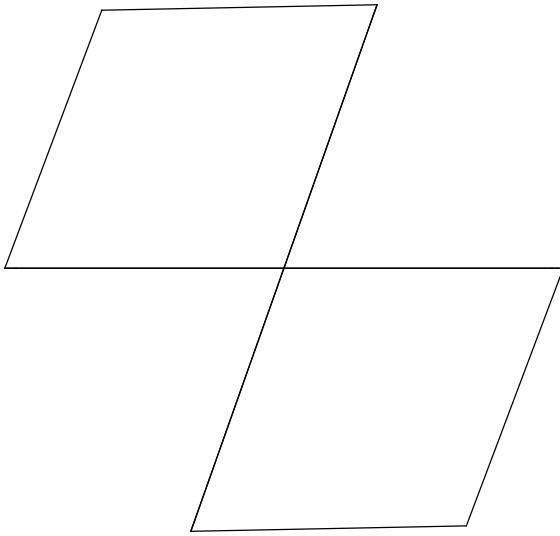
$$\left\{ \frac{8}{3}, 3, \frac{8}{3}, 4 \right\}$$

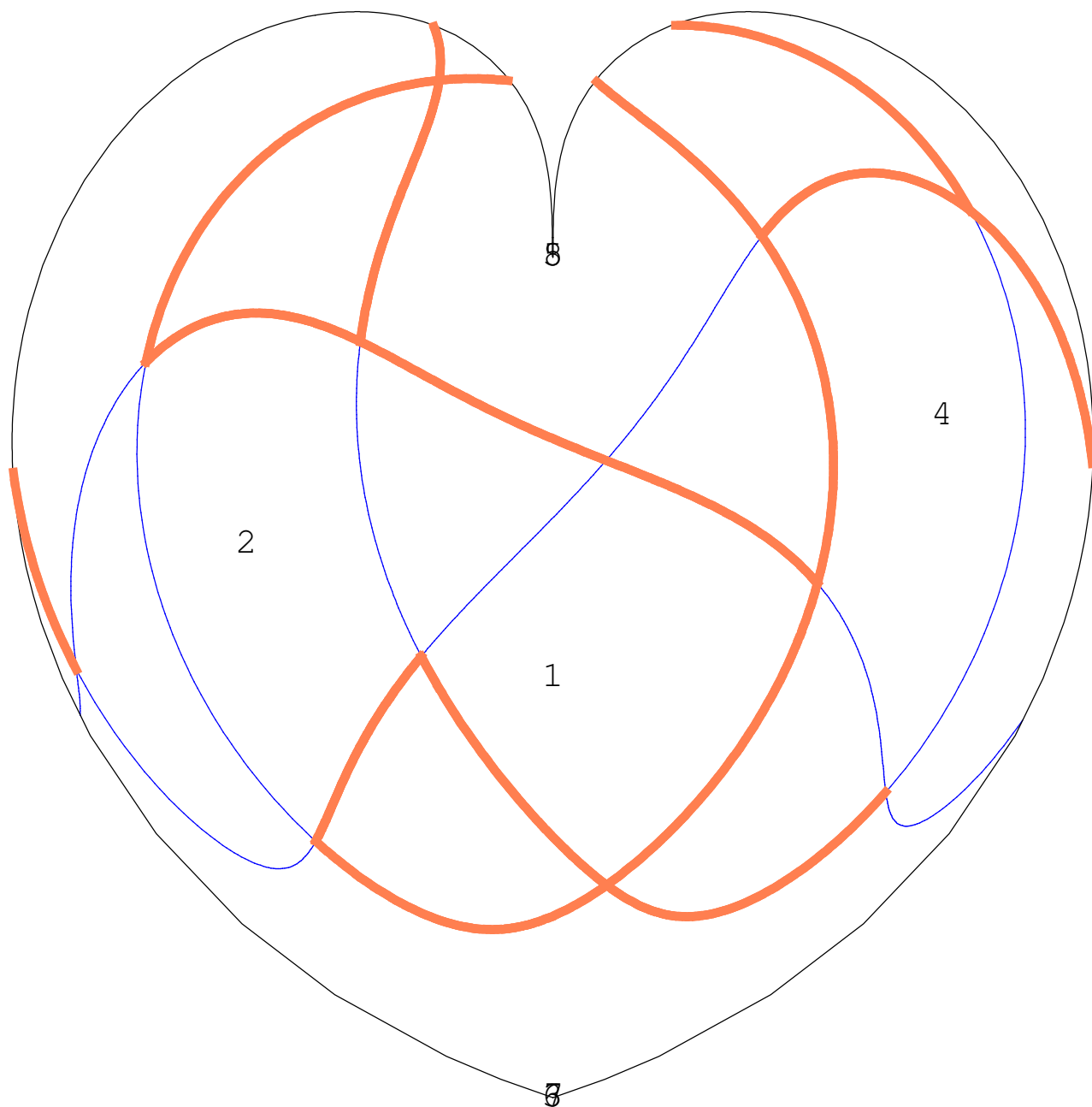


6.

cubohemioctahedron

$$\left\{6, \frac{4}{3}, 6, 4\right\}$$

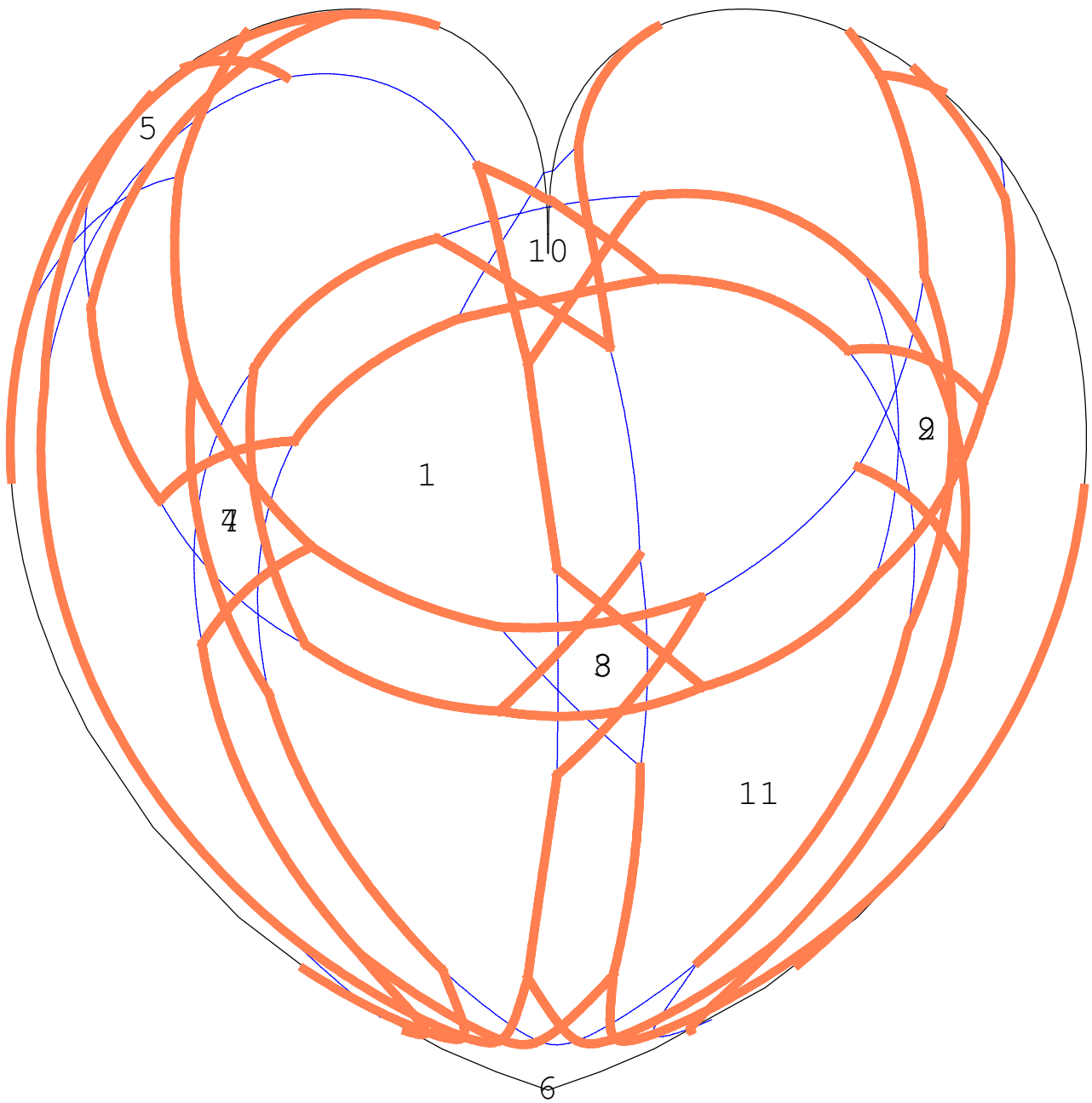
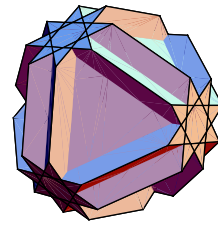
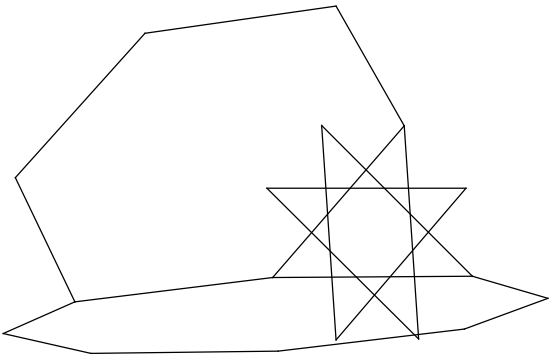




7.

cubitruncated cuboctahedron

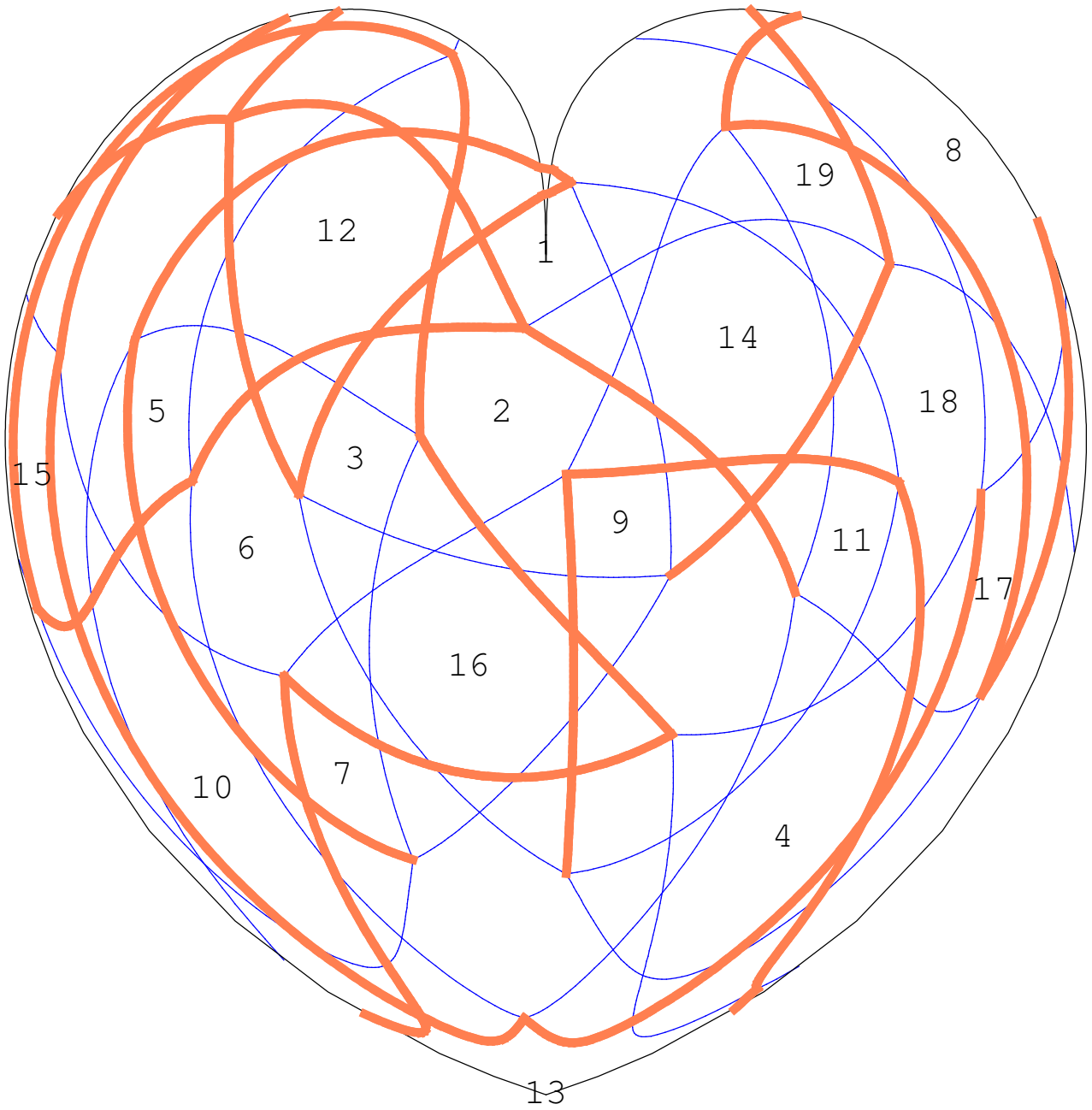
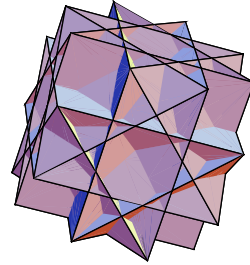
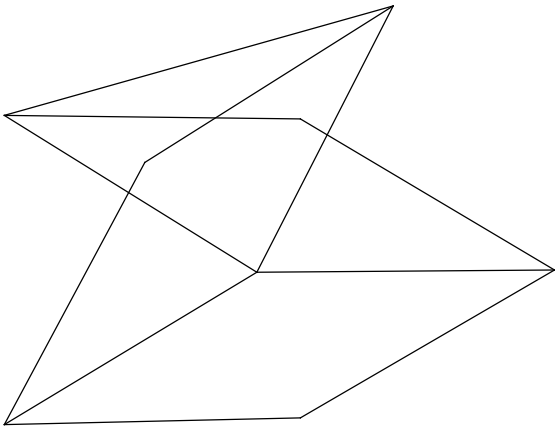
$$\left\{ \frac{8}{3}, 6, 8 \right\}$$



8.

great rhombicuboctahedron

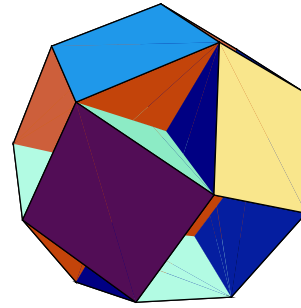
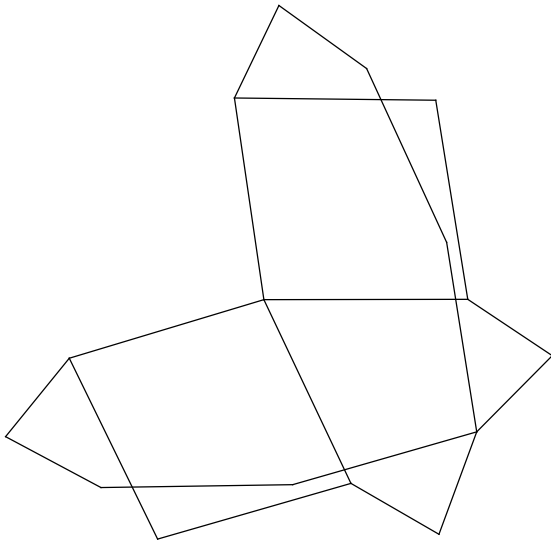
$$\{4, \frac{3}{2}, 4, 4\}$$

