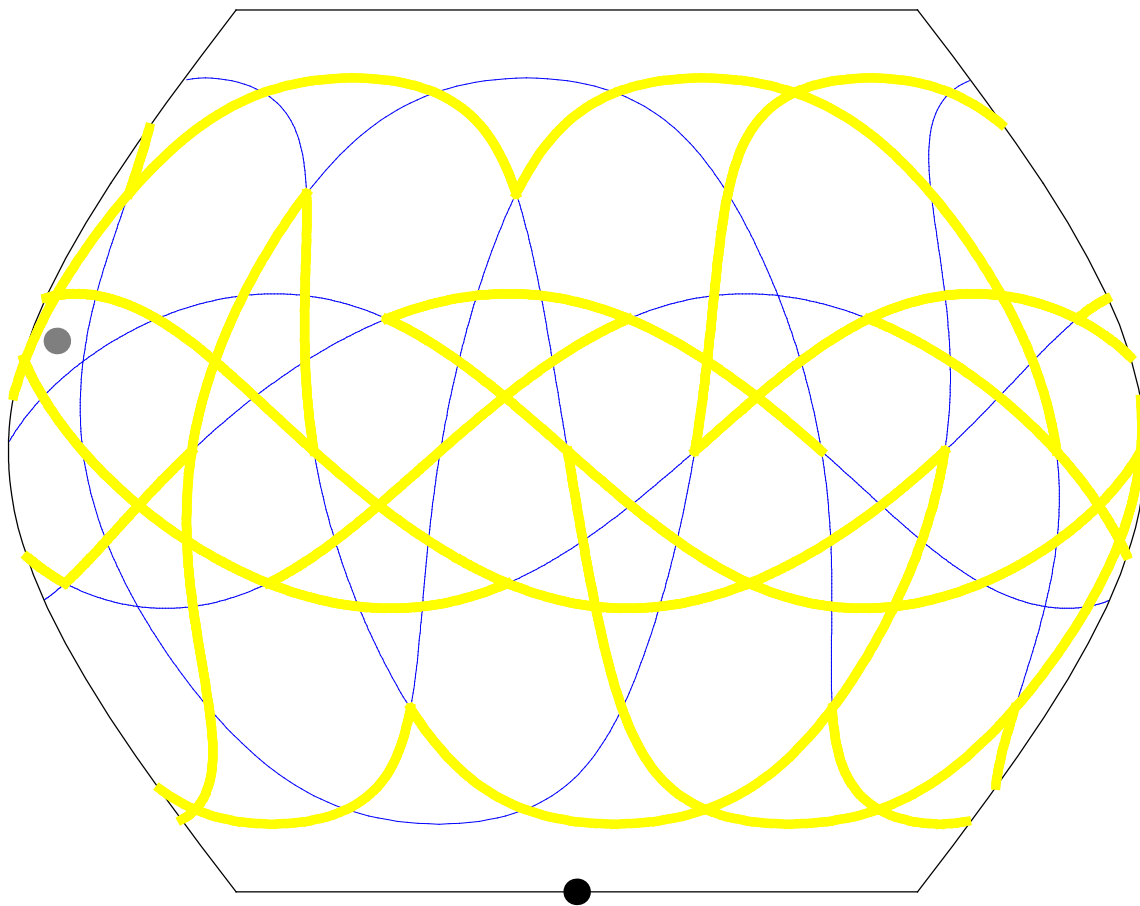


Izidor Hafner

# Mazes on Uniform Polyhedra

Ginzburg projection

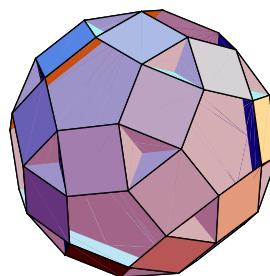
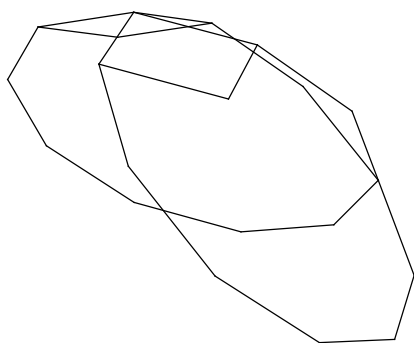


# Introduction

Let us take an example. We are given a uniform polyhedron.

small rhombidodecahedron

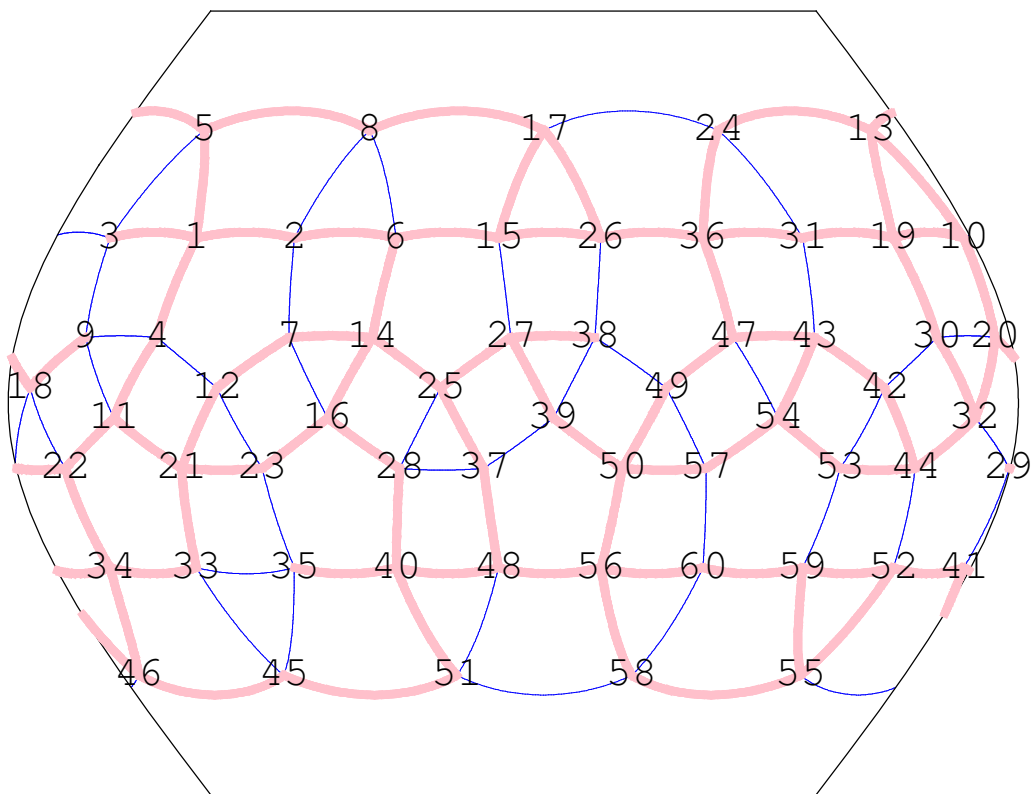
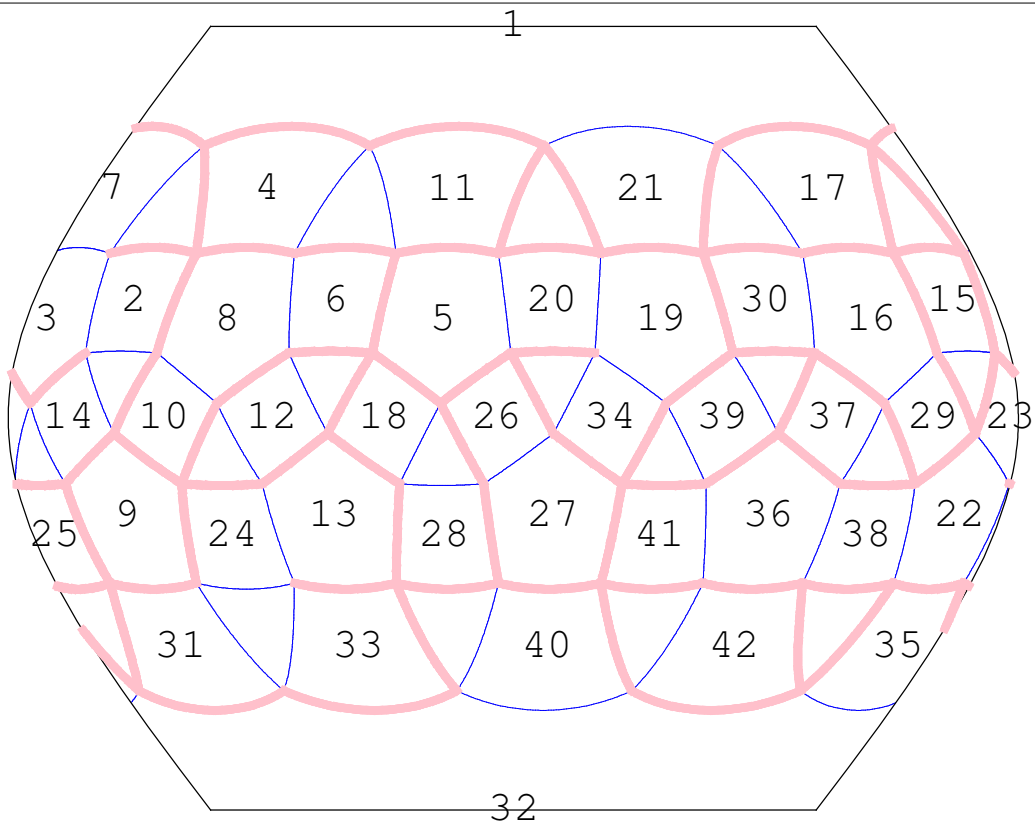
$$\left\{10, 4, \frac{10}{9}, \frac{4}{3}\right\}$$



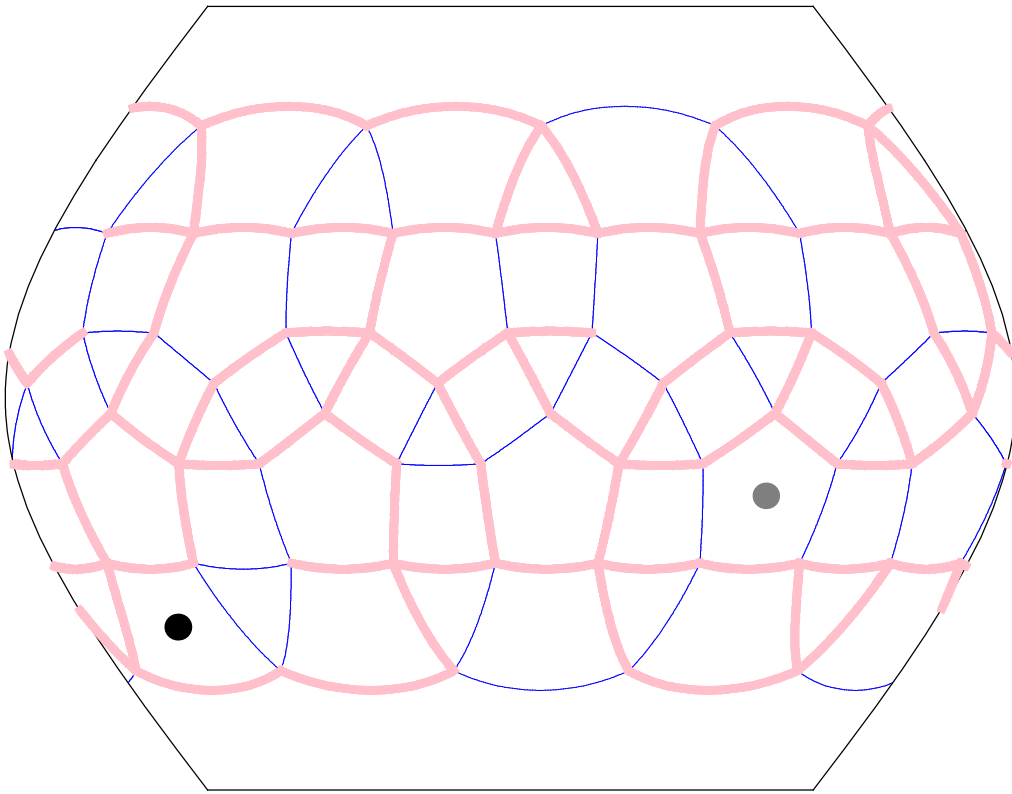
In Mathematica the polyhedron is given by a list of faces and with a list of coordinates of vertices [Roman E. Maeder, The Mathematica Programmer II, Academic Press 1996]. The list of faces consists of a list of lists, where a face is represented by a list of vertices, which is given by a matrix. Let us show the first five faces:

```
{1, 2, 6, 15, 26, 36, 31, 19, 10, 3}
  {1, 3, 9, 4}
{1, 4, 11, 22, 29, 32, 30, 19, 13, 5}
  {1, 5, 8, 2}
{2, 8, 17, 26, 38, 39, 37, 28, 16, 7}
```