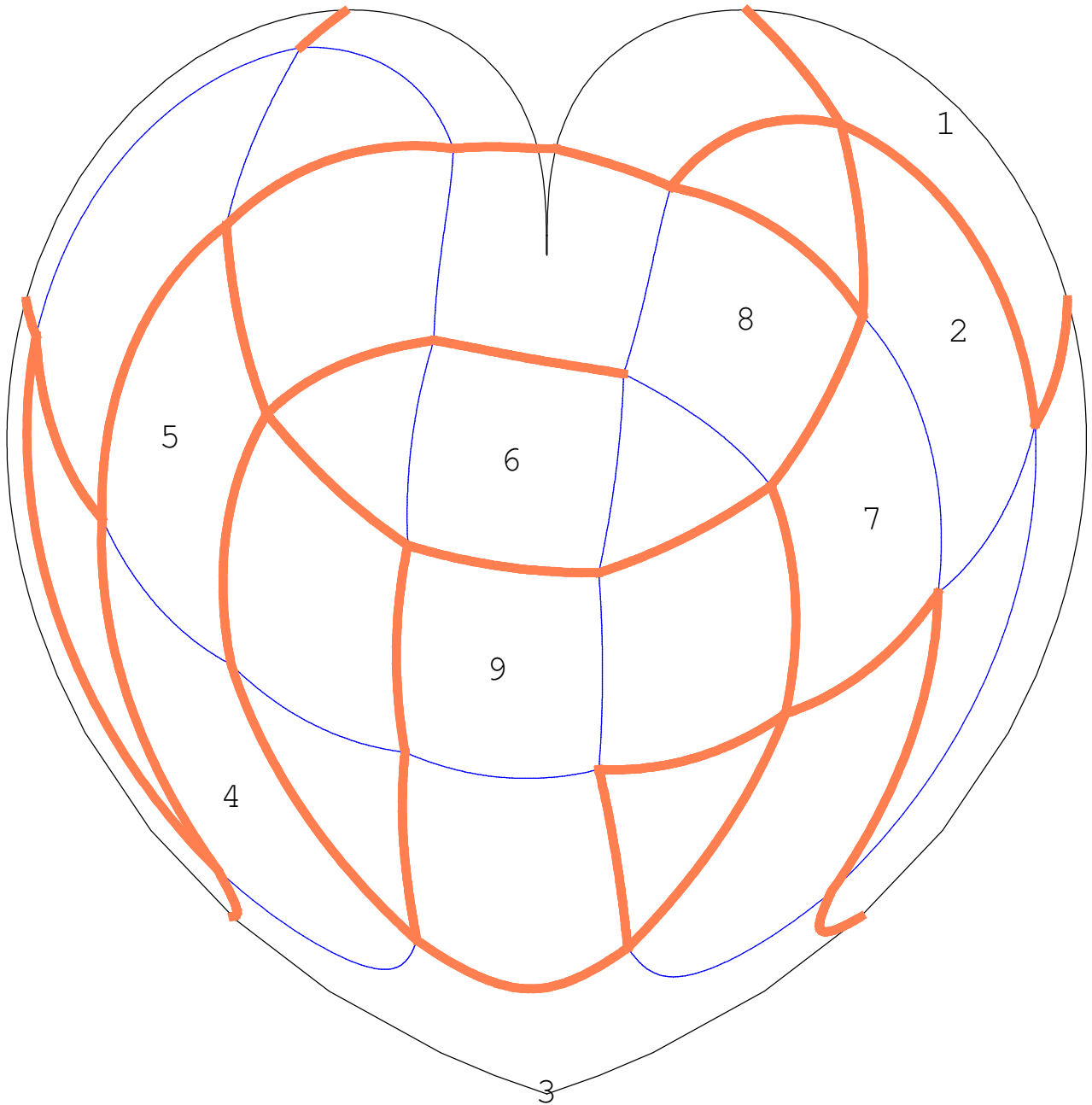


9.

small rhombihexahedron

$$\left\{8, 4, \frac{8}{7}, \frac{4}{3}\right\}$$

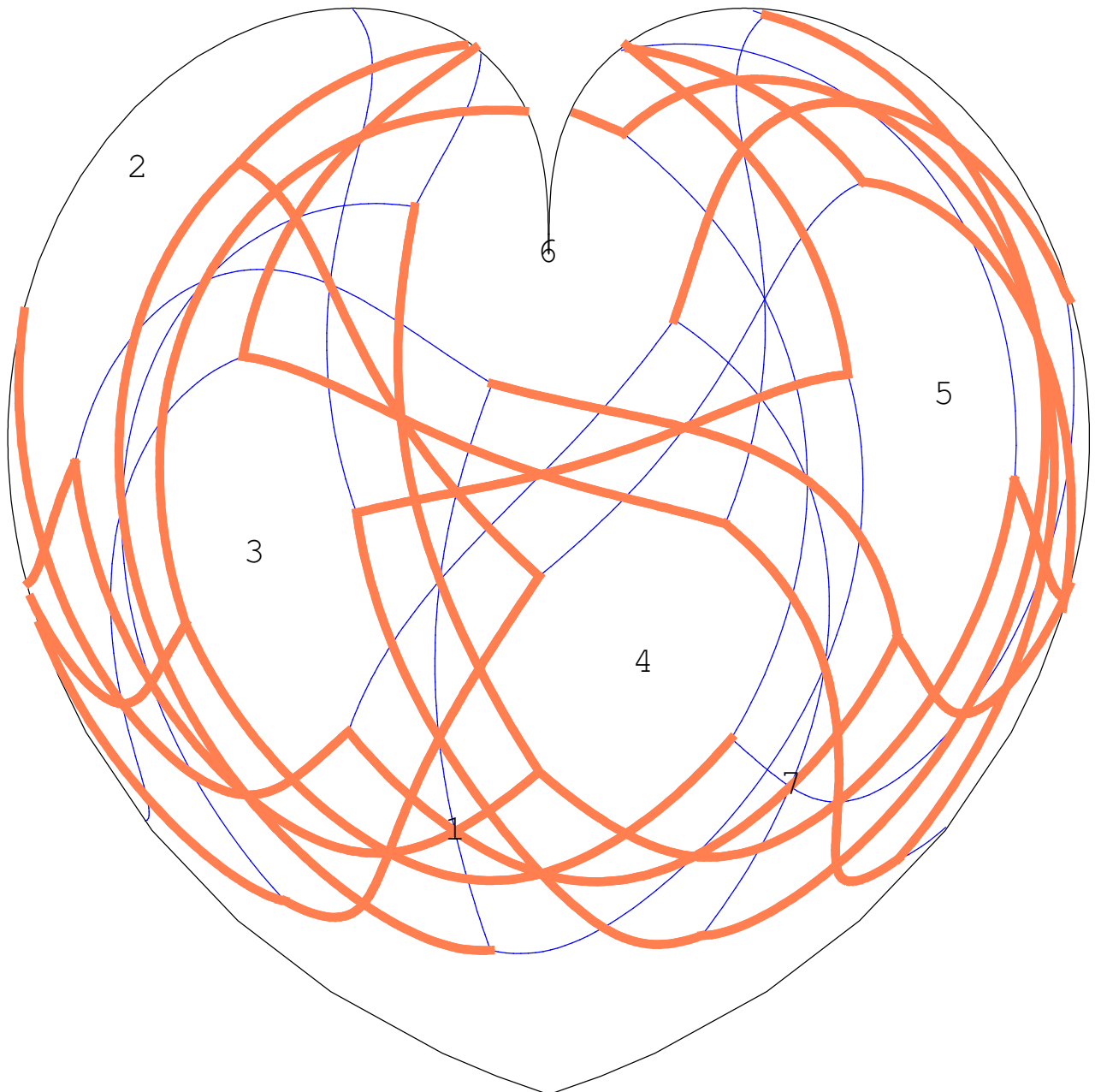
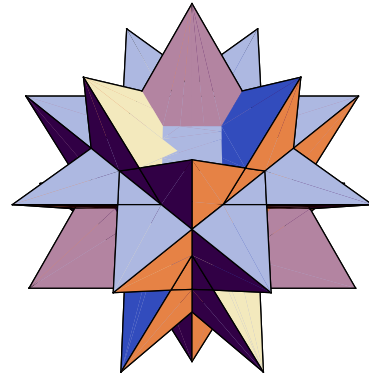
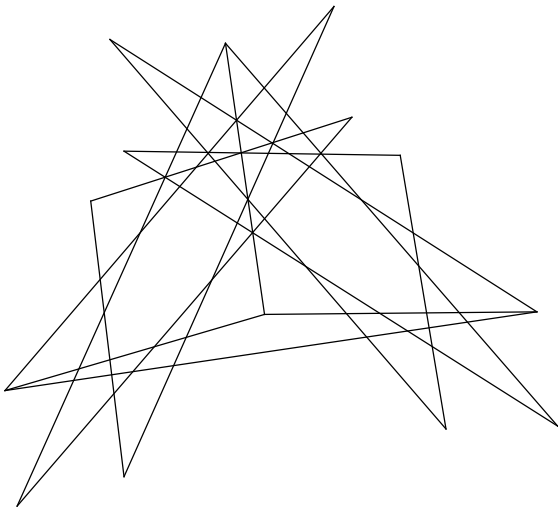




10.

stellated truncated hexahedron

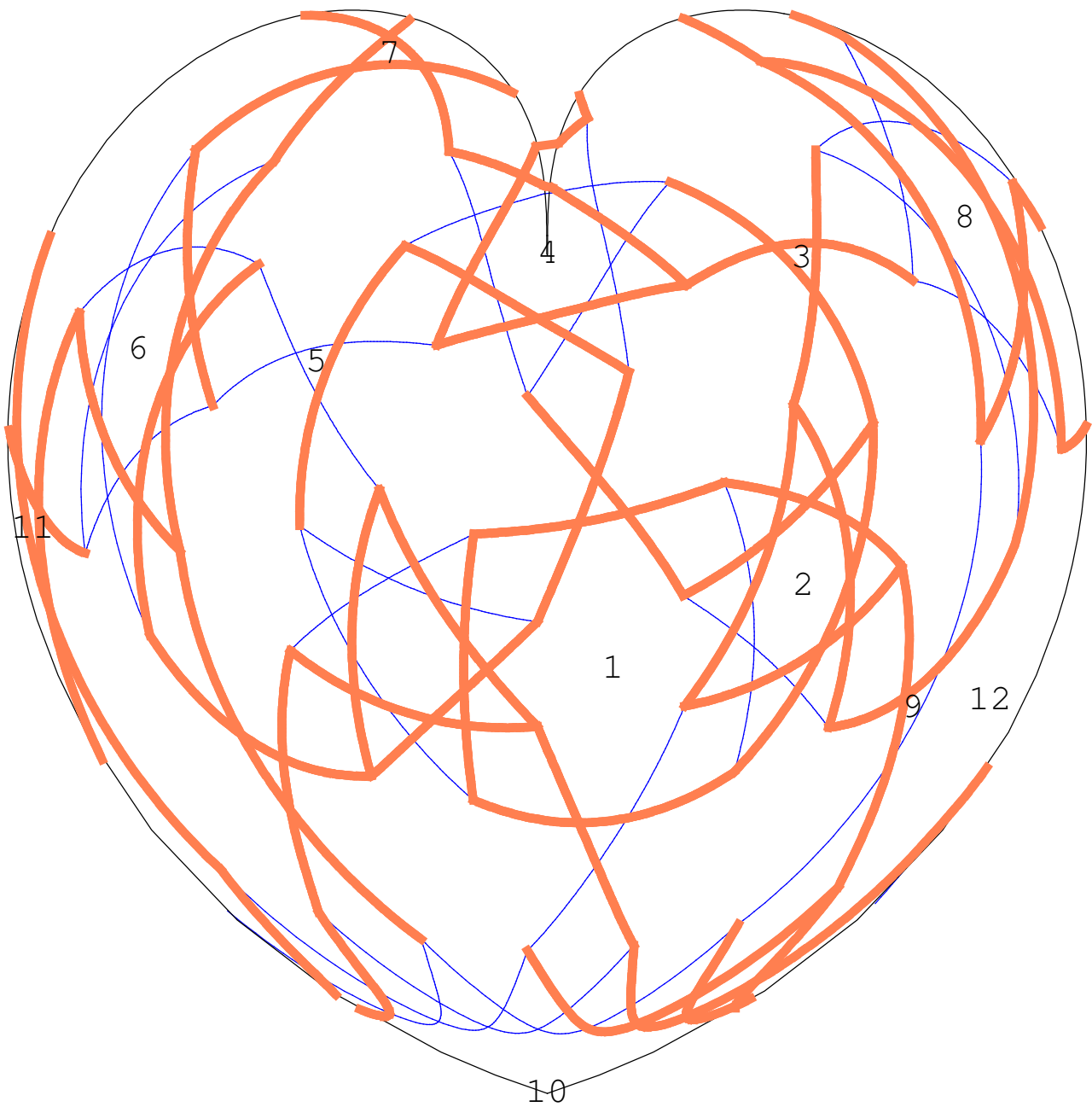
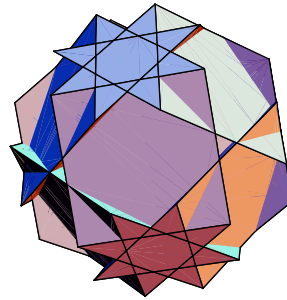
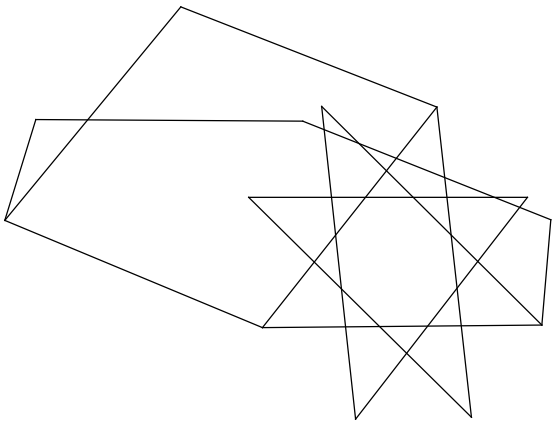
$$\left\{ \frac{8}{3}, \frac{8}{3}, 3 \right\}$$



11.

great truncated cuboctahedron

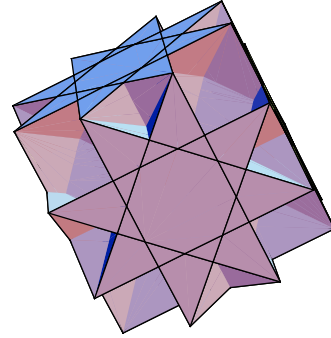
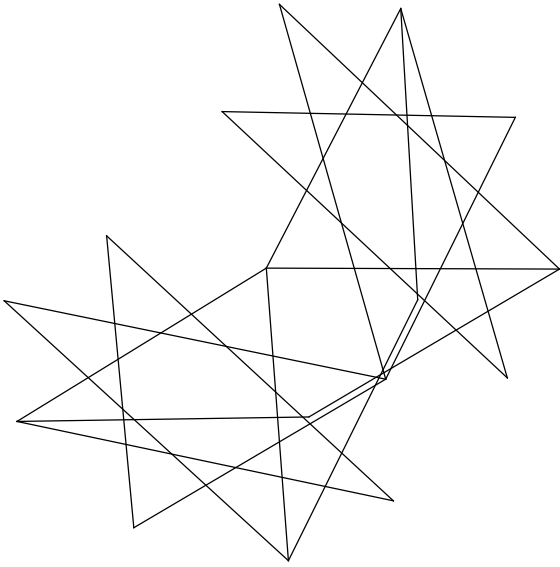
$$\left\{ \frac{8}{3}, 4, 6 \right\}$$