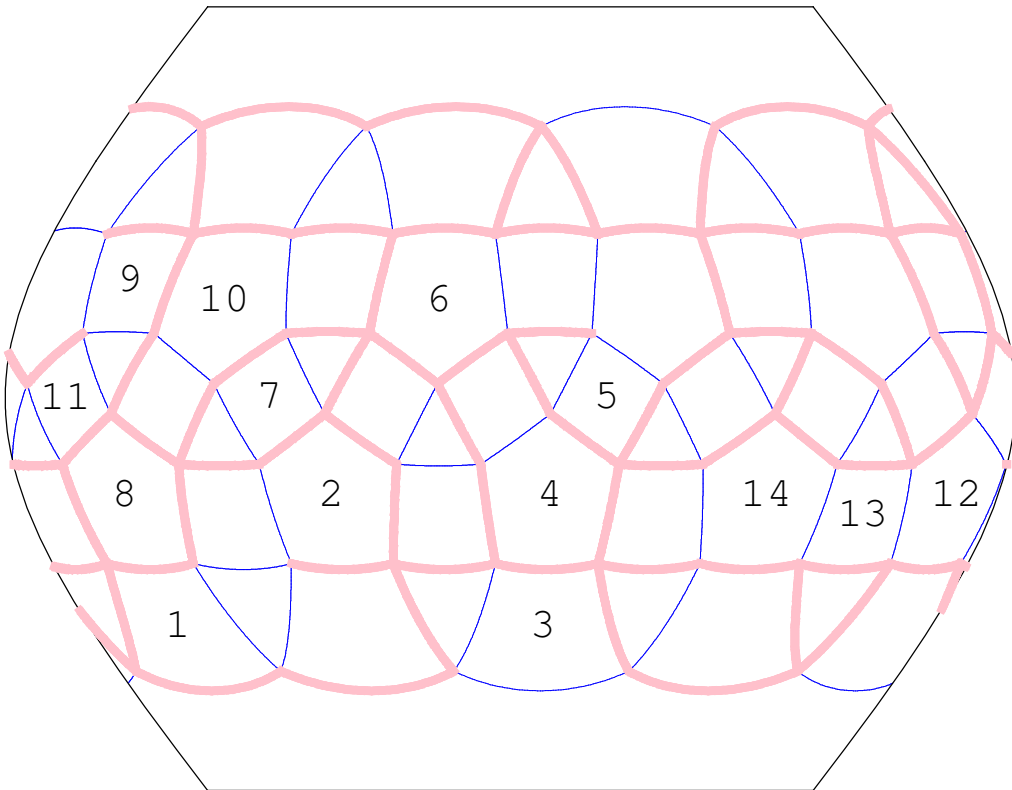
The next two figures represent faces and vertices. The polyhedron is projected onto a sphere and the sphere is projected by a cartographic projection.



The problem is to find the path from the black dot to gray dot, where thick lines represent walls of a maze.



The solution is given by a list of faces passed from the black to gray dot.

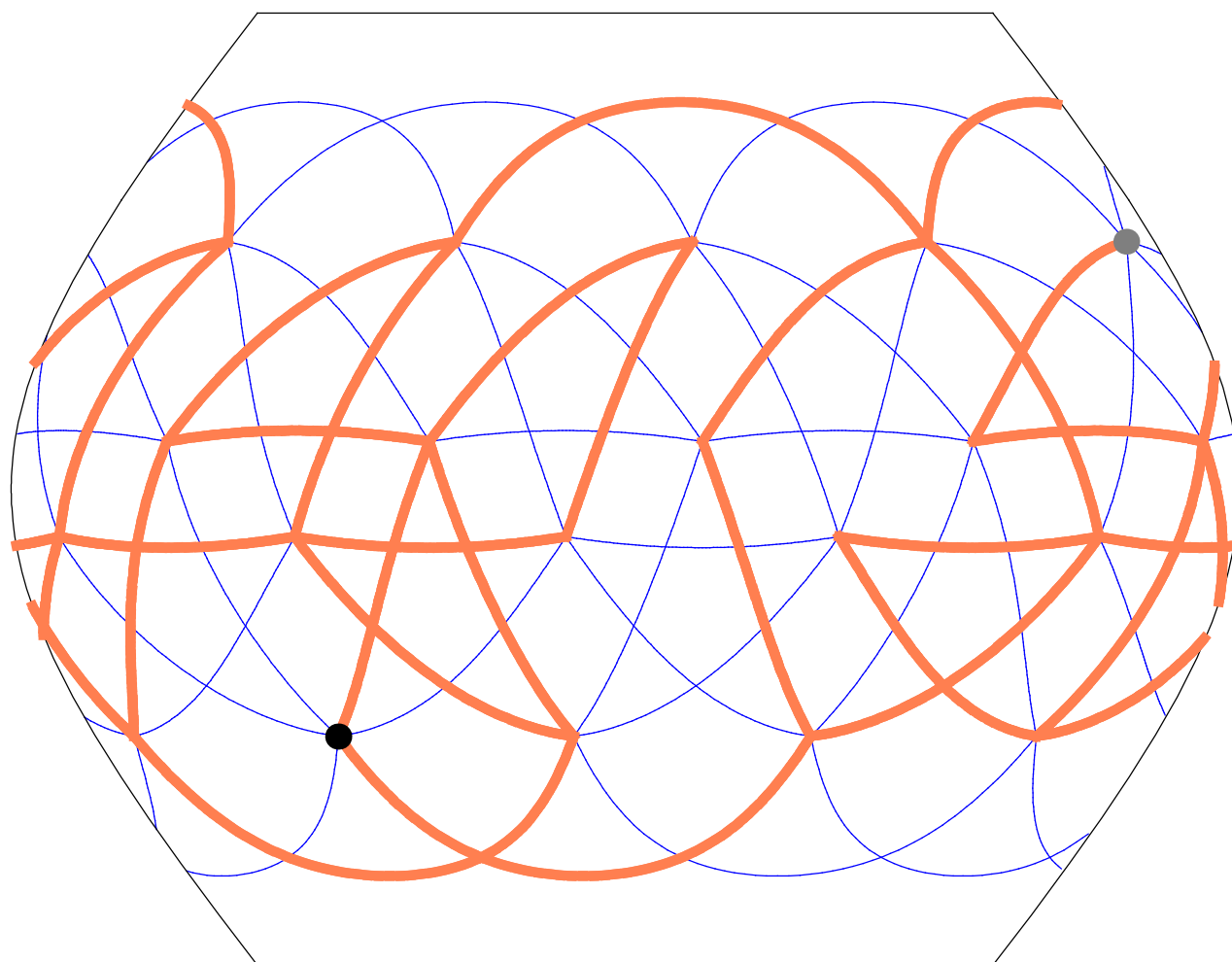
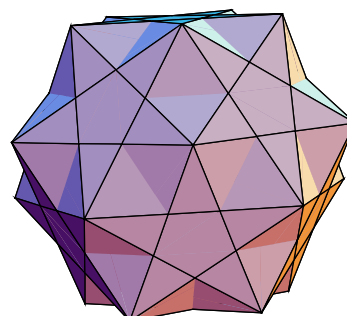
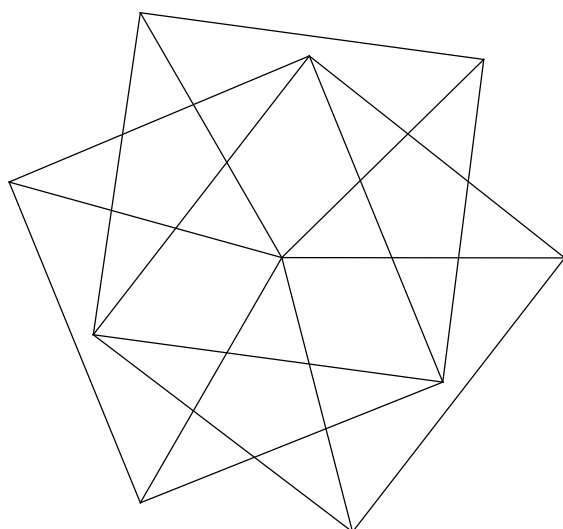


# Problems

1.

small ditrigonal icosidodecahedron

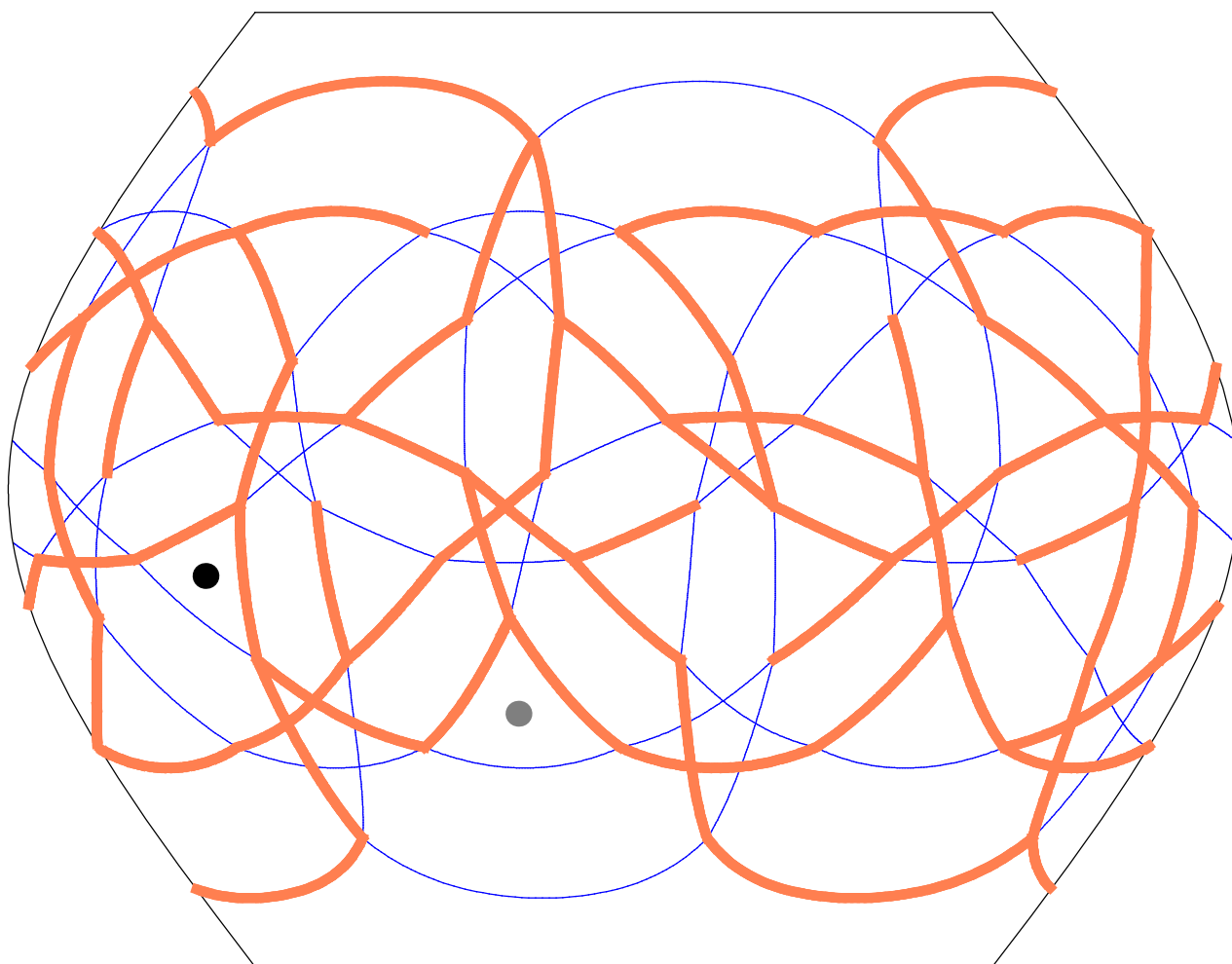
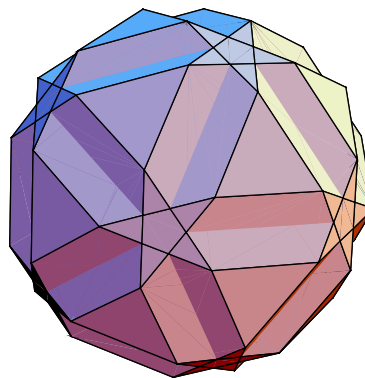
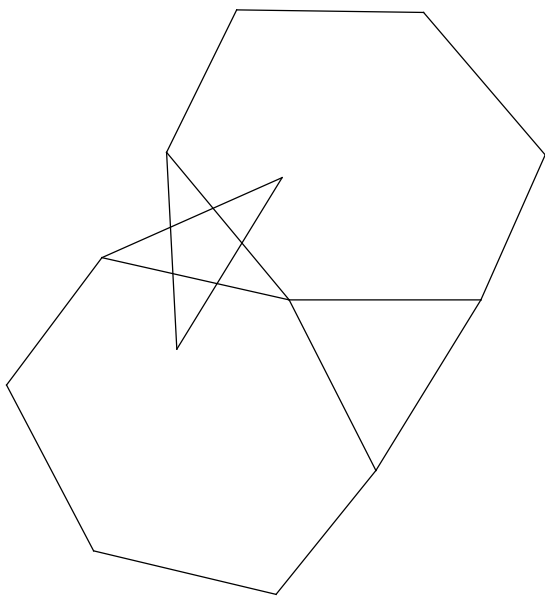
$$\left\{ \frac{5}{2}, 3, \frac{5}{2}, 3, \frac{5}{2}, 3 \right\}$$



2.

small icosicosidodecahedron

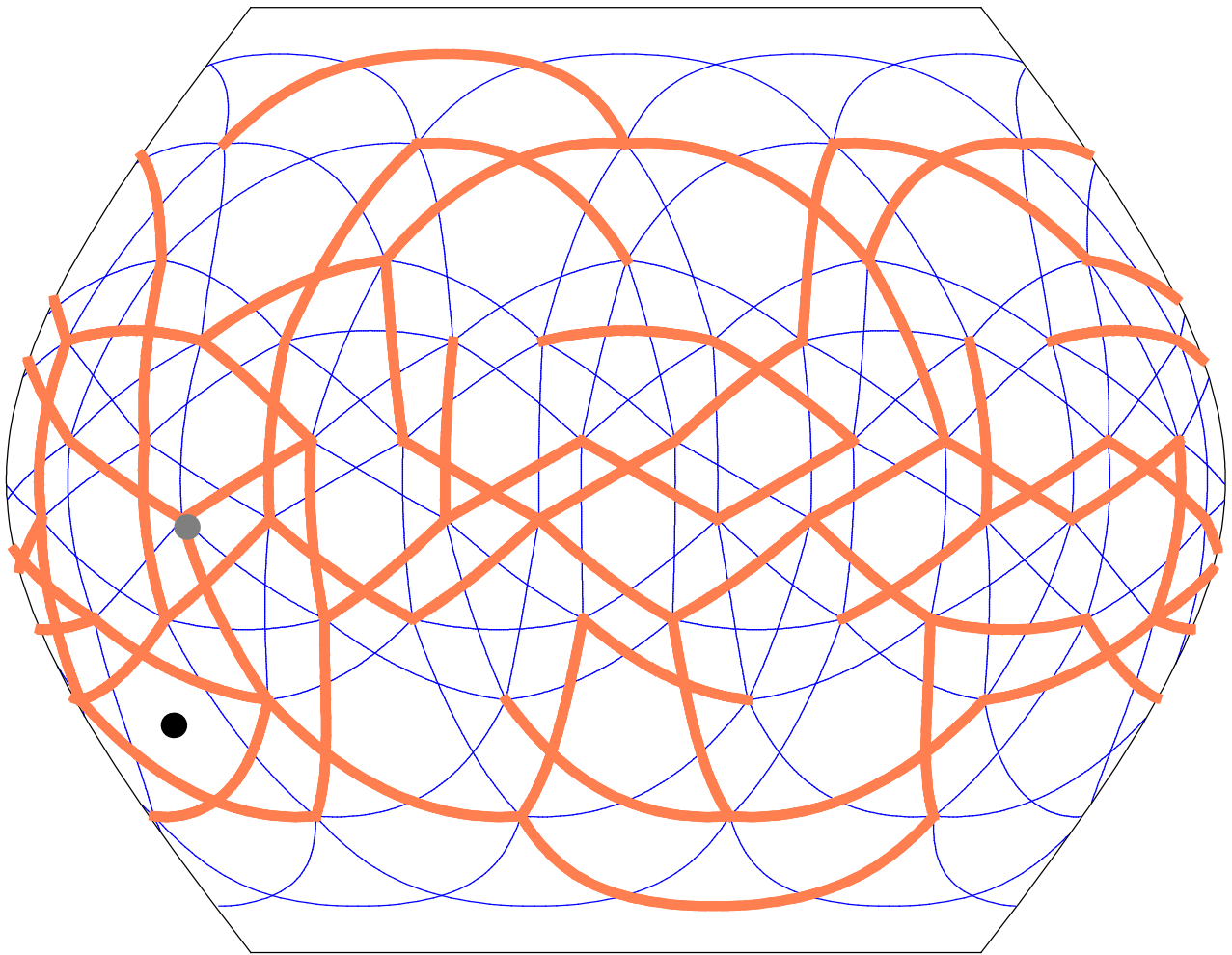
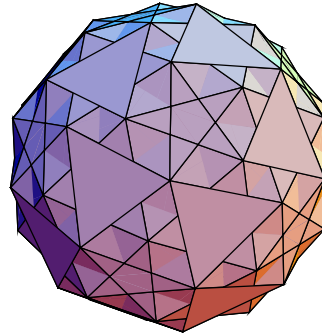
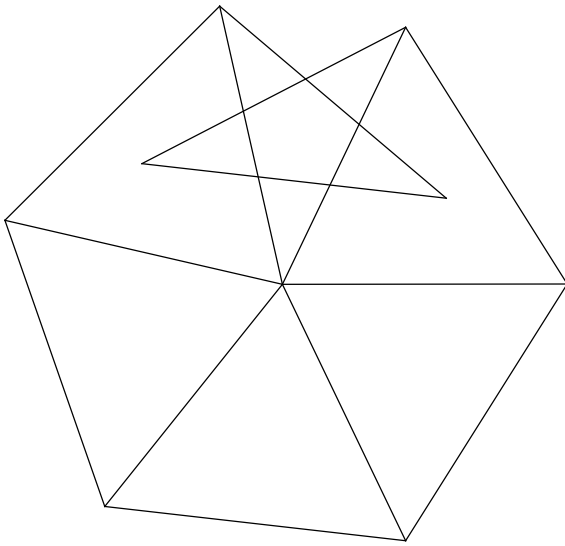
$$\left\{6, \frac{5}{2}, 6, 3\right\}$$



3.

small snub icosicosidodecahedron

$$\left\{3, \frac{5}{2}, 3, 3, 3, 3\right\}$$

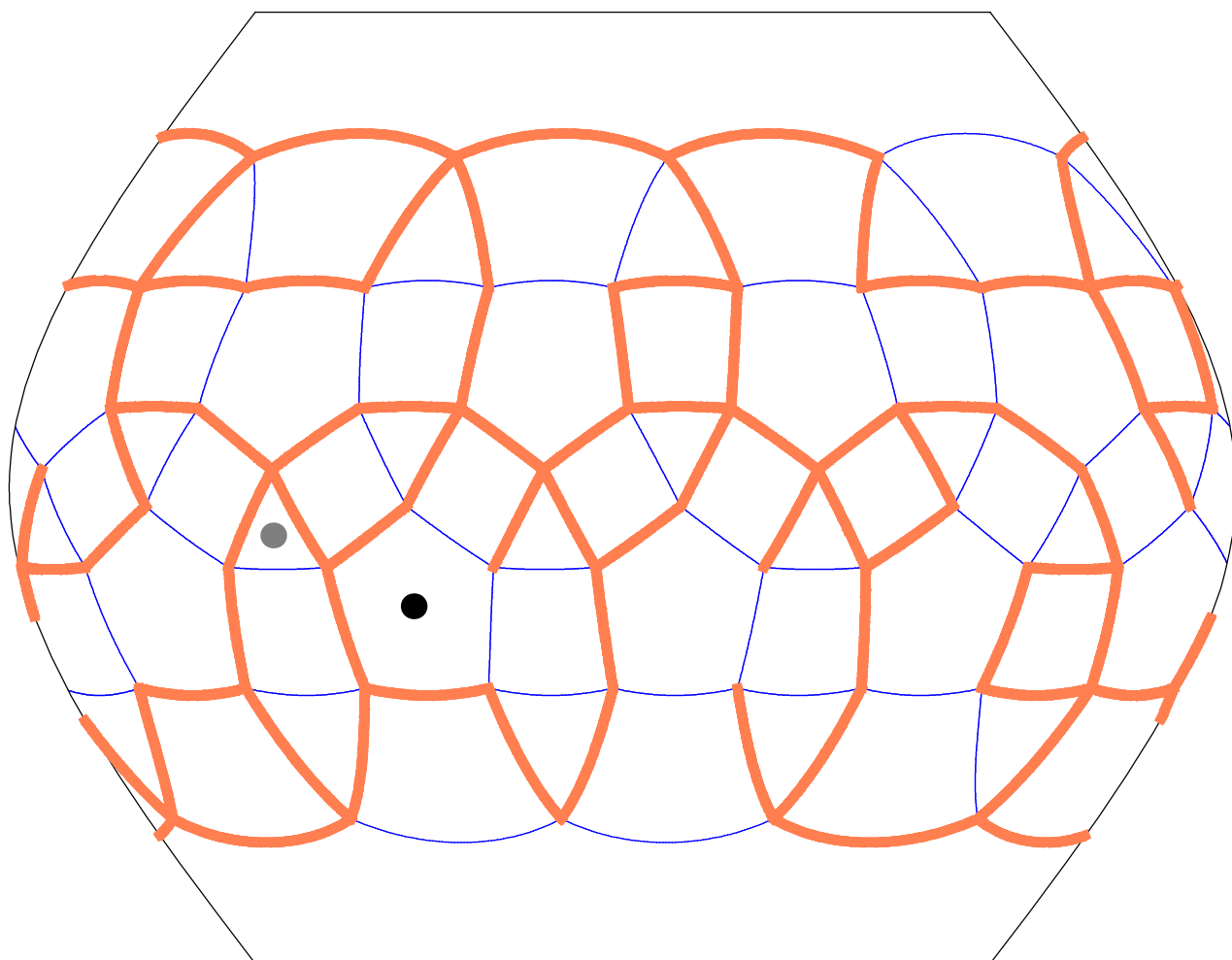
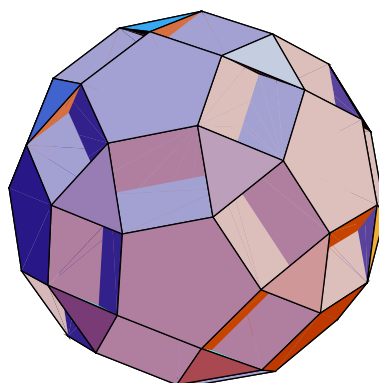
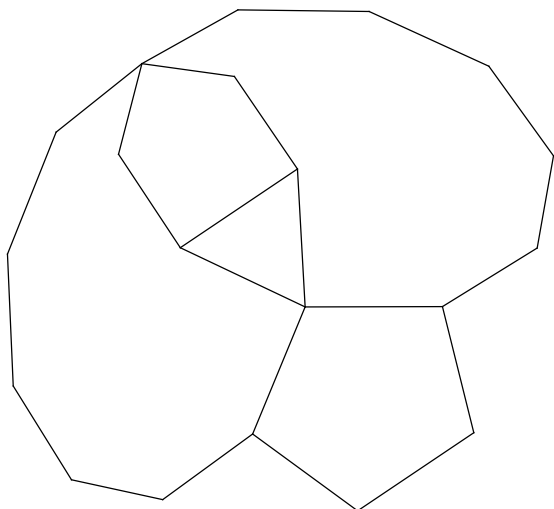




4.