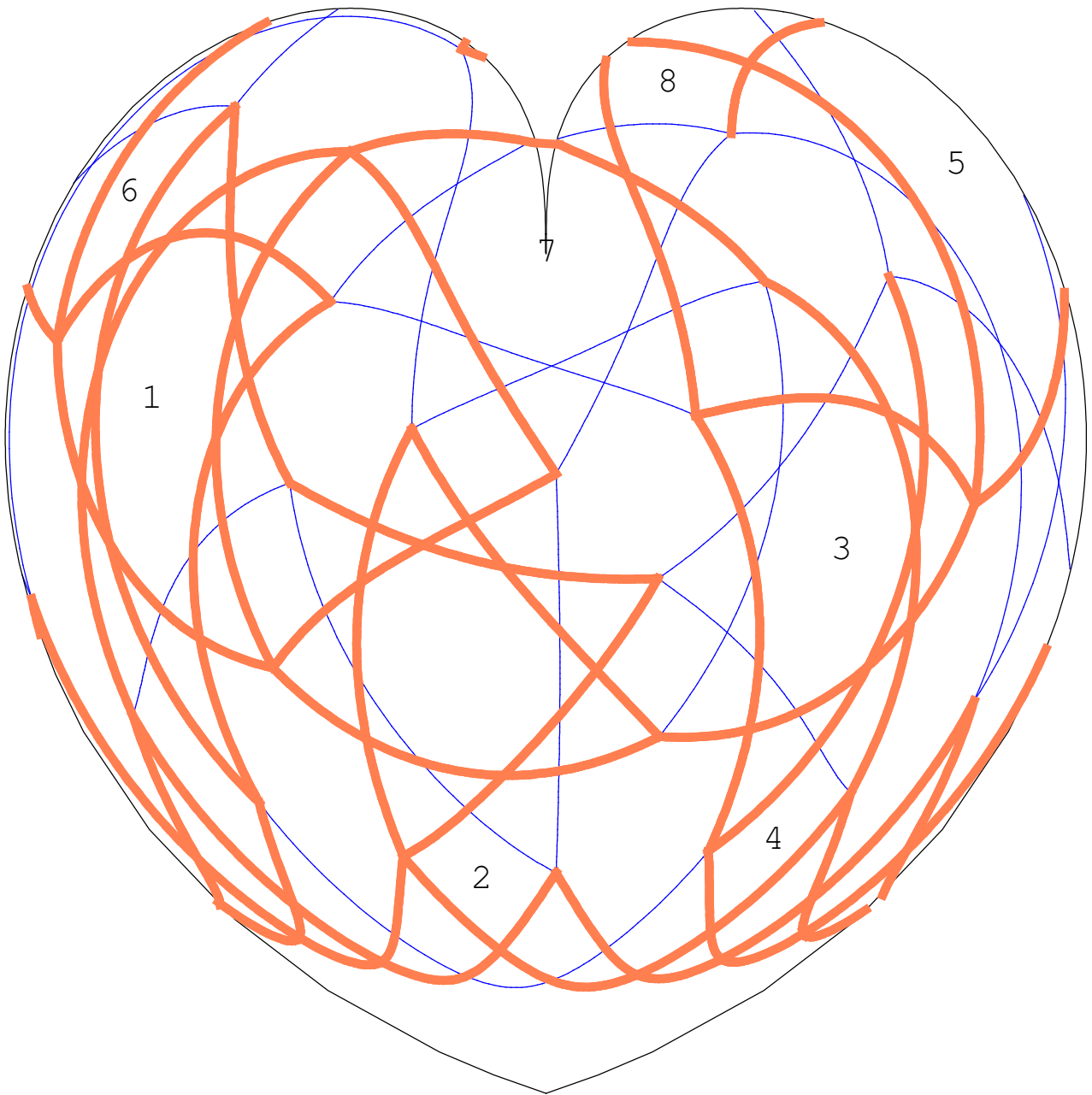


12.

great rhombihexahedron

$$\left\{4, \frac{8}{3}, \frac{4}{3}, \frac{8}{5}\right\}$$

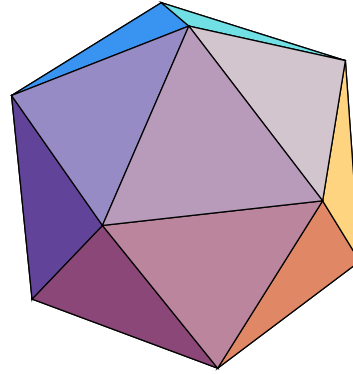
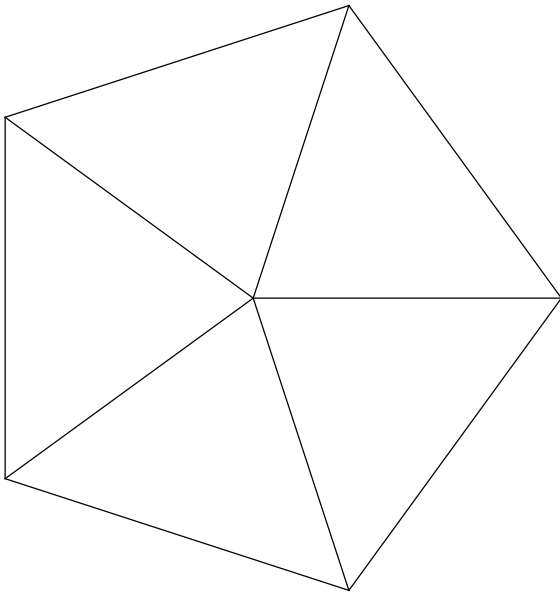


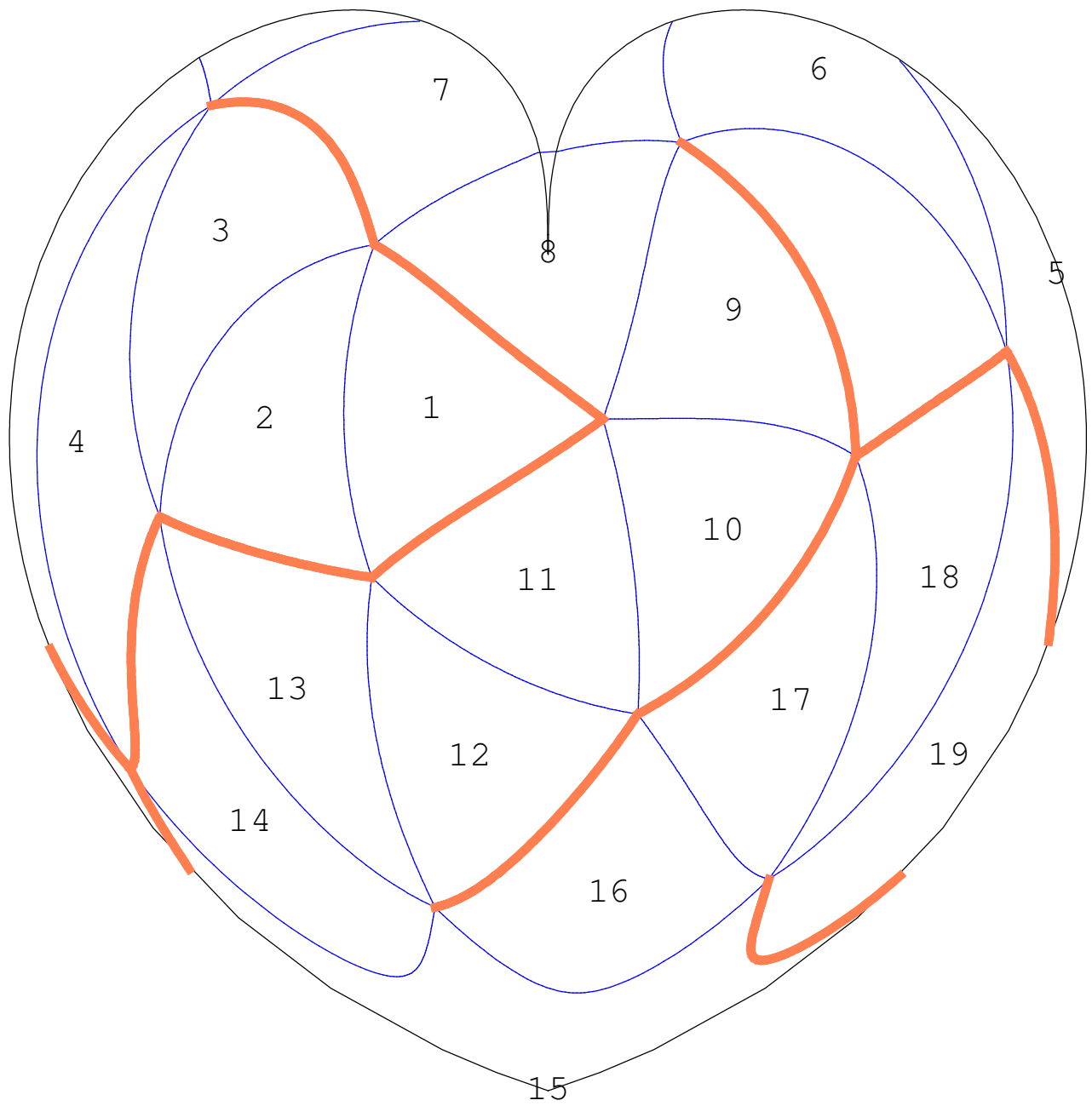


13.

icosahedron

{3, 3, 3, 3, 3}

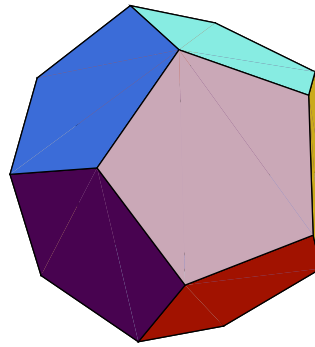
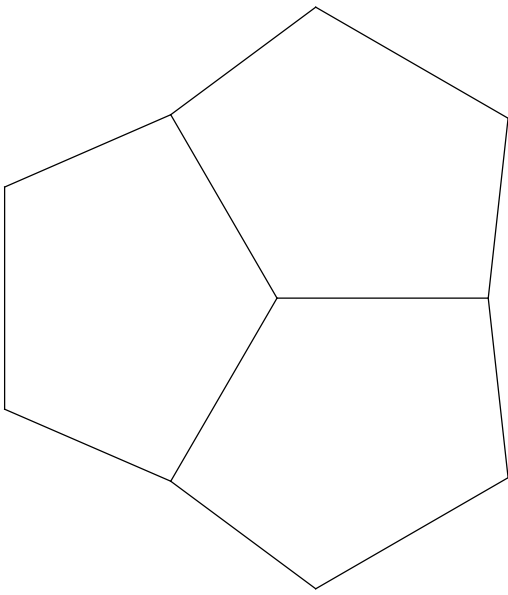


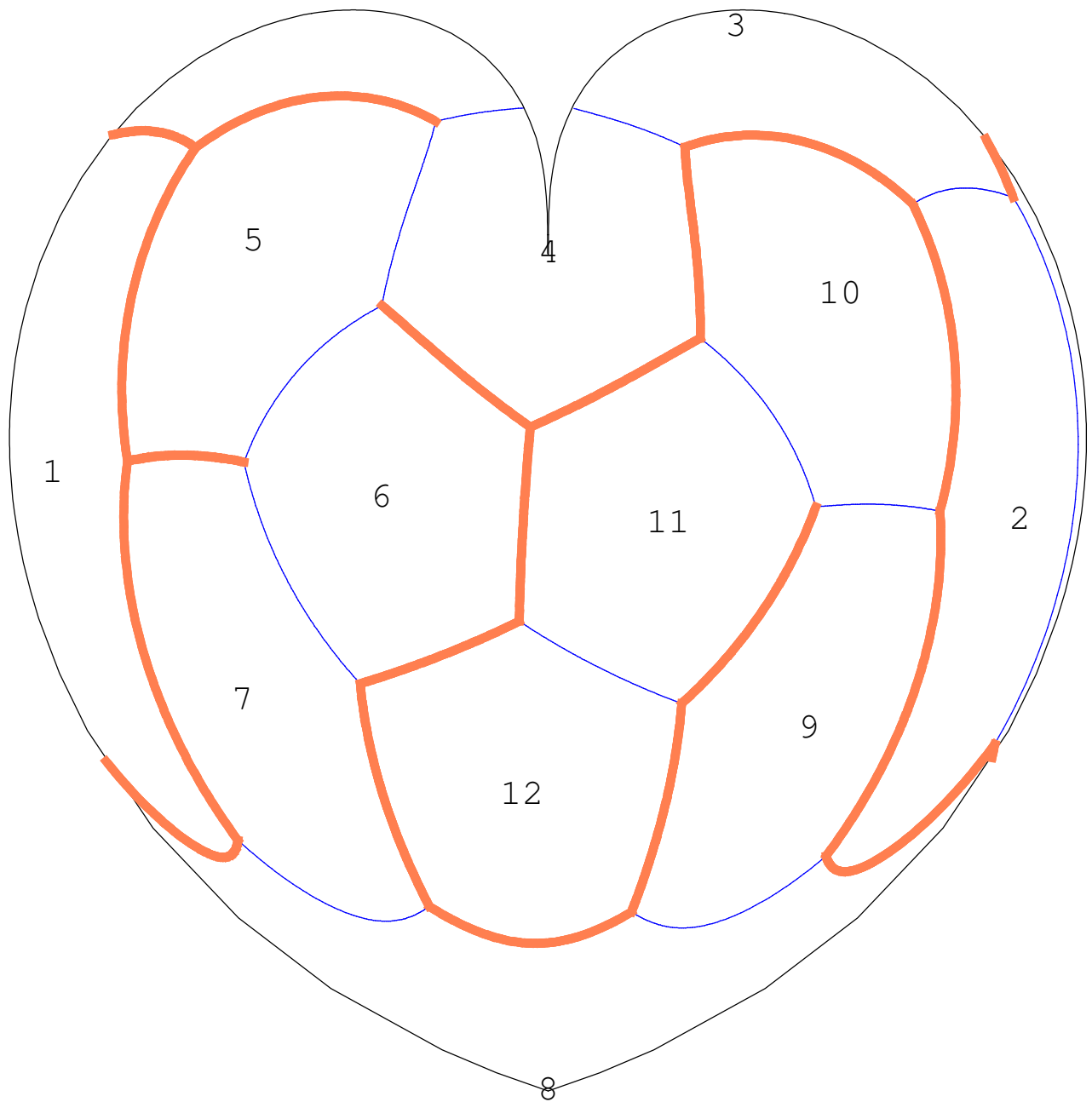


14.

dodecahedron

{5, 5, 5}

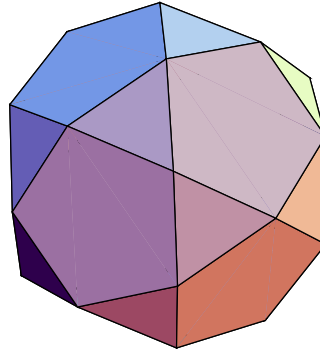
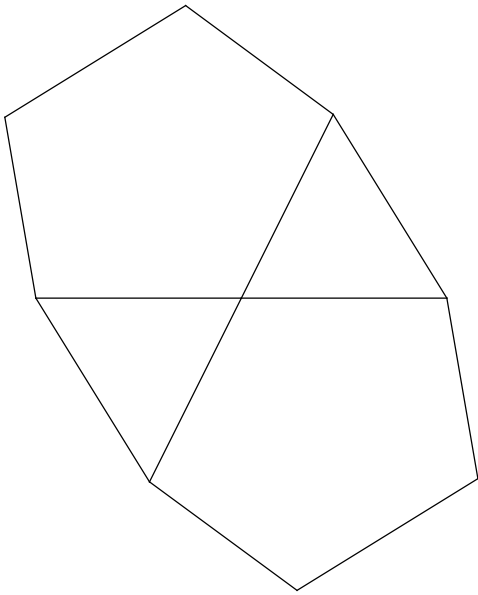


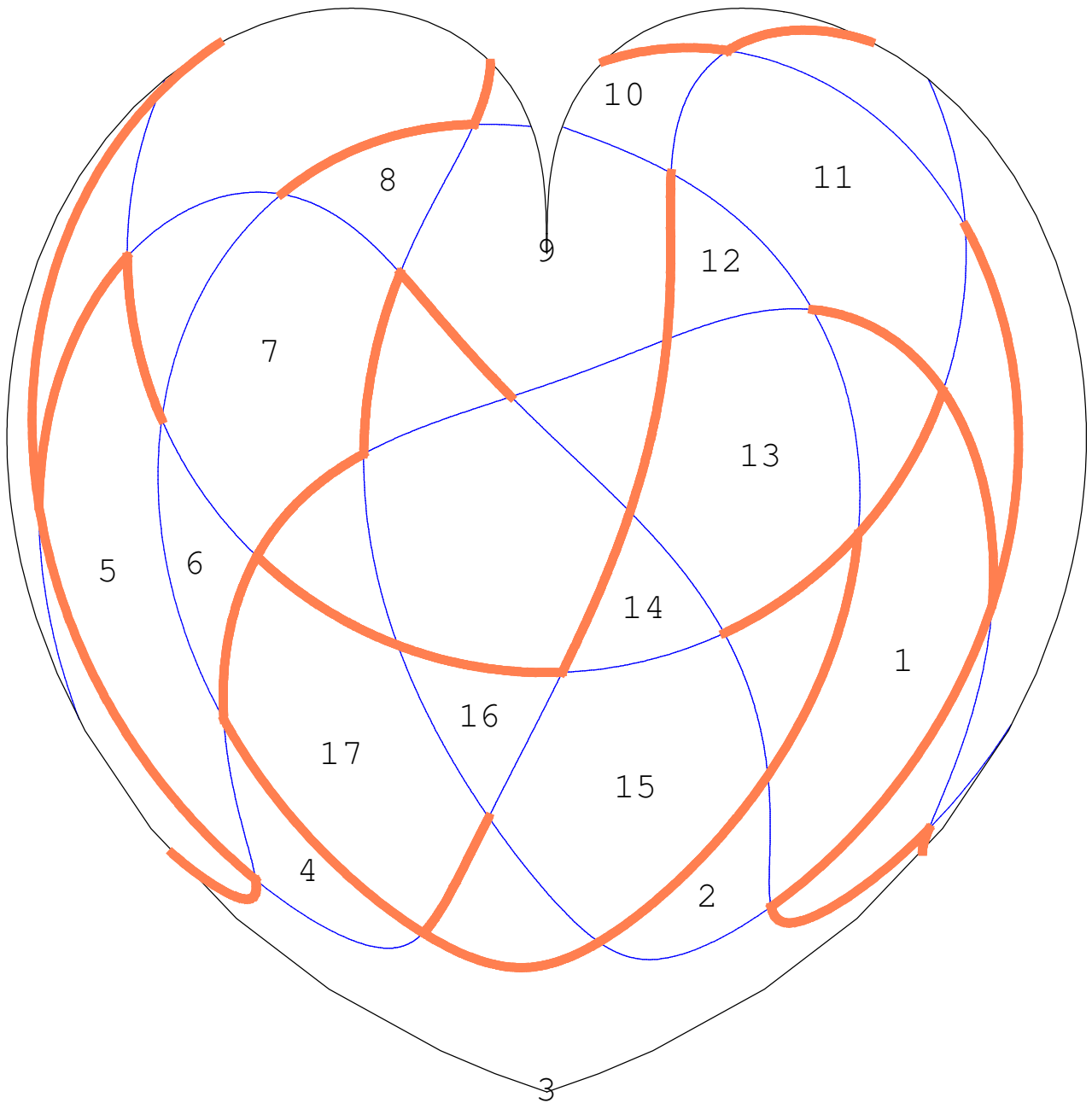


15.

icosidodecahedron

{3, 5, 3, 5}

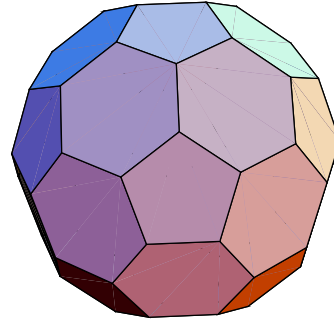
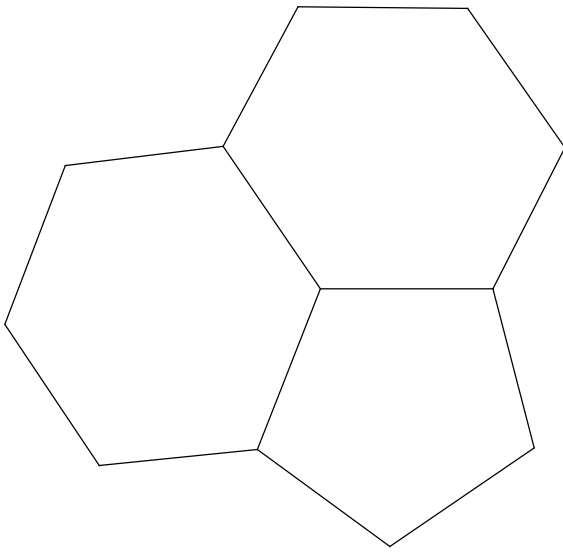


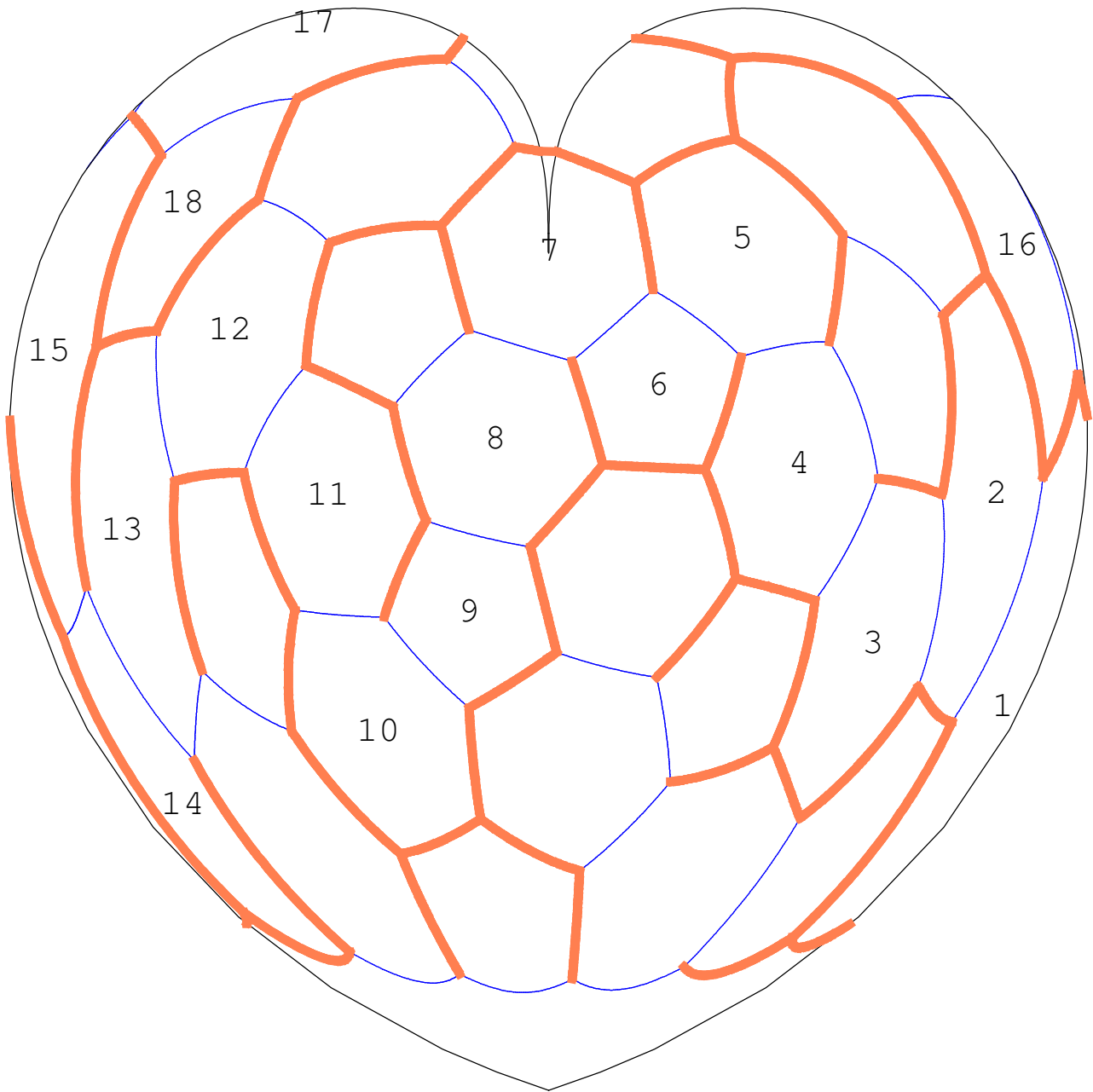


16.

truncated icosahedron

{6, 6, 5}

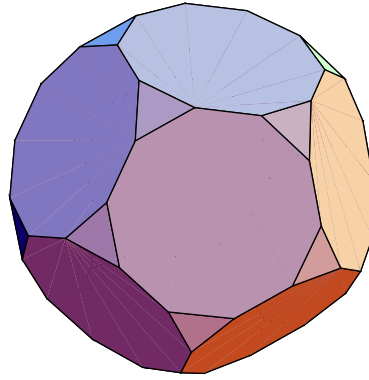
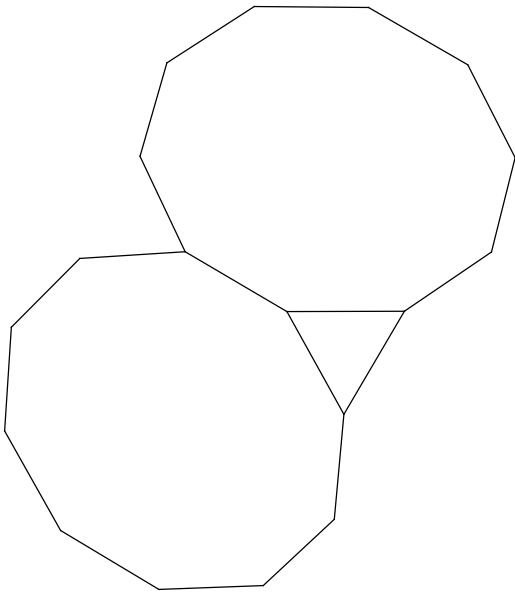


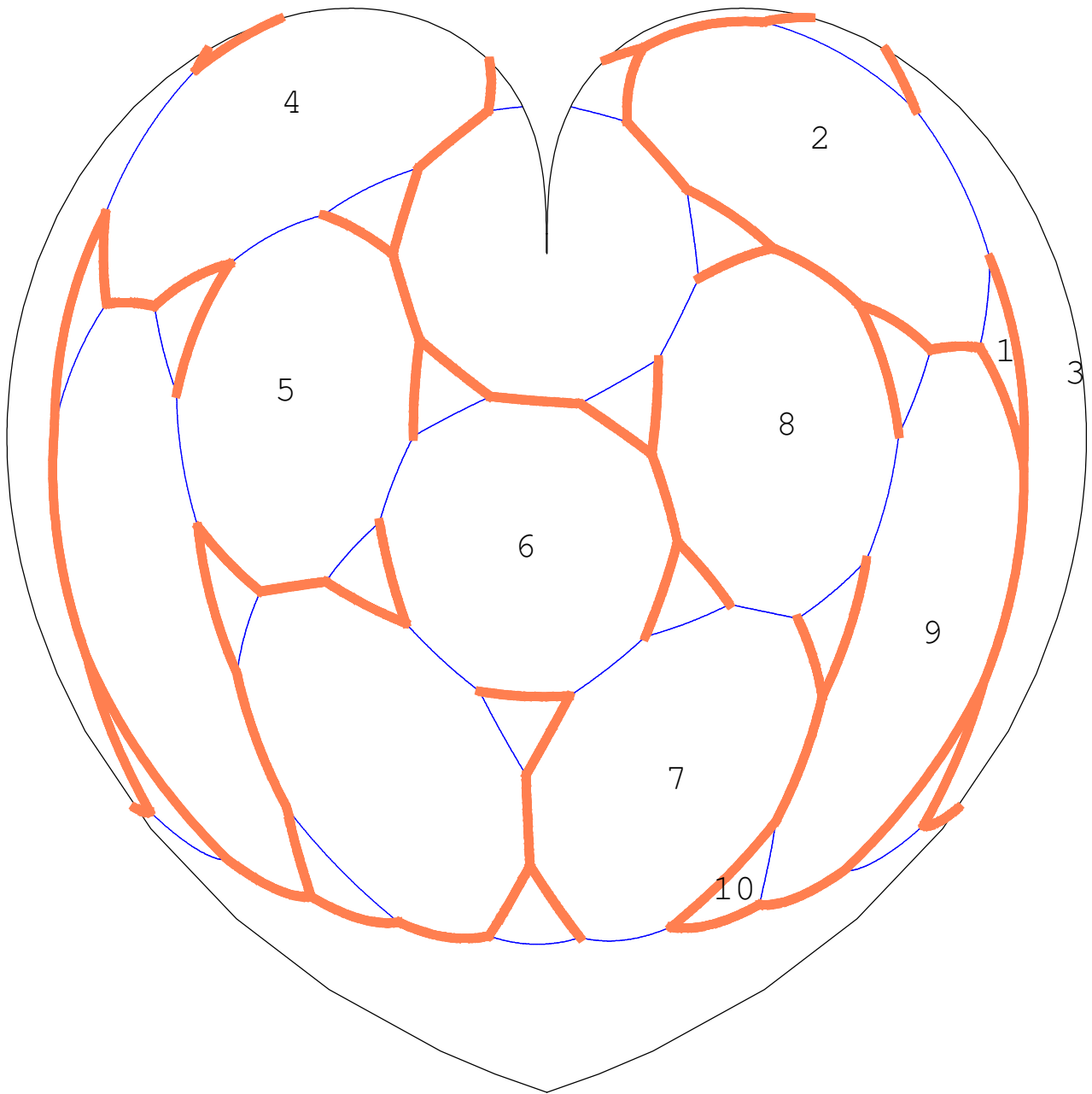


17.

truncated dodecahedron

{10, 10, 3}

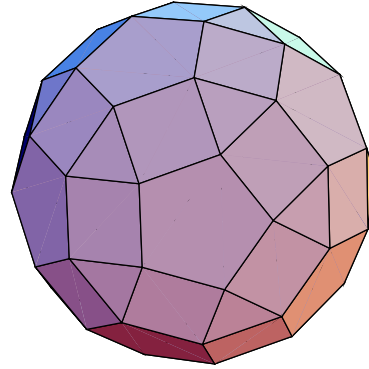
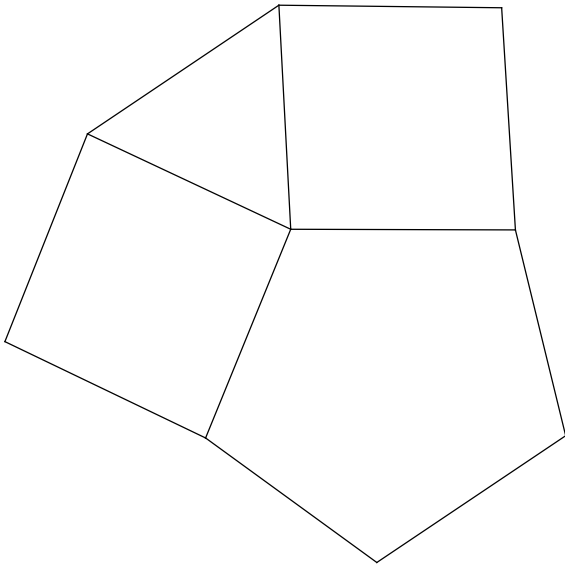