small dodecicosidodecahedron

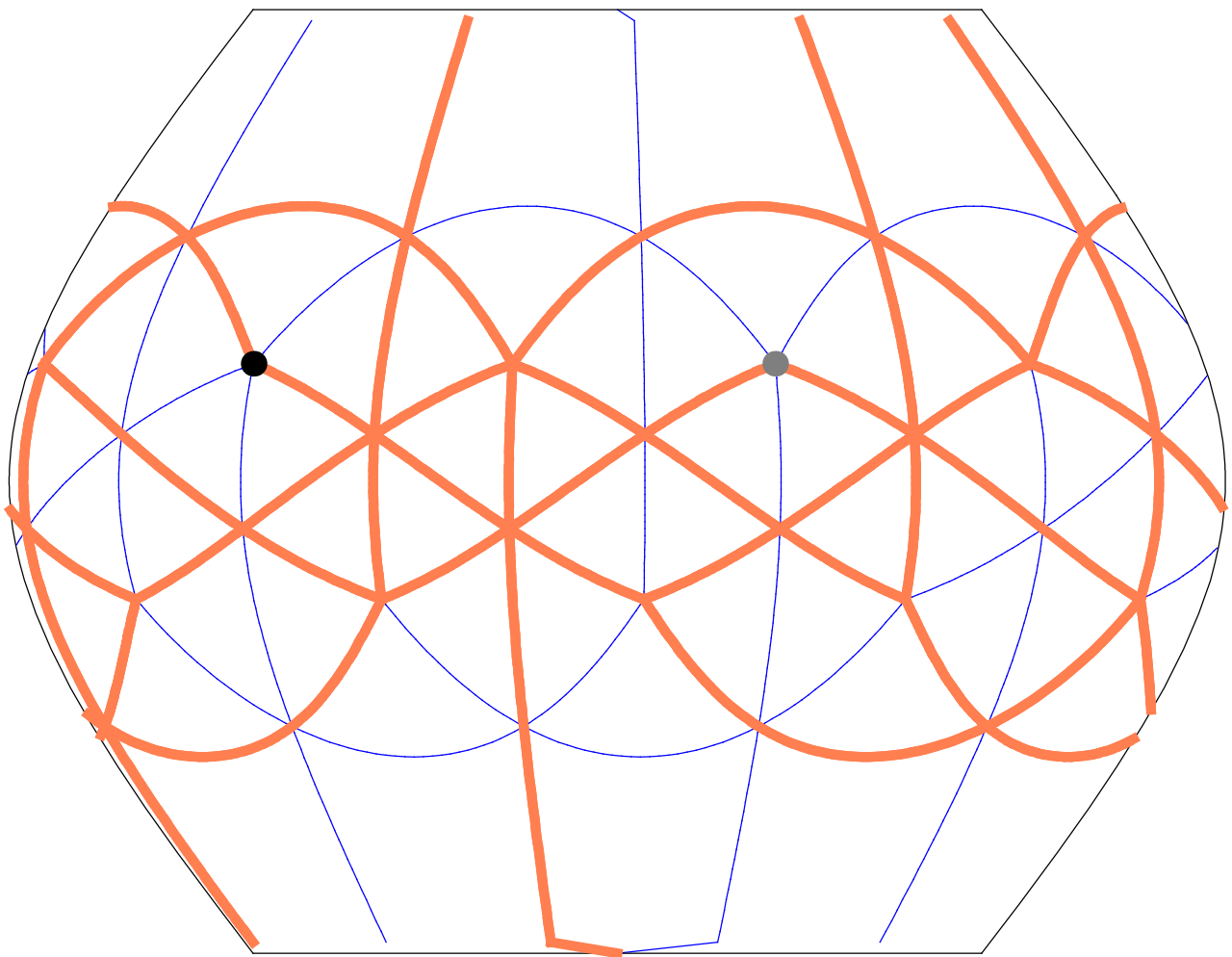
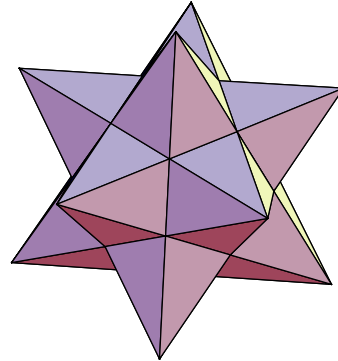
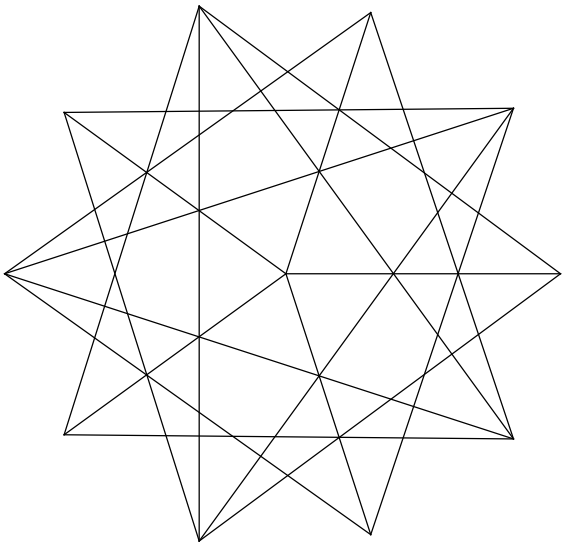
$$\left\{10, \frac{3}{2}, 10, 5\right\}$$



5.

small stellated dodecahedron

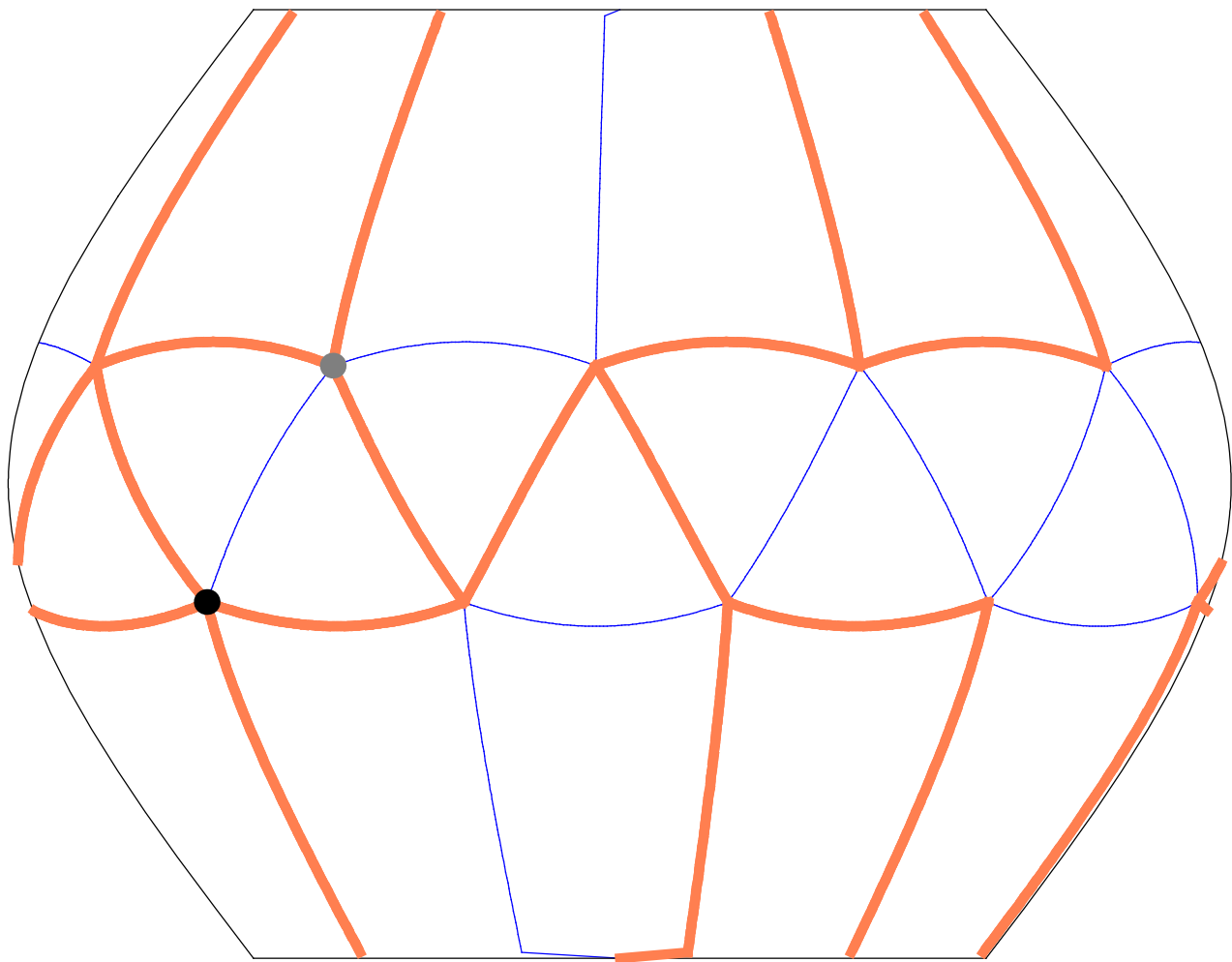
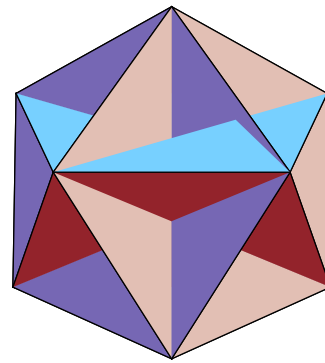
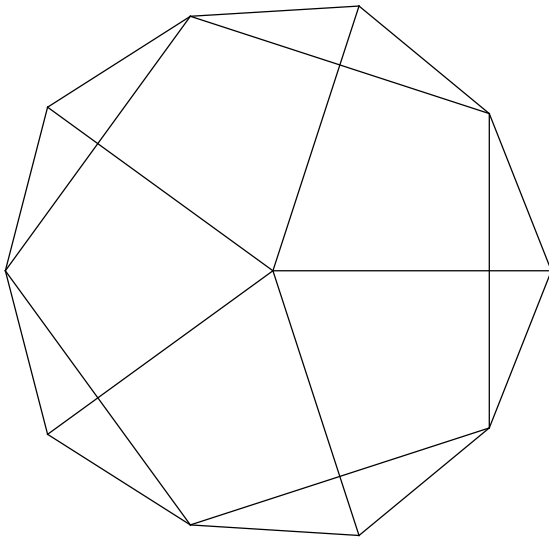
$$\left\{ \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2} \right\}$$



6.

great dodecahedron

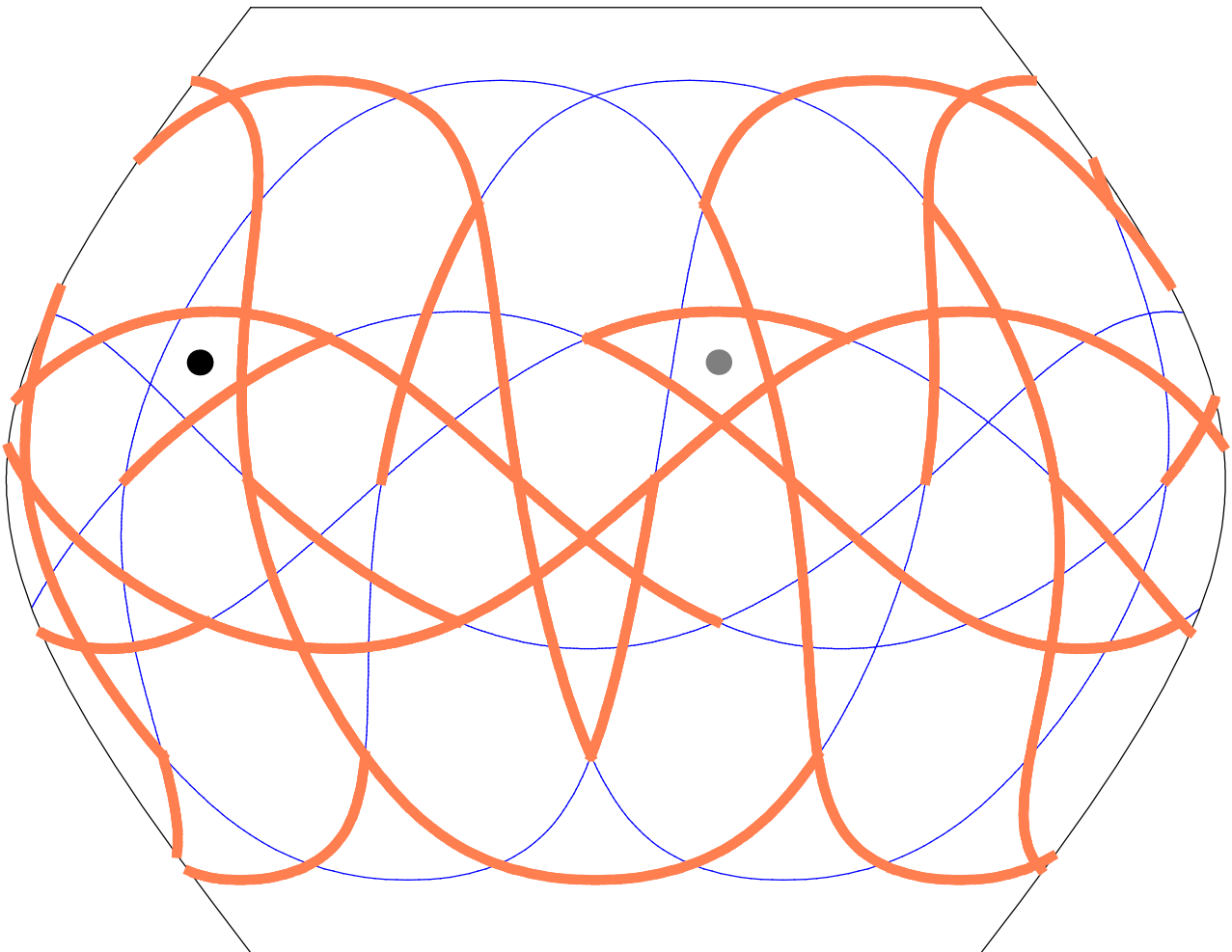
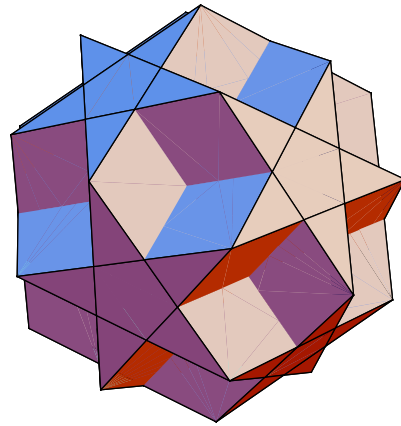
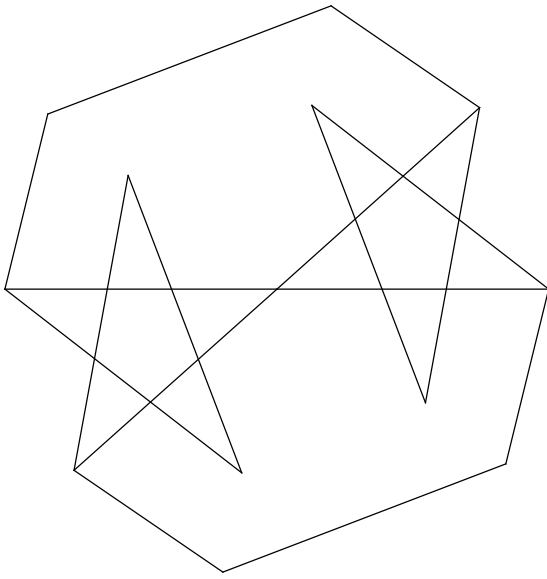
$$\frac{1}{2} \{5, 5, 5, 5, 5\}$$



7.

dodecadodecahedron

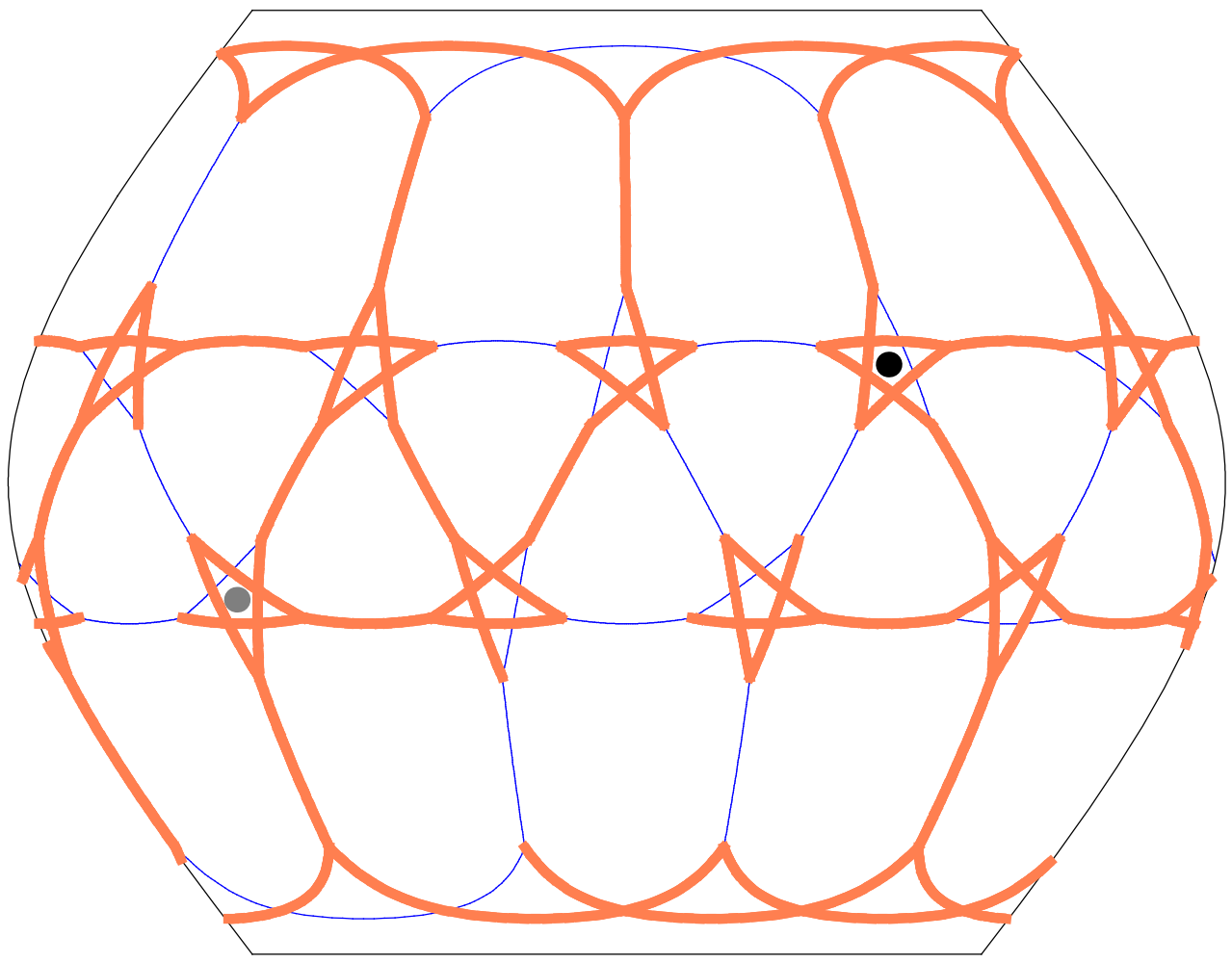
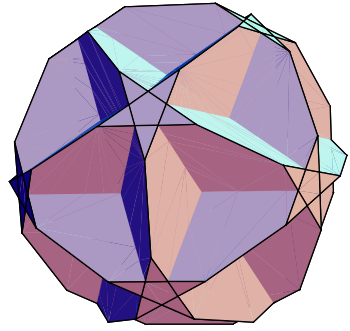
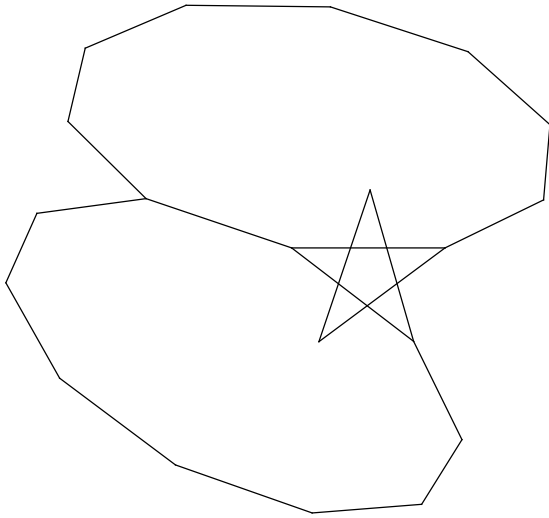
$$\left\{ \frac{5}{2}, 5, \frac{5}{2}, 5 \right\}$$



8.

truncated great dodecahedron

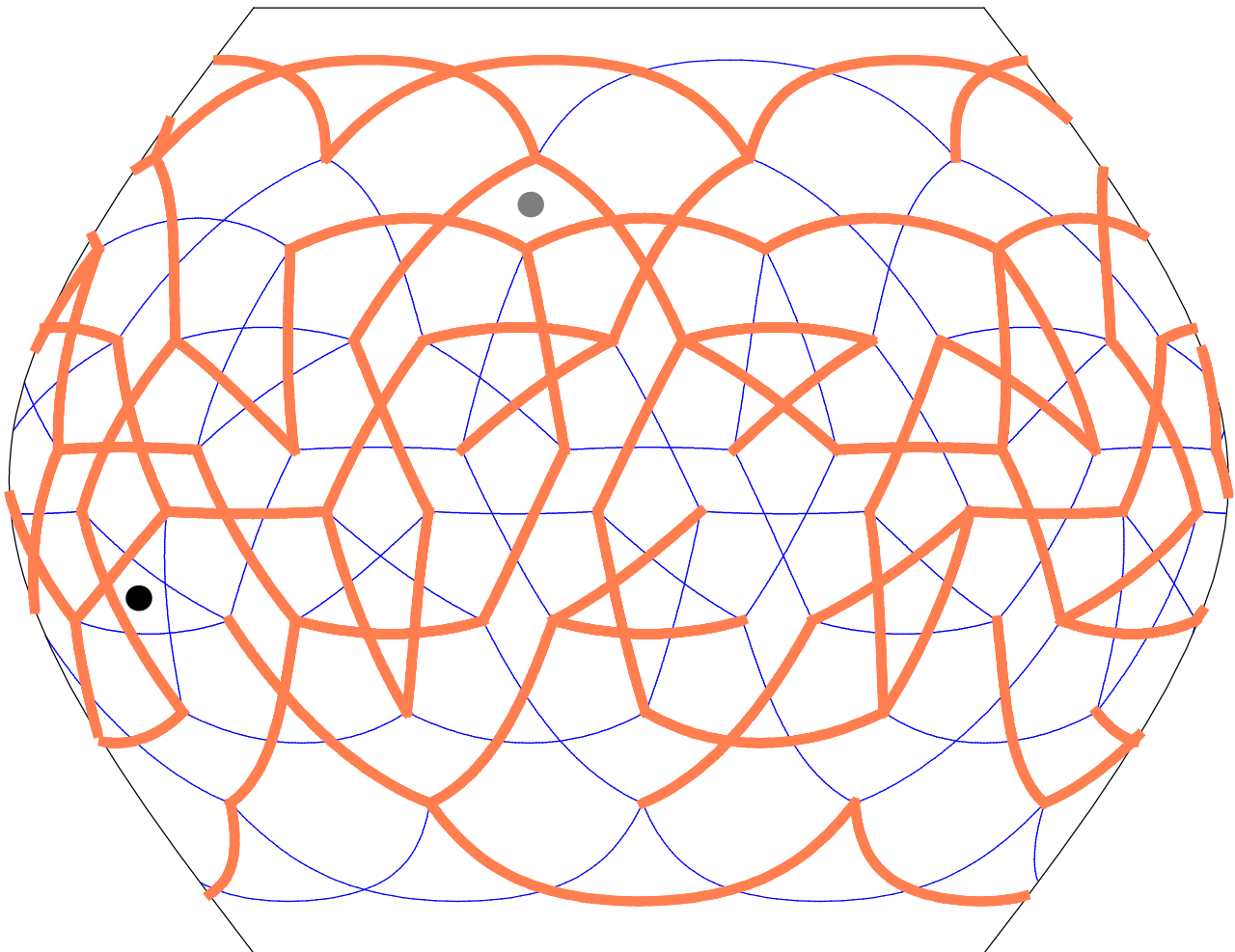
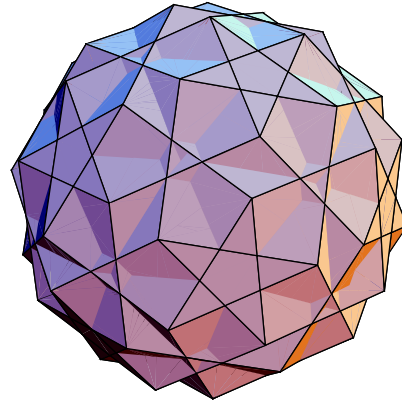
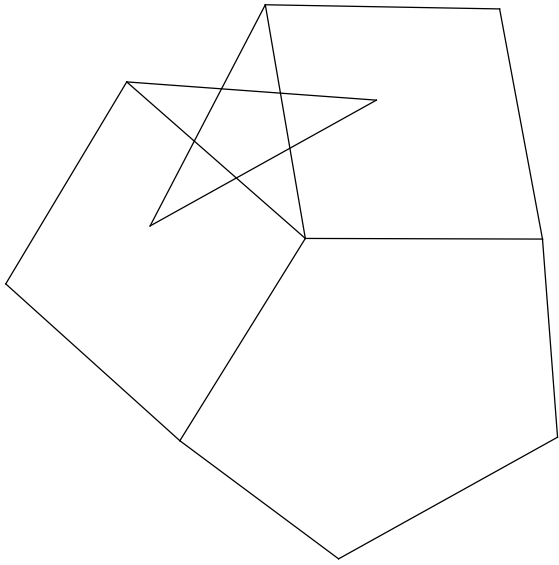
$$\{10, 10, \frac{5}{2}\}$$