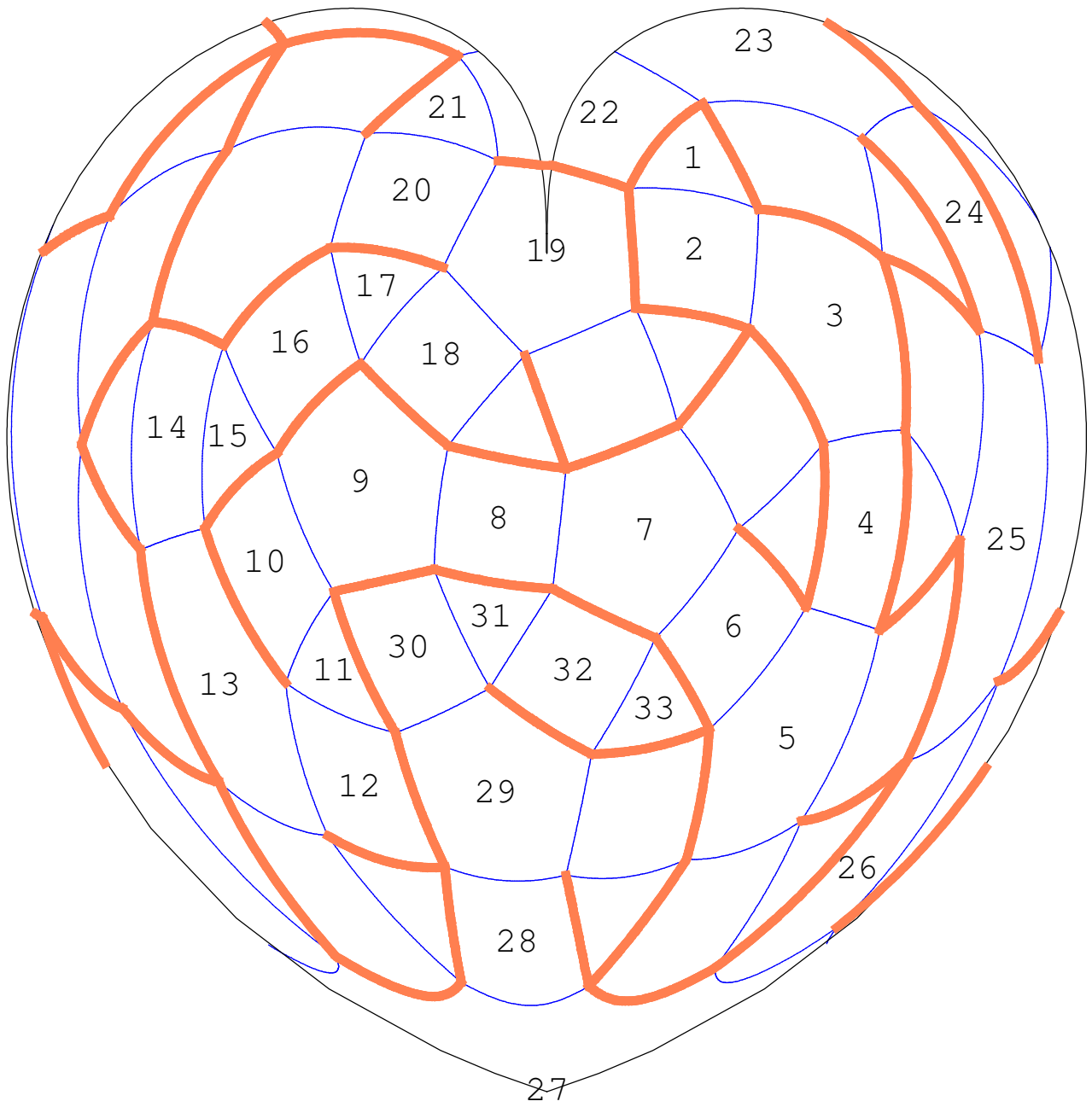


18.

rhombicosidodecahedron

{4, 3, 4, 5}

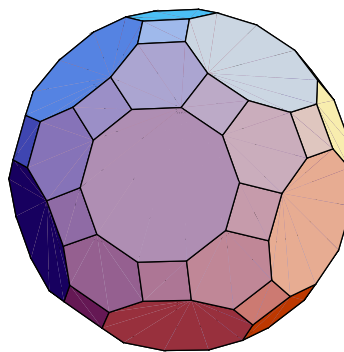
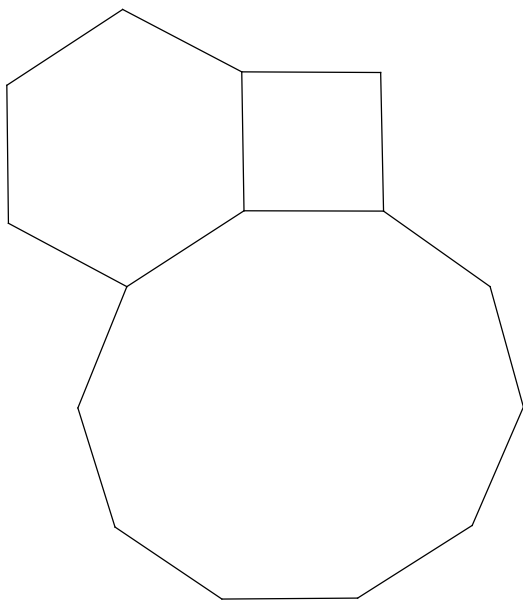


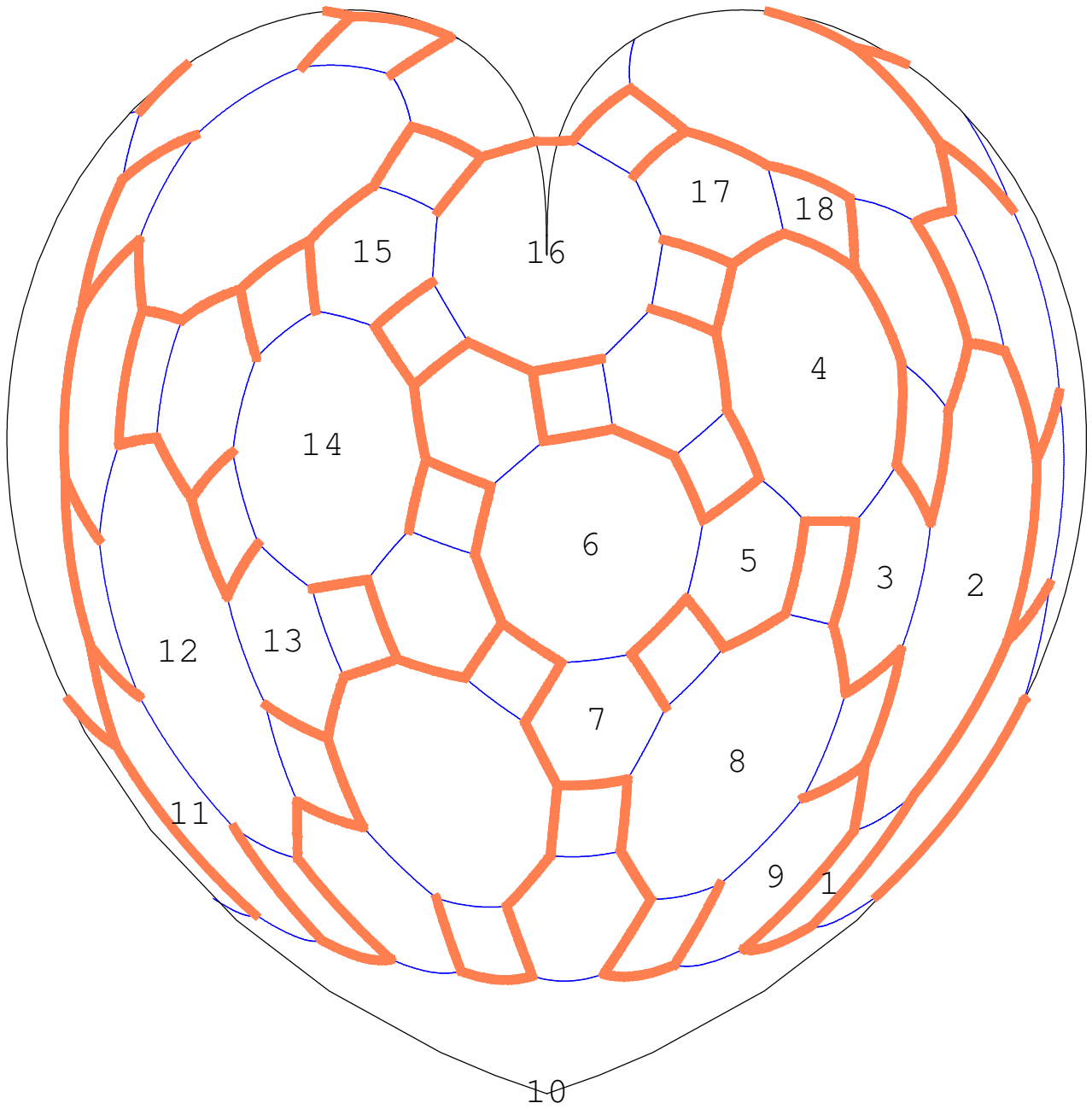


19.

truncated icosidodecahedron

{4, 6, 10}

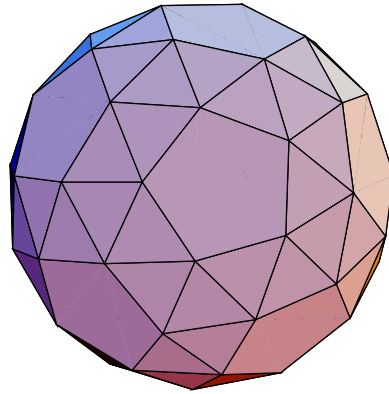
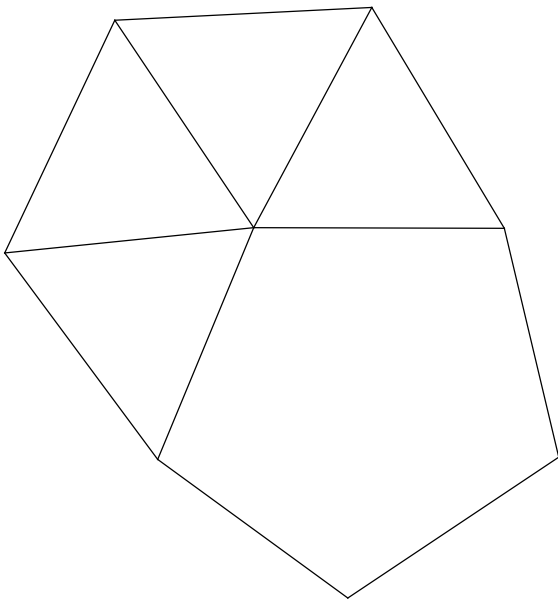


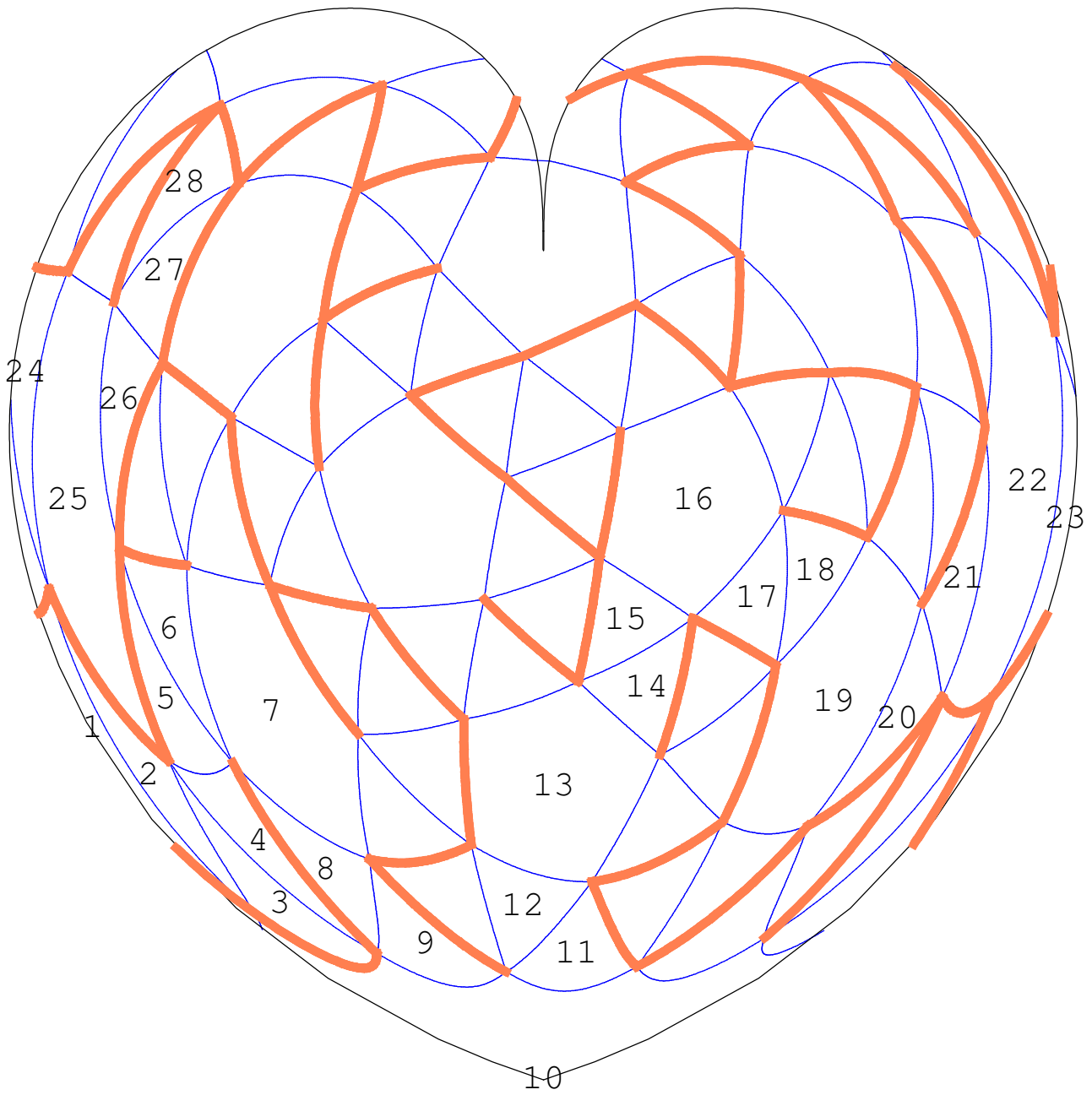


20.

snub dodecahedron

$\{3, 3, 3, 3, 5\}$

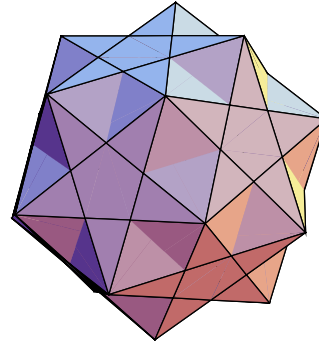
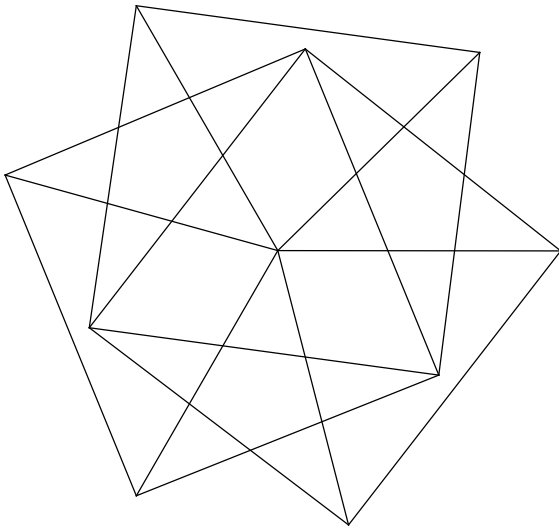