9.

rhombidodecadodecahedron

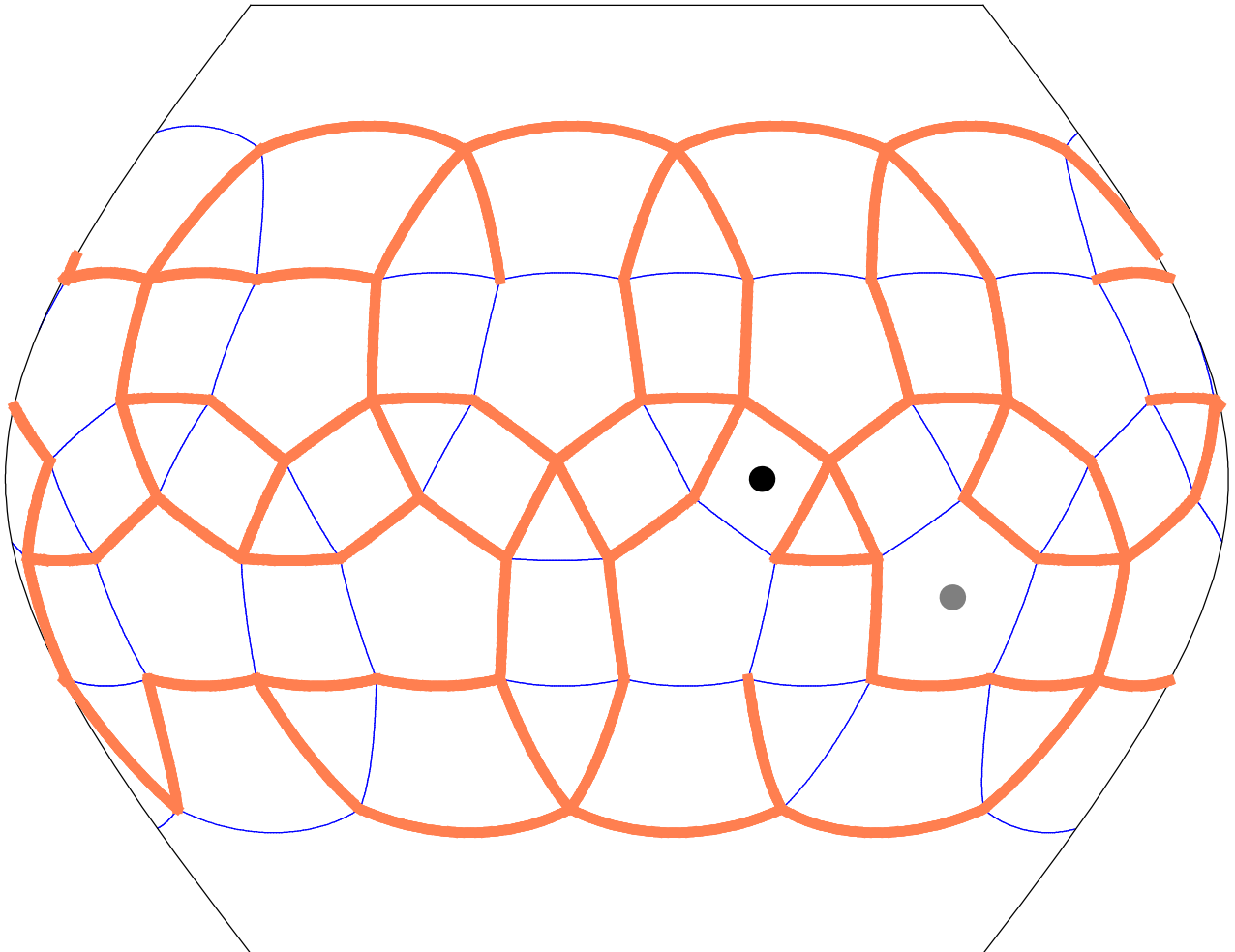
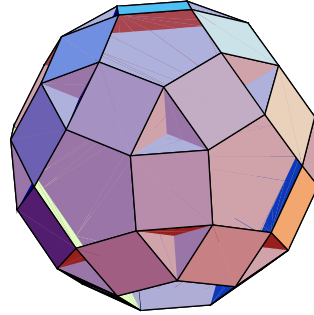
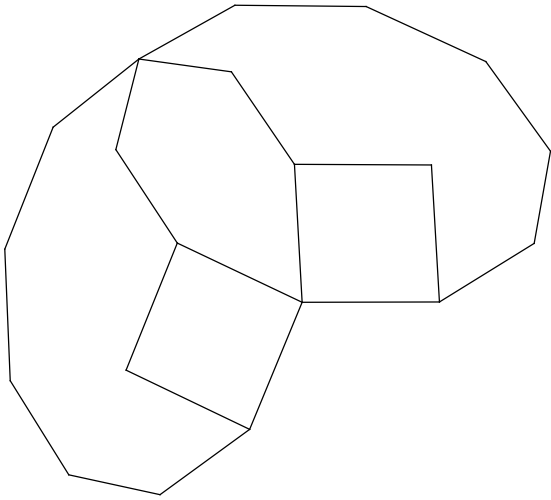
$$\{4, \frac{5}{2}, 4, 5\}$$



10.

small rhombidodecahedron

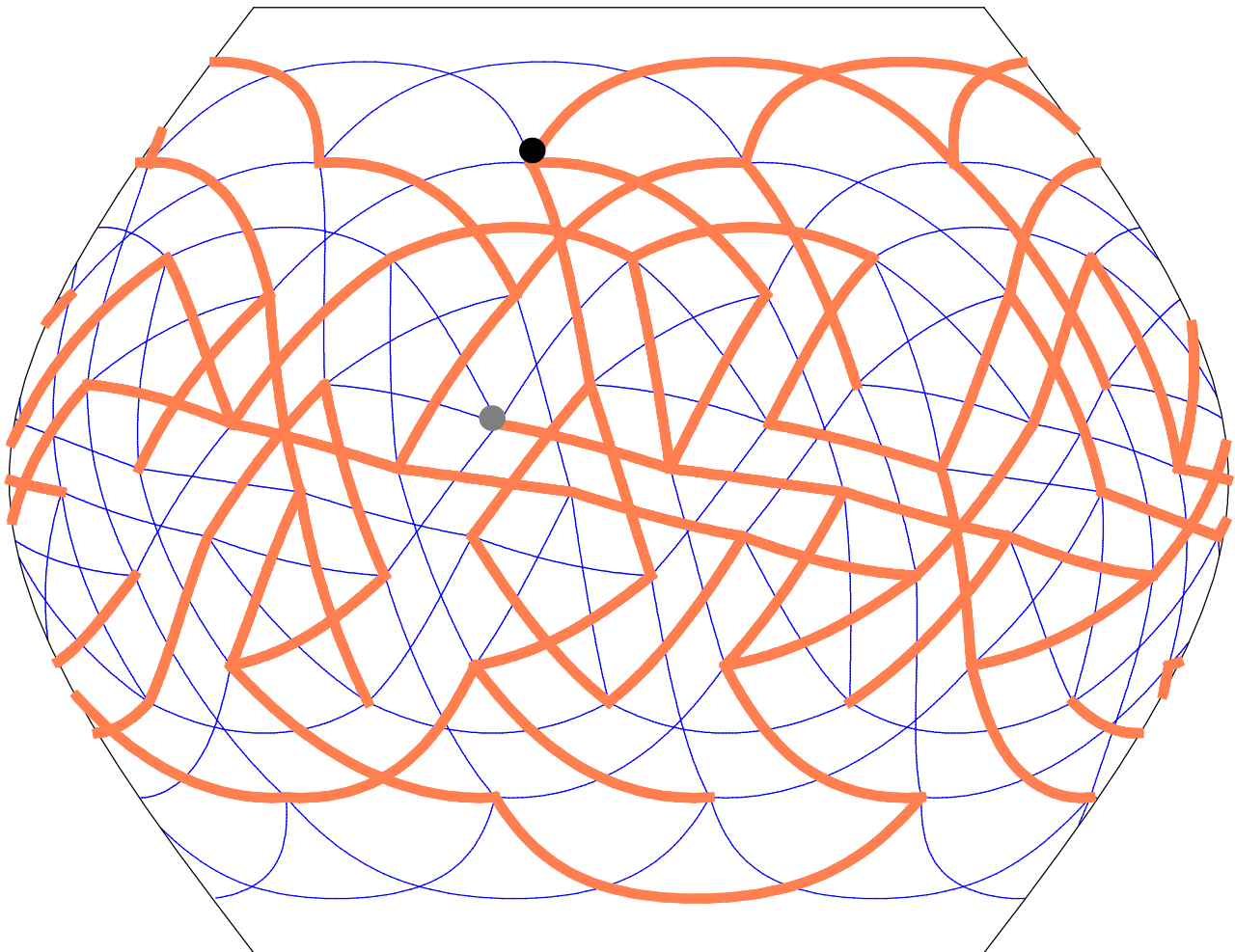
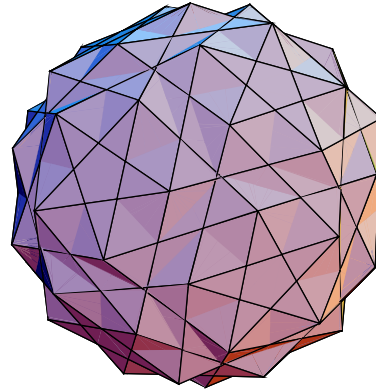
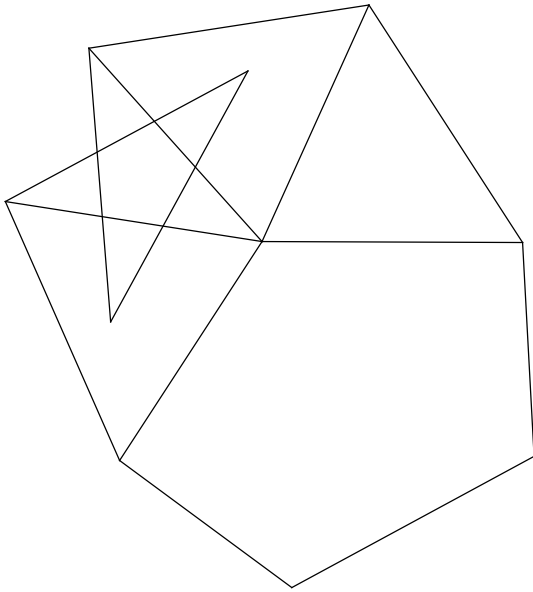
$$\left\{10, 4, \frac{10}{9}, \frac{4}{3}\right\}$$



11.