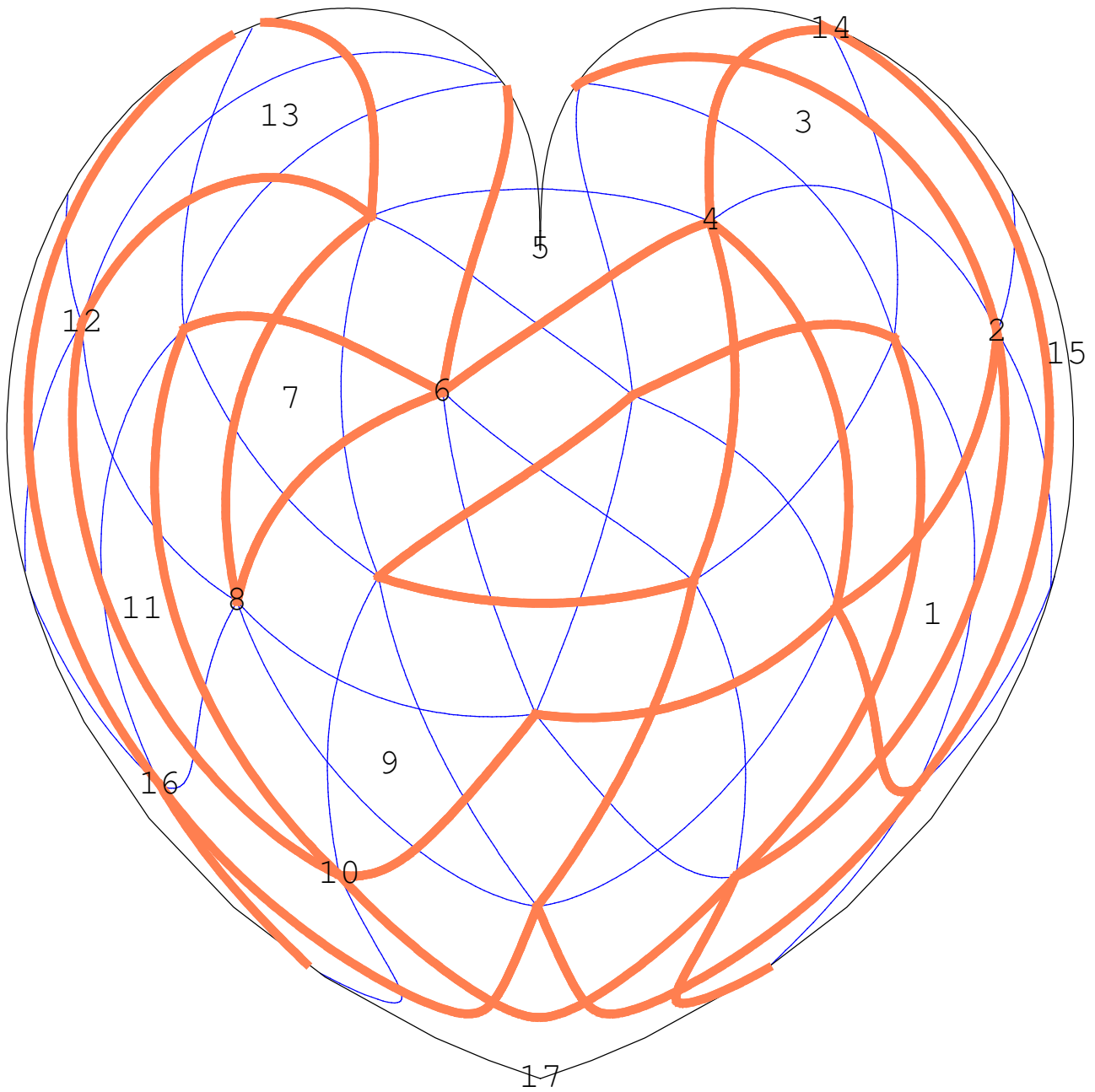


21.

small ditrigonal icosidodecahedron

$$\left\{ \frac{5}{2}, 3, \frac{5}{2}, 3, \frac{5}{2}, 3 \right\}$$

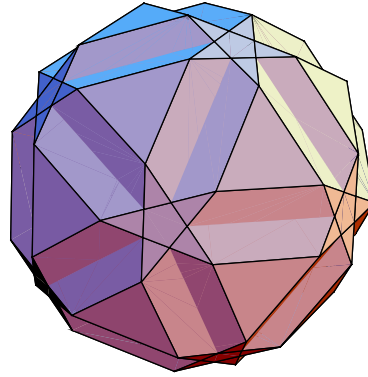
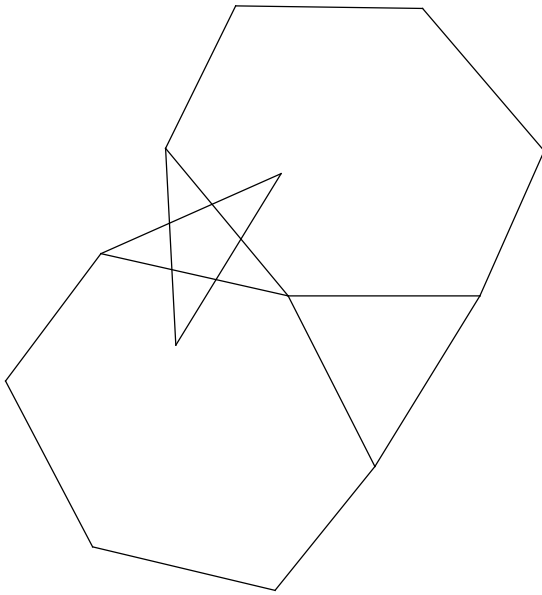


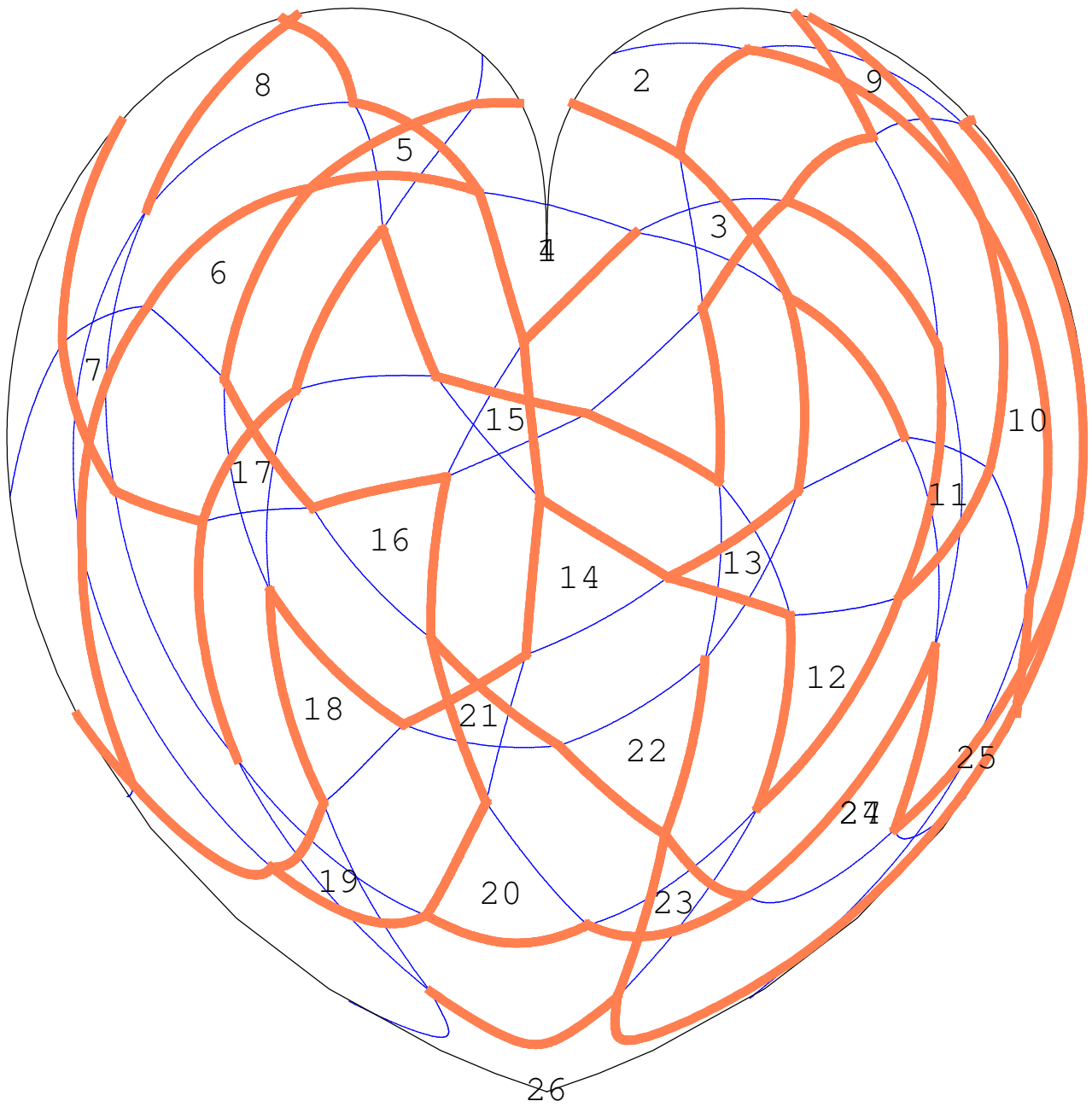


22.

small icosicosidodecahedron

$$\left\{6, \frac{5}{2}, 6, 3\right\}$$

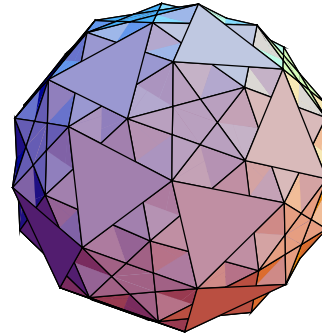
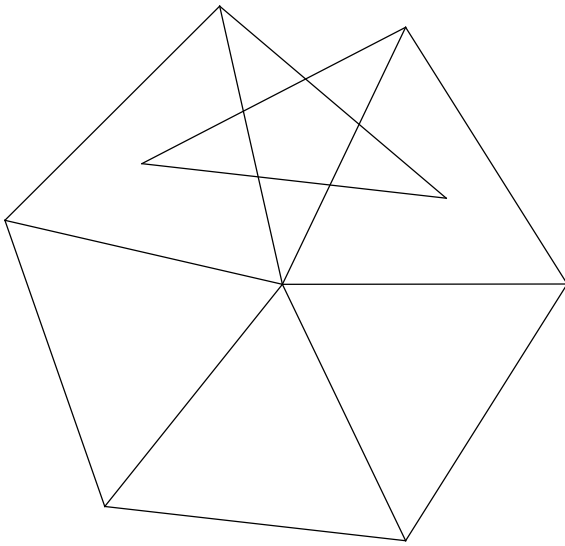


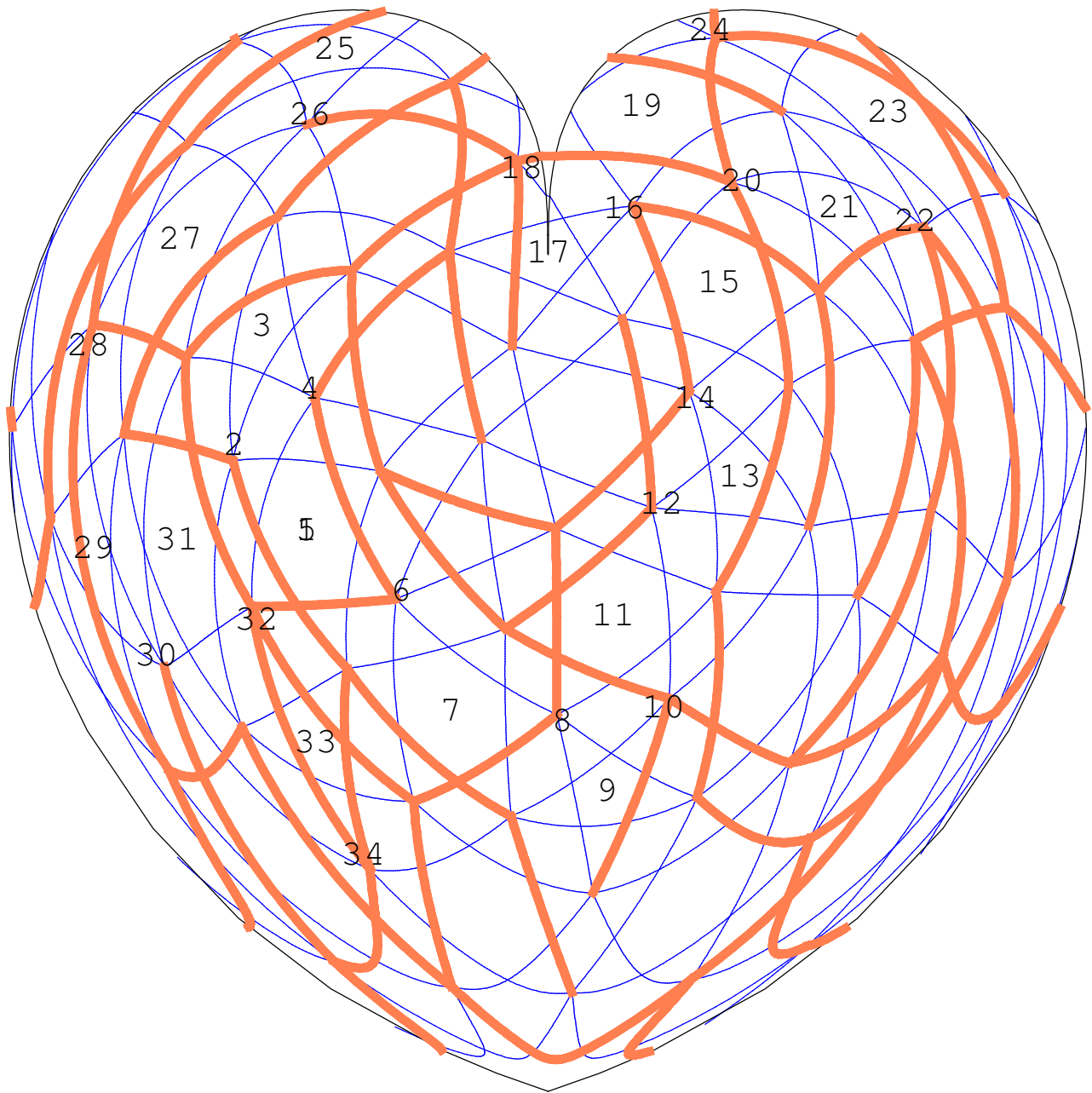


23.

small snub icosicosidodecahedron

$$\left\{3, \frac{5}{2}, 3, 3, 3, 3\right\}$$

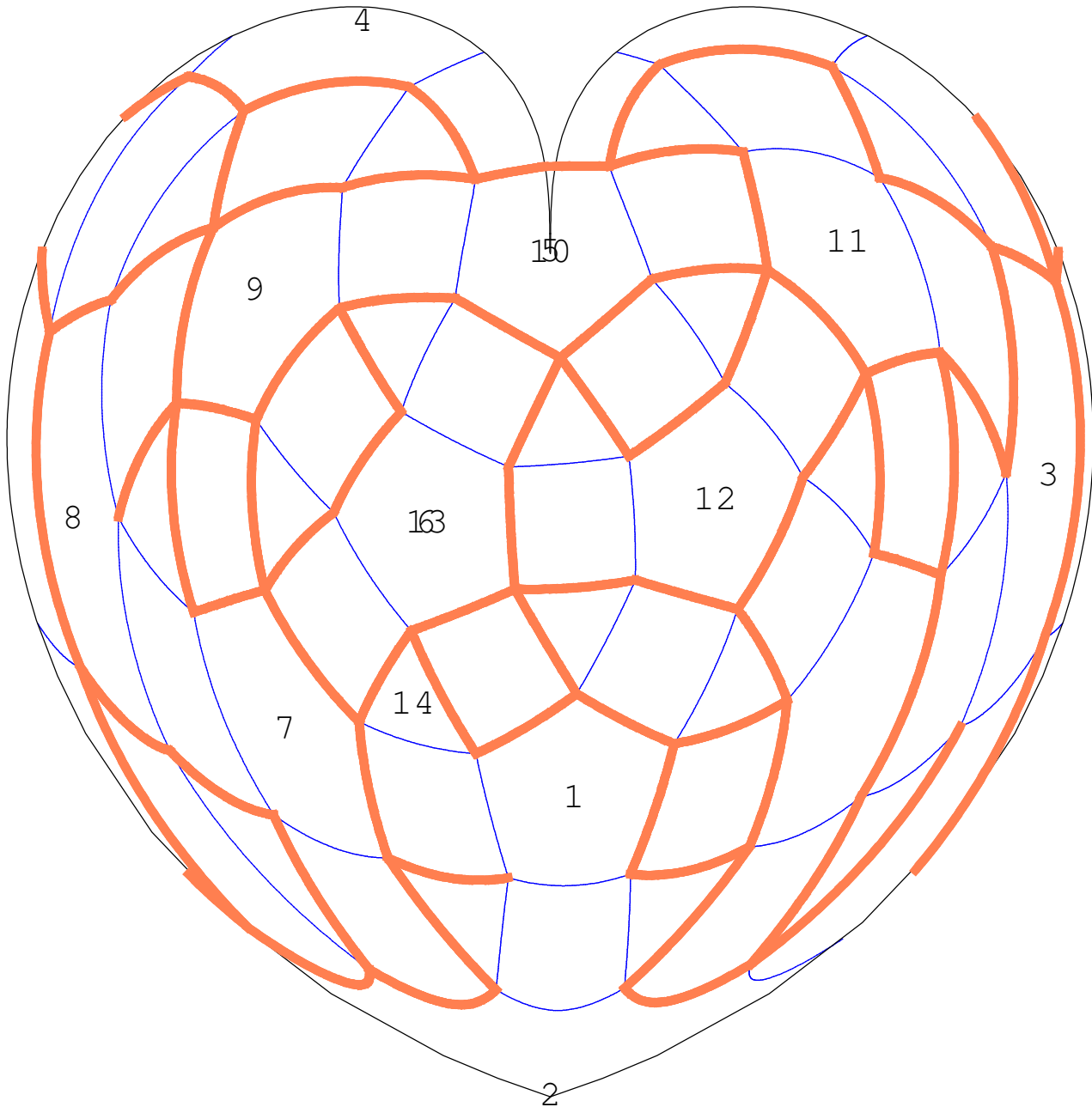
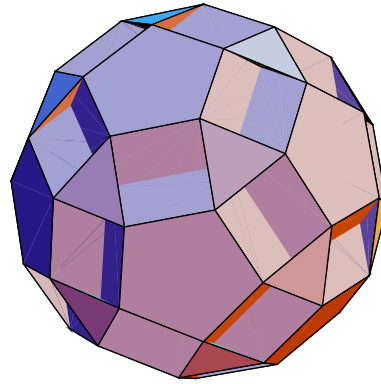
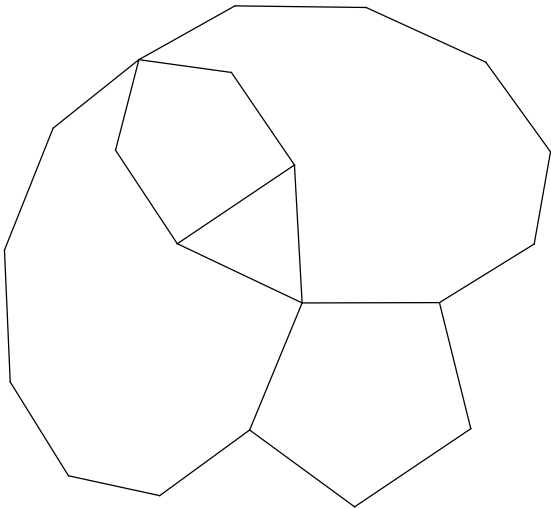




24.

small dodecicosidodecahedron

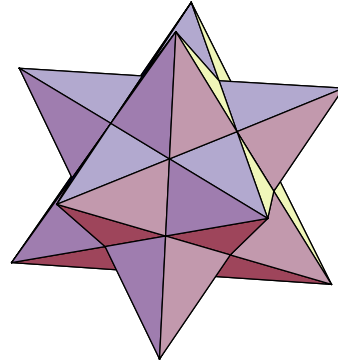
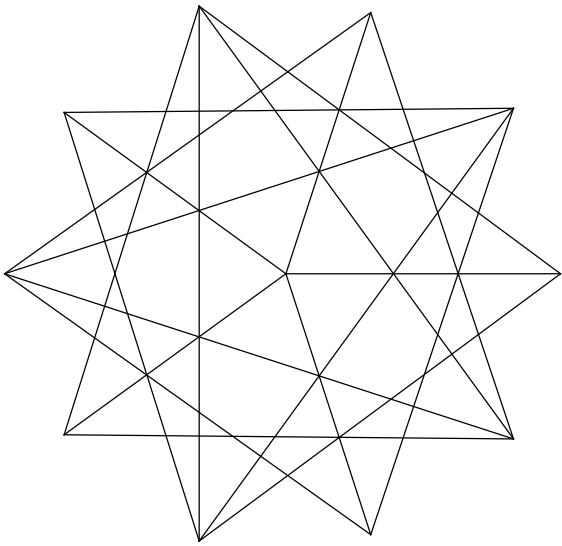
$$\left\{10, \frac{3}{2}, 10, 5\right\}$$

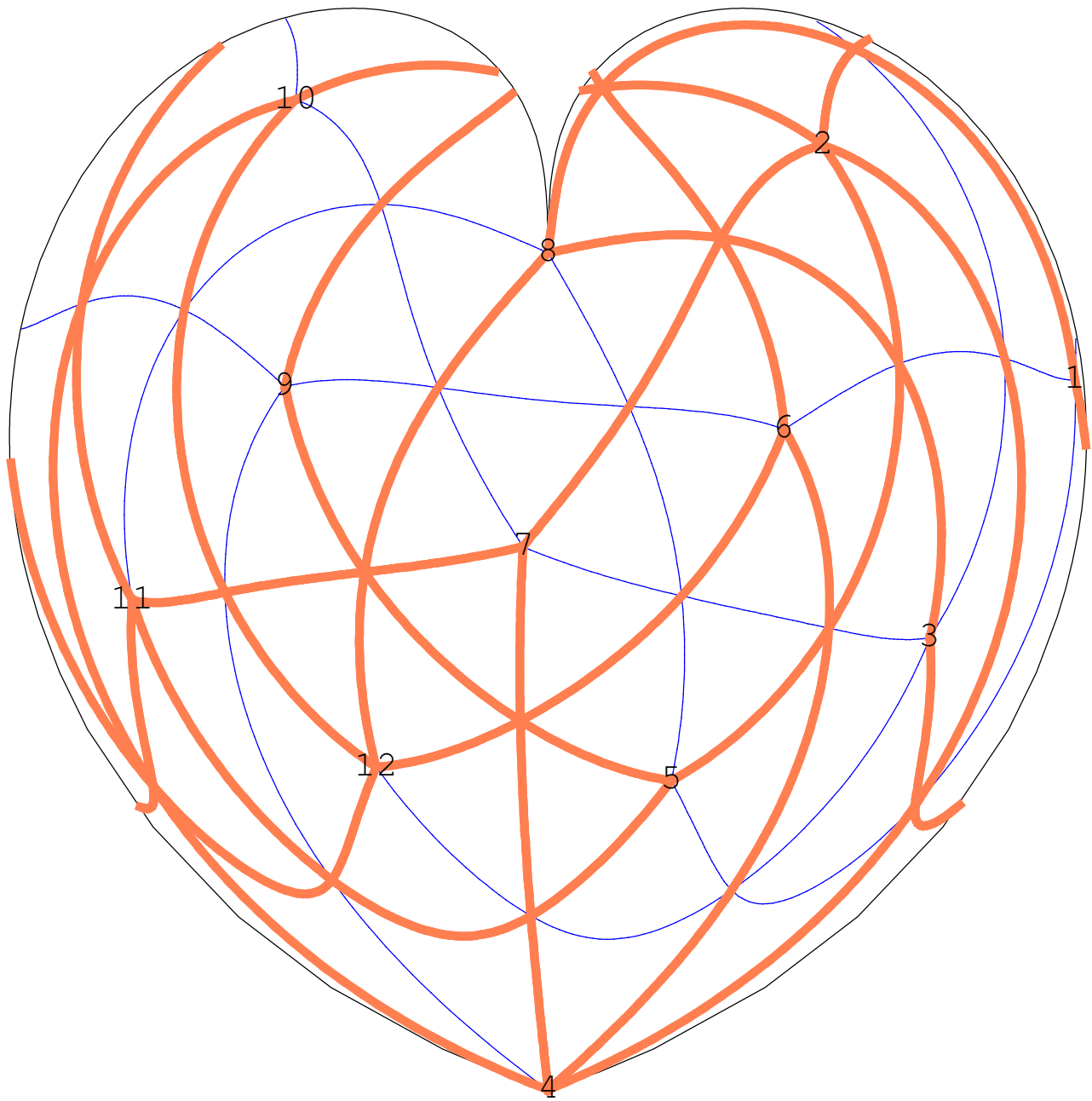


25.

small stellated dodecahedron

$$\left\{ \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2} \right\}$$

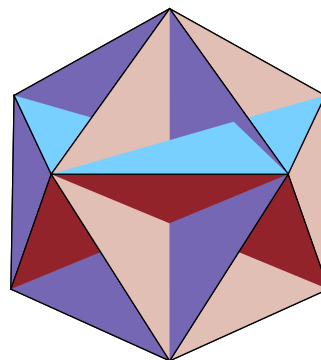
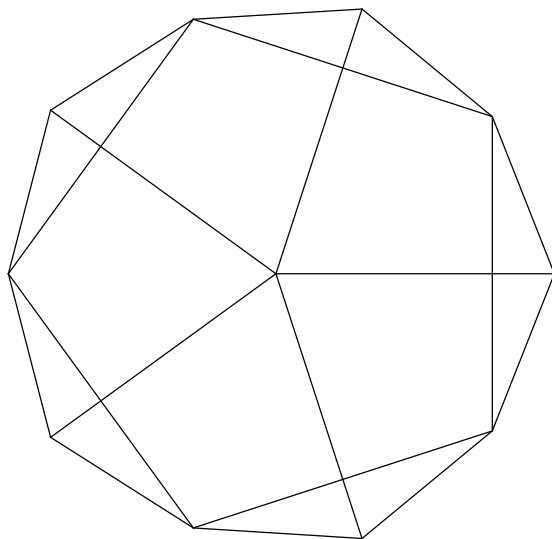


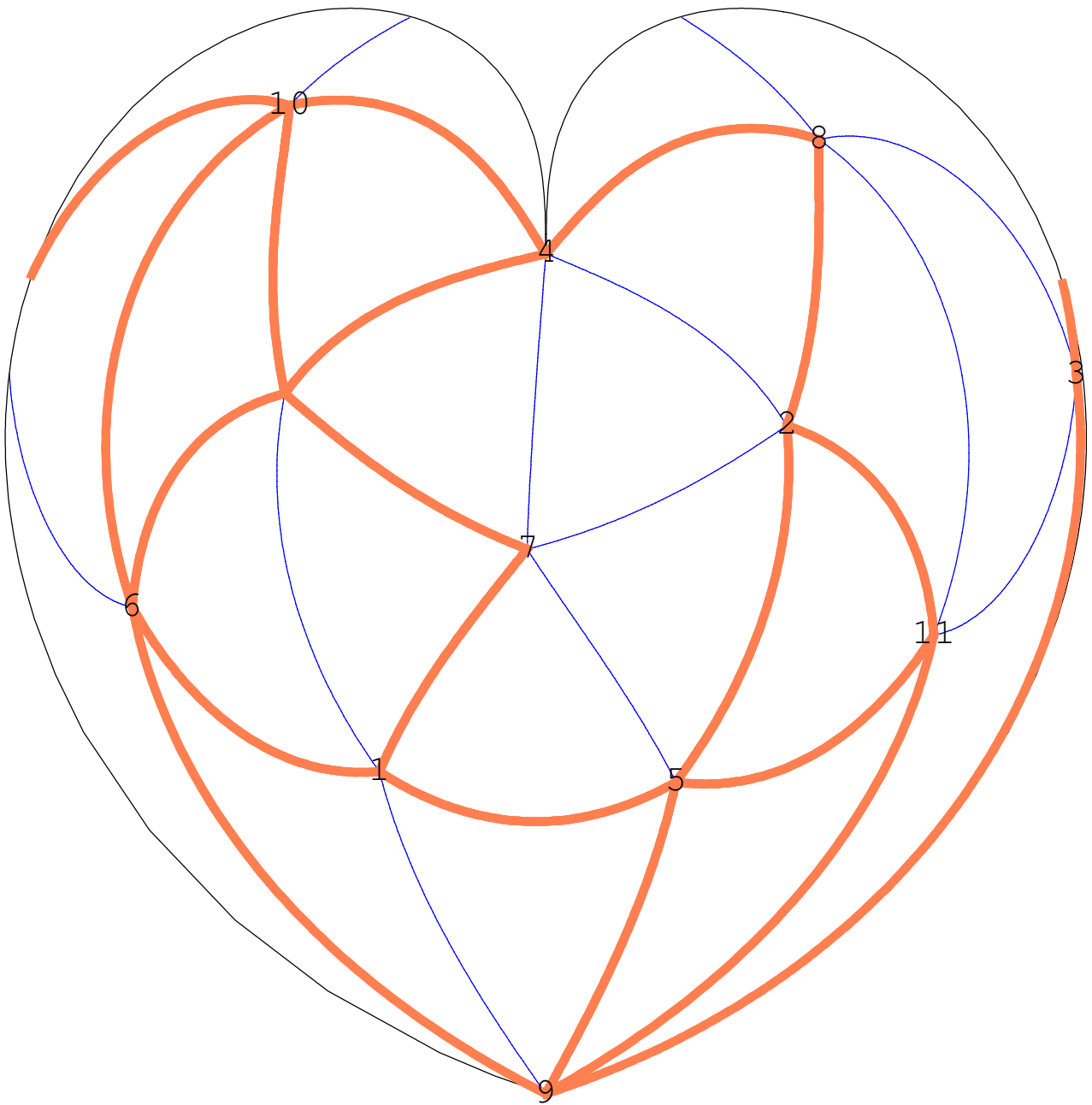


26.