snub dodecadodecahedron

$$\{3, 3, \frac{5}{2}, 3, 5\}$$

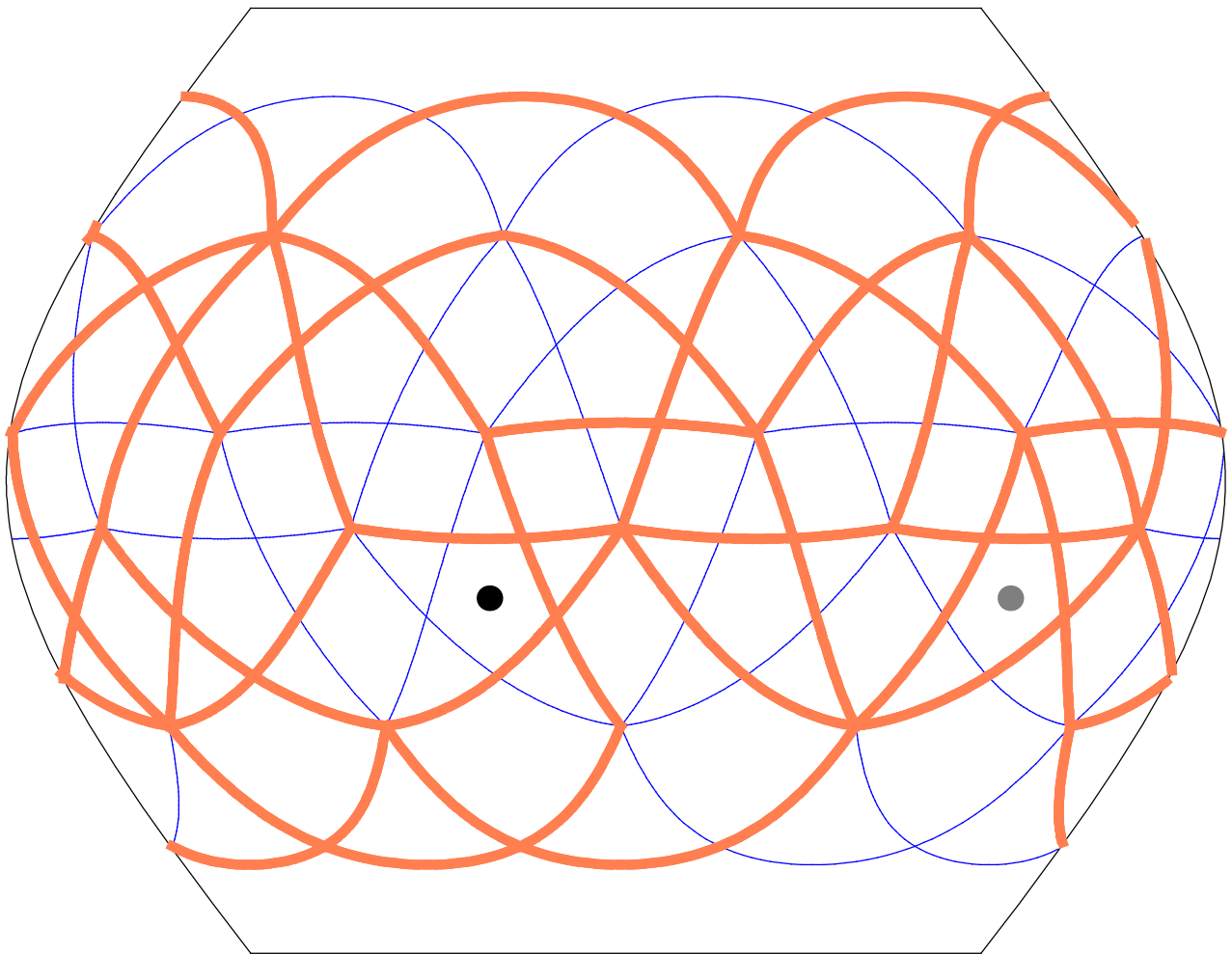
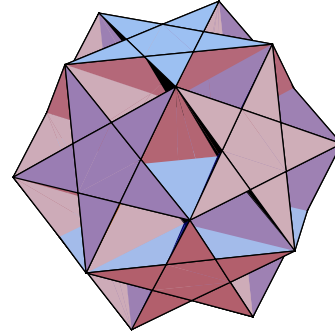
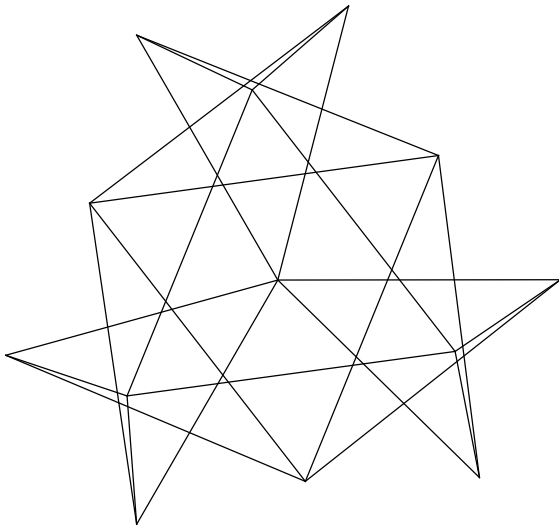




12.

ditrigrinal dodecadodecahedron

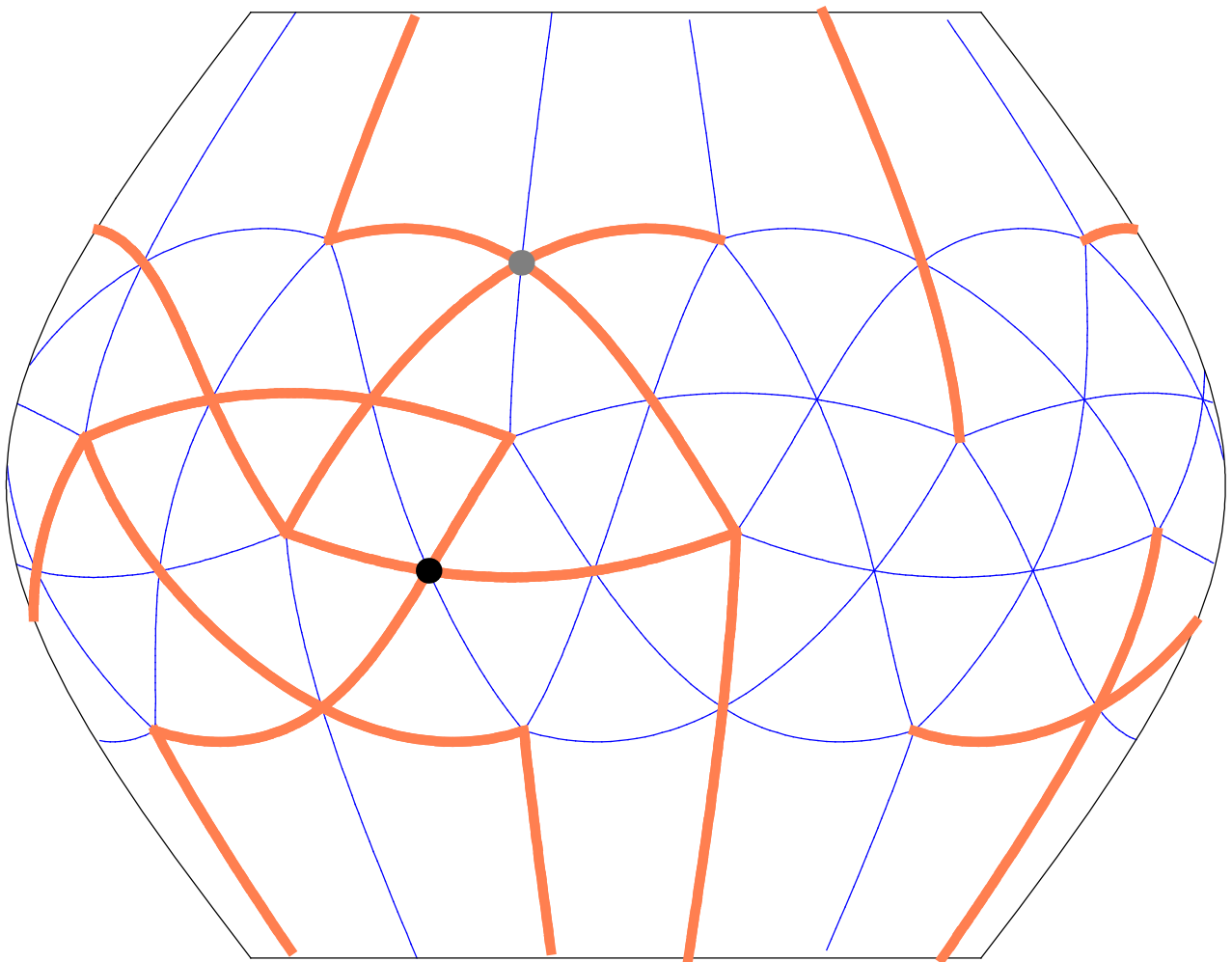
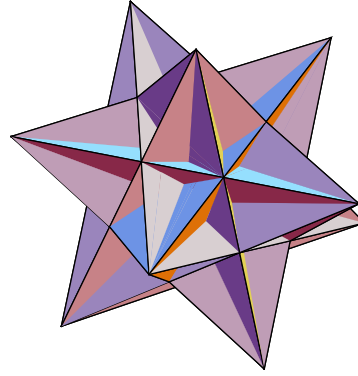
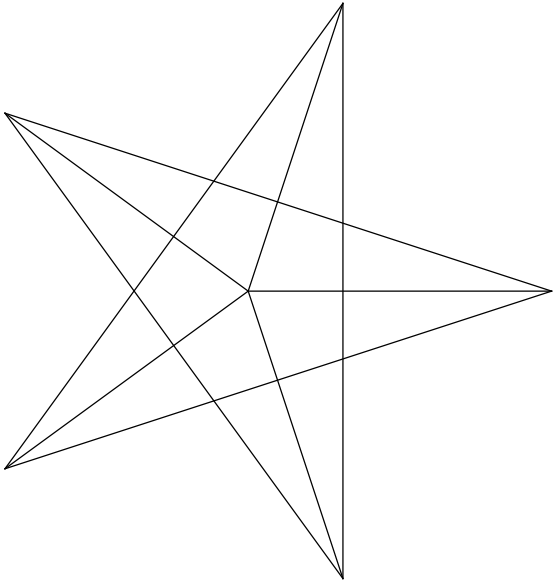
$$\left\{\frac{5}{3}, 5, \frac{5}{3}, 5, \frac{5}{3}, 5\right\}$$



13.

great icosahedron

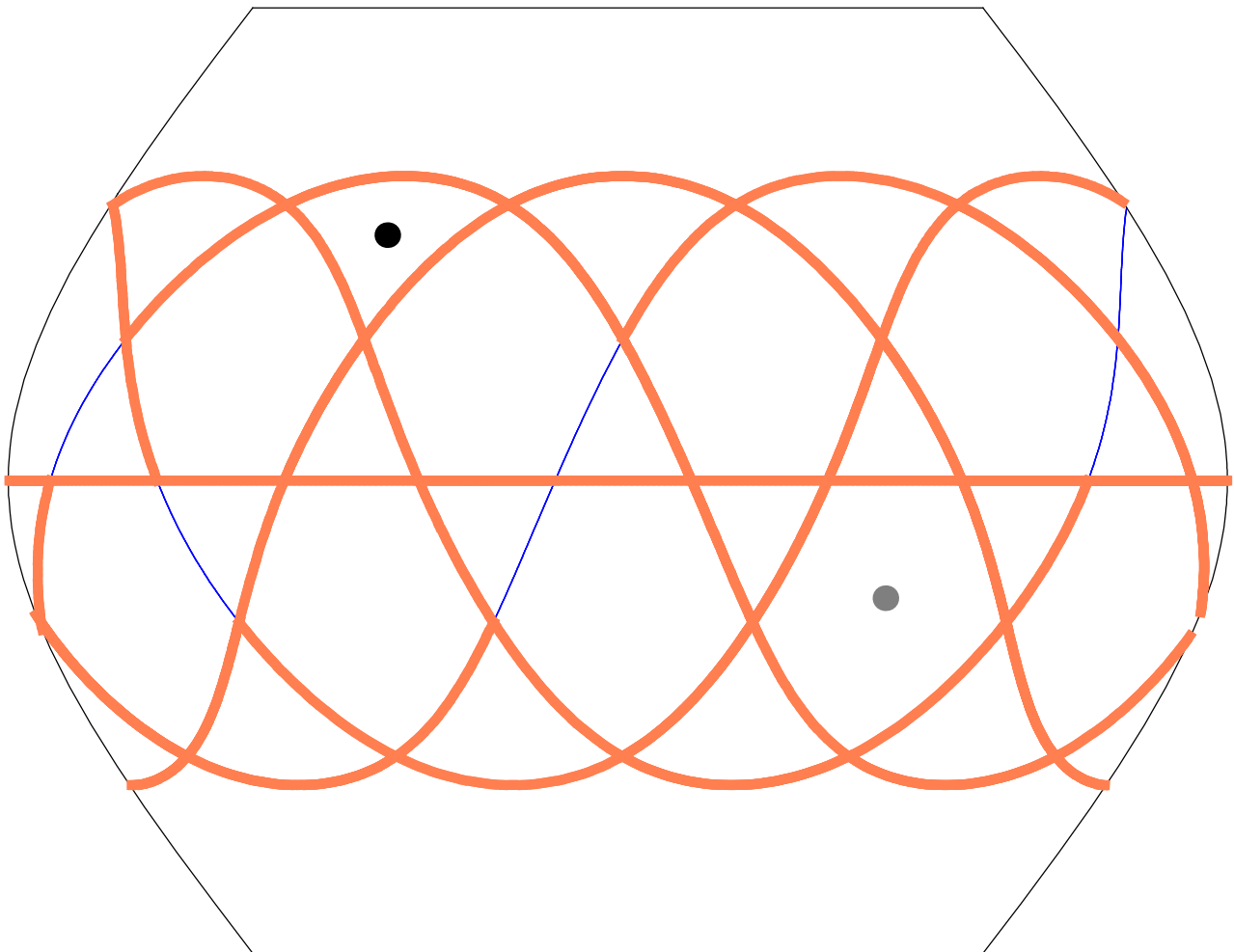
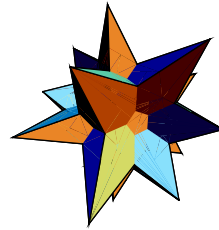
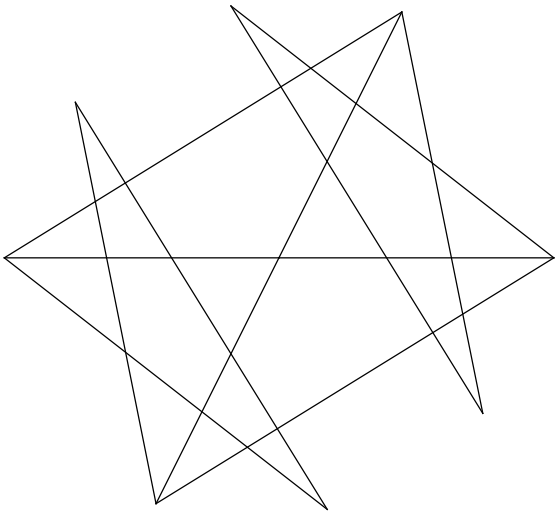
$$\frac{1}{2} \{3, 3, 3, 3, 3\}$$