great dodecahedron

$$\frac{1}{2} \{5, 5, 5, 5, 5\}$$

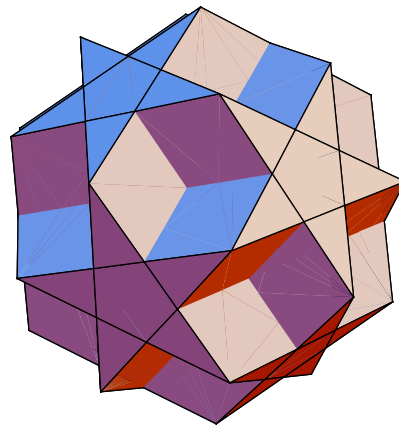
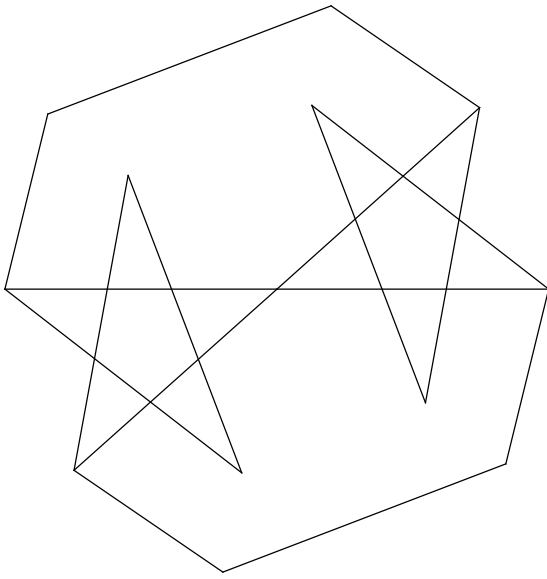


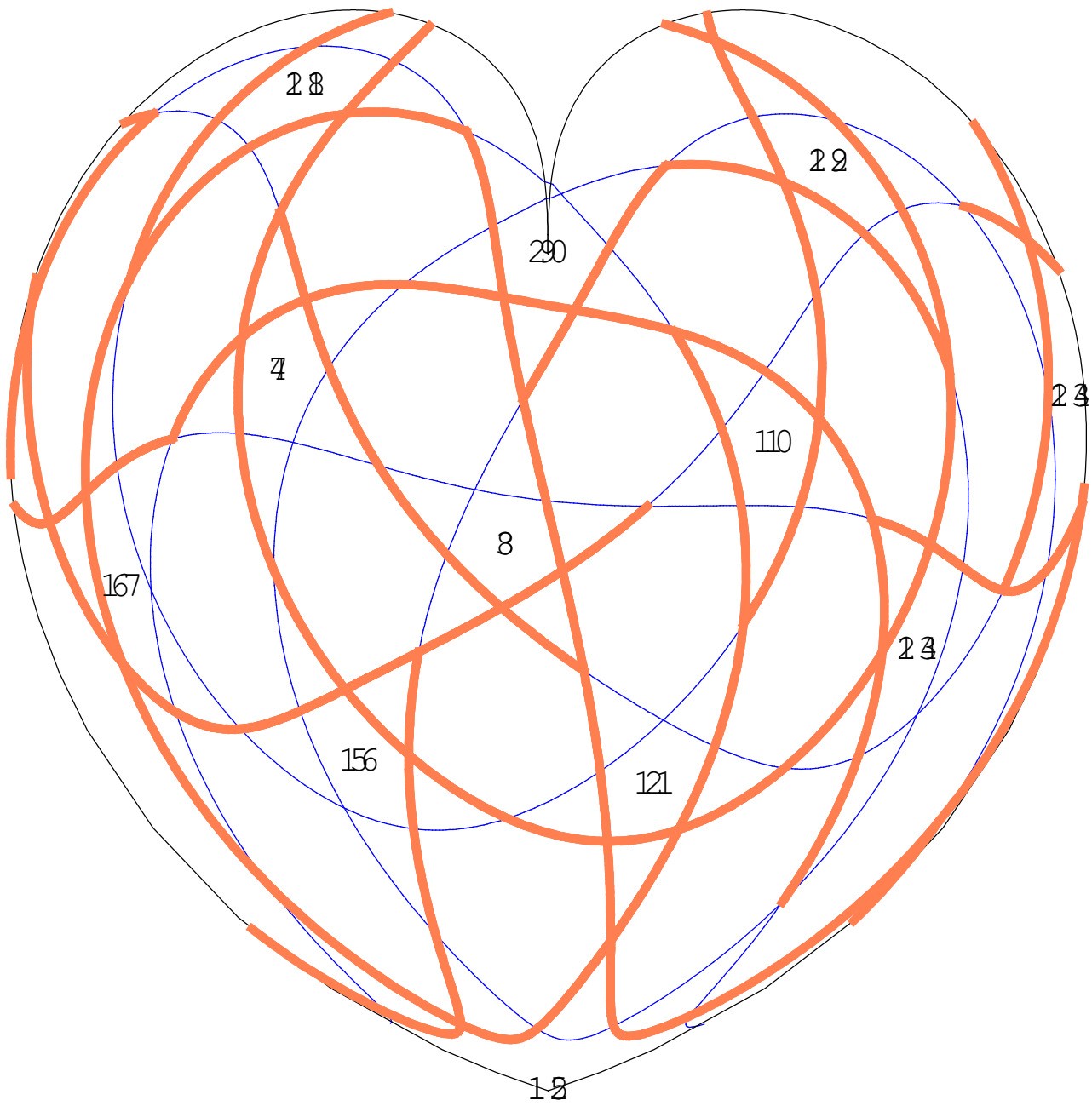


27.

dodecadodecahedron

$$\left\{\frac{5}{2}, 5, \frac{5}{2}, 5\right\}$$

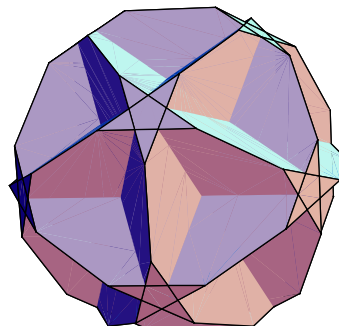
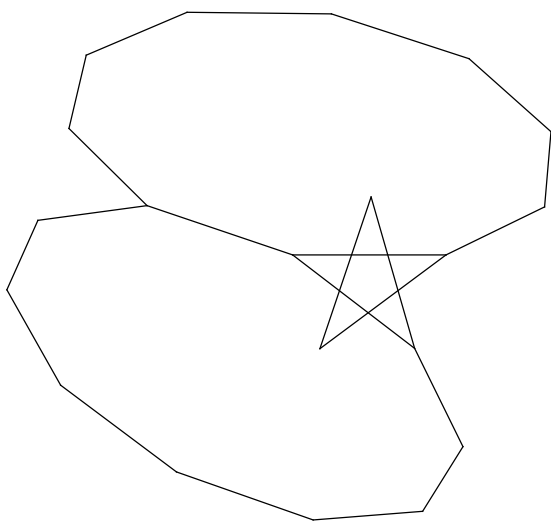


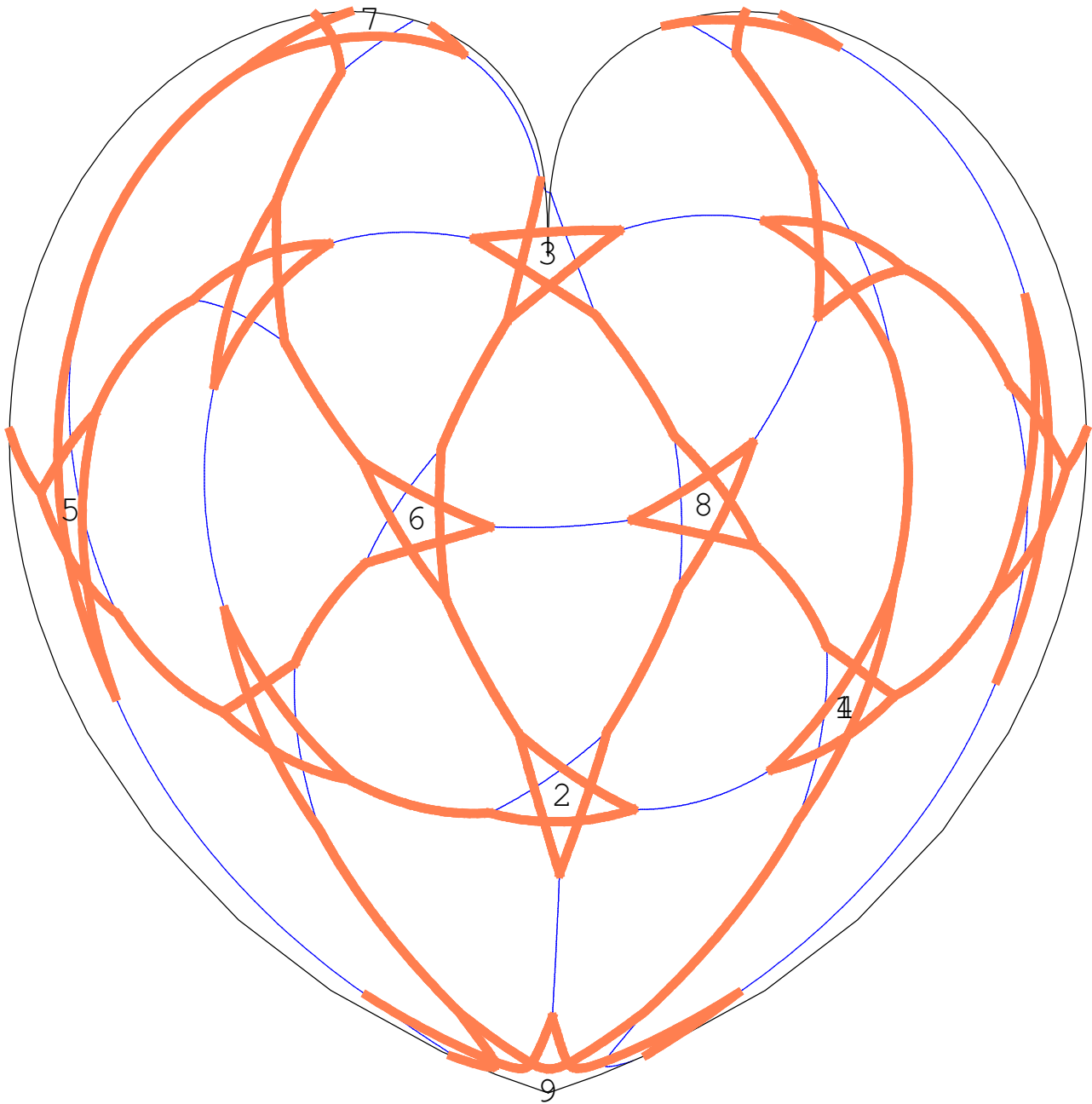


28.

truncated great dodecahedron

$$\{10, 10, \frac{5}{2}\}$$

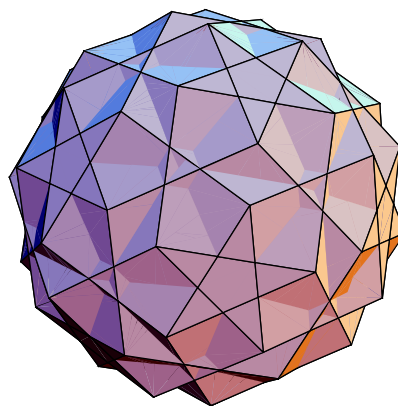
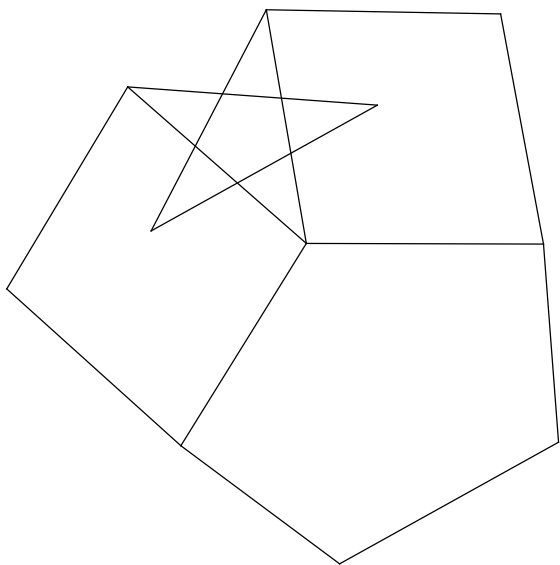


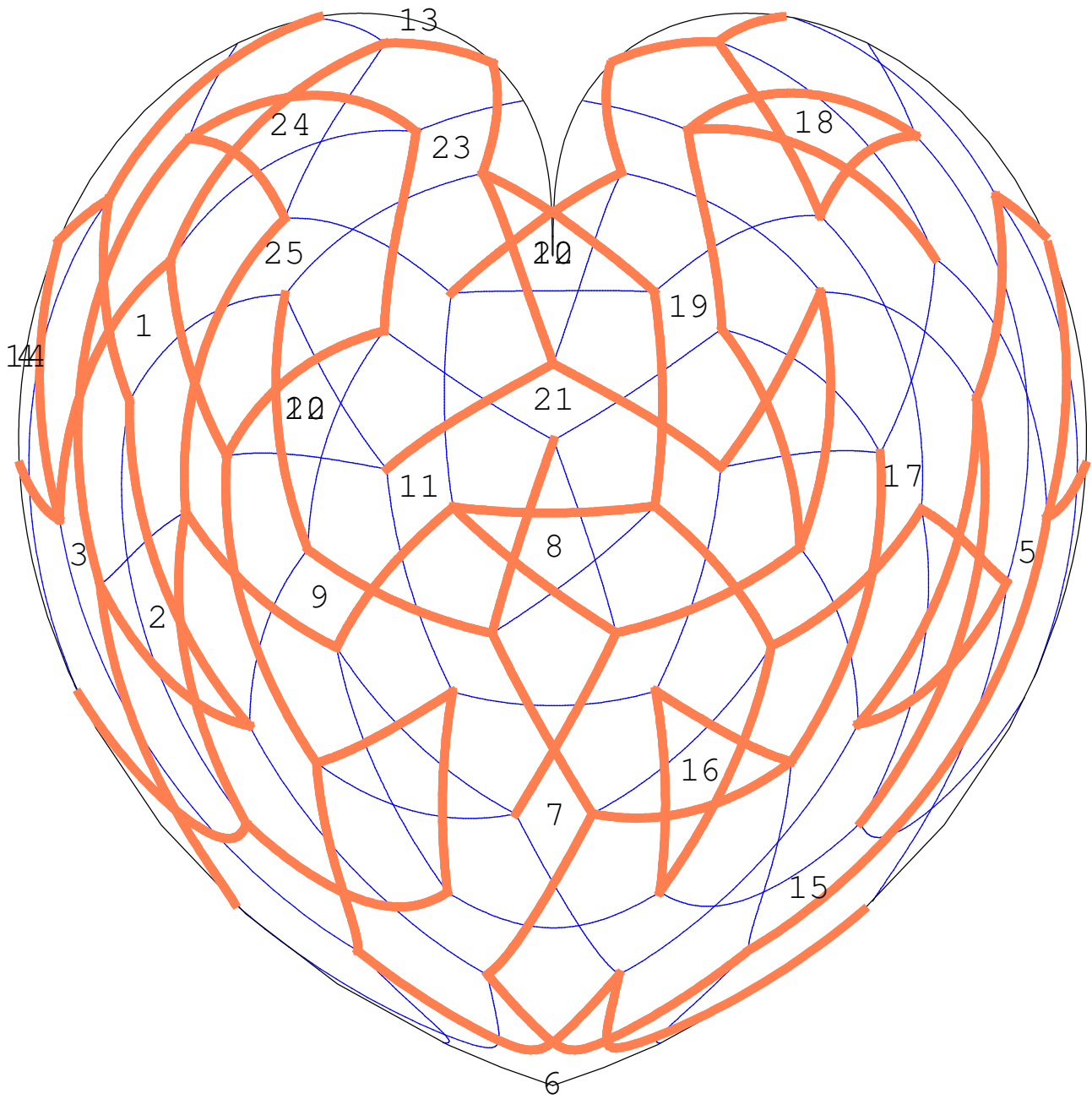


29.

rhombidodecadodecahedron

$$\left\{4, \frac{5}{2}, 4, 5\right\}$$





30.

small rhombidodecahedron

$$\left\{10, 4, \frac{10}{9}, \frac{4}{3}\right\}$$