14.

great icosidodecahedron

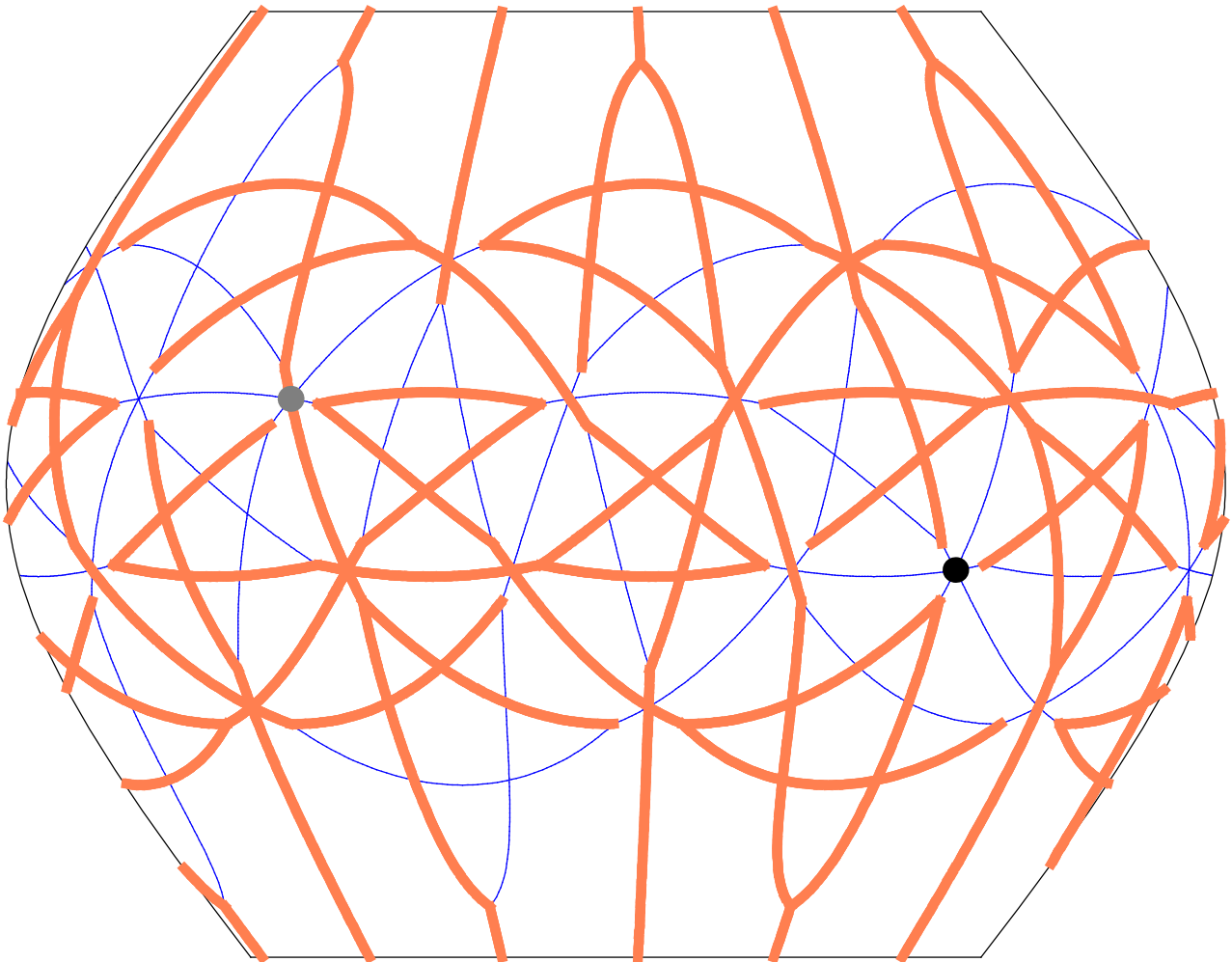
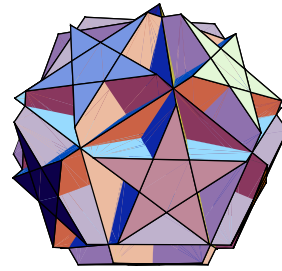
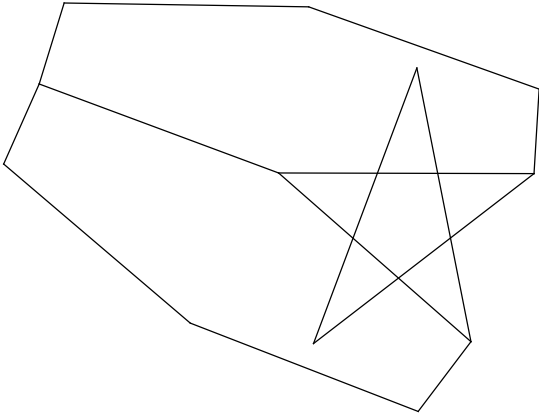
$$\left\{ \frac{5}{2}, 3, \frac{5}{2}, 3 \right\}$$



15.

great truncated icosahedron

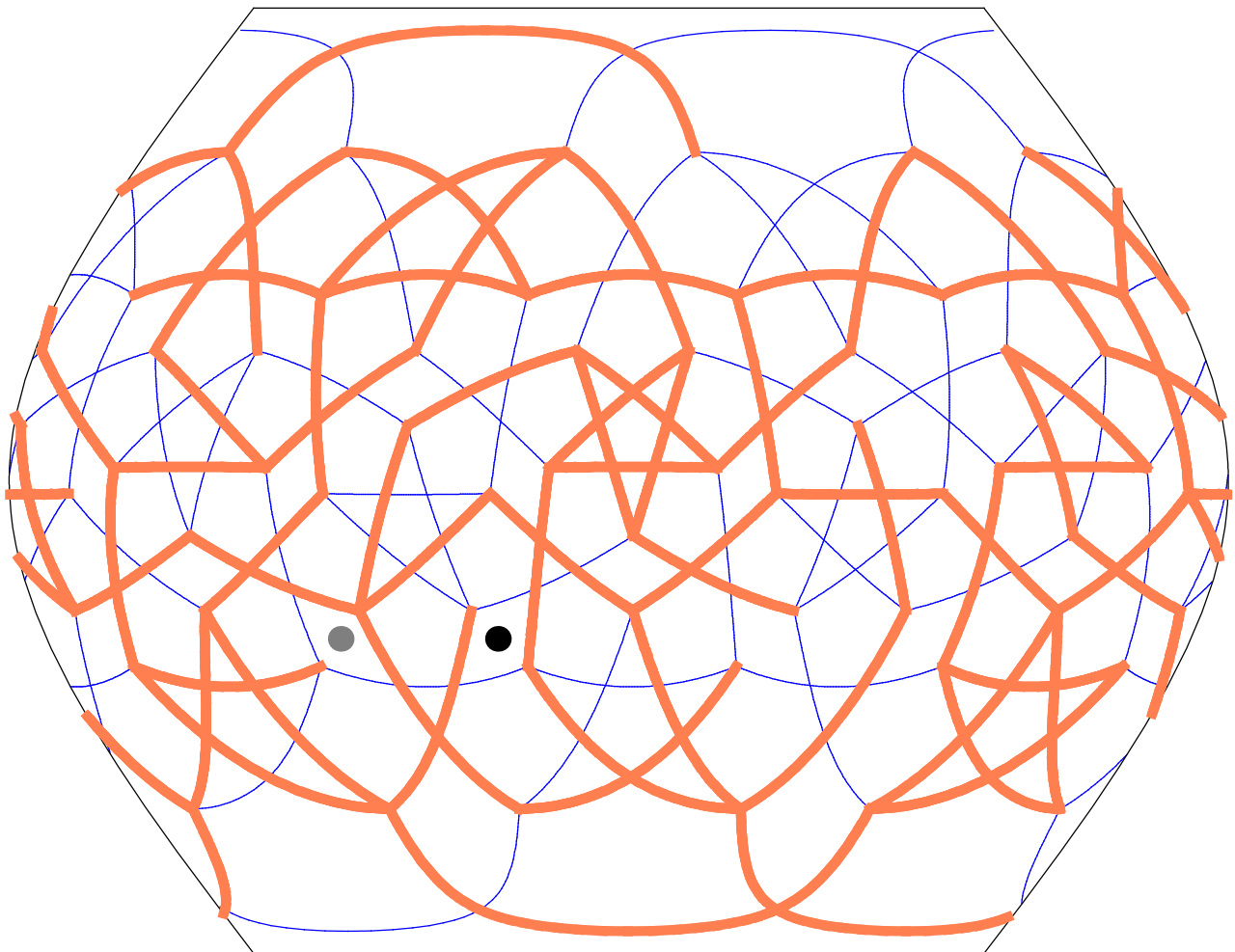
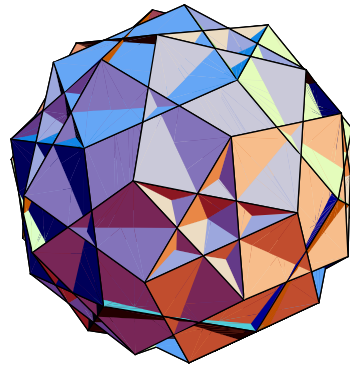
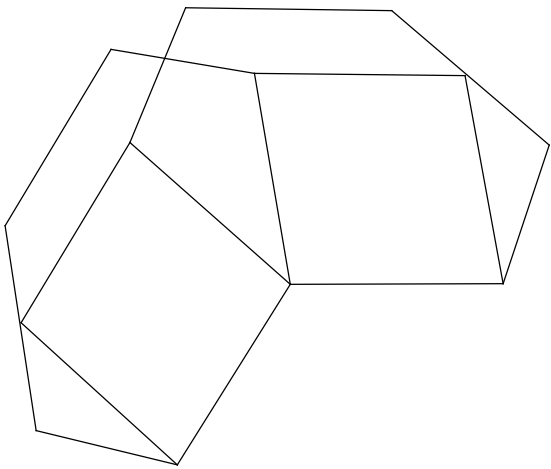
$$\{6, 6, \frac{5}{2}\}$$



16.

rhombicosahedron

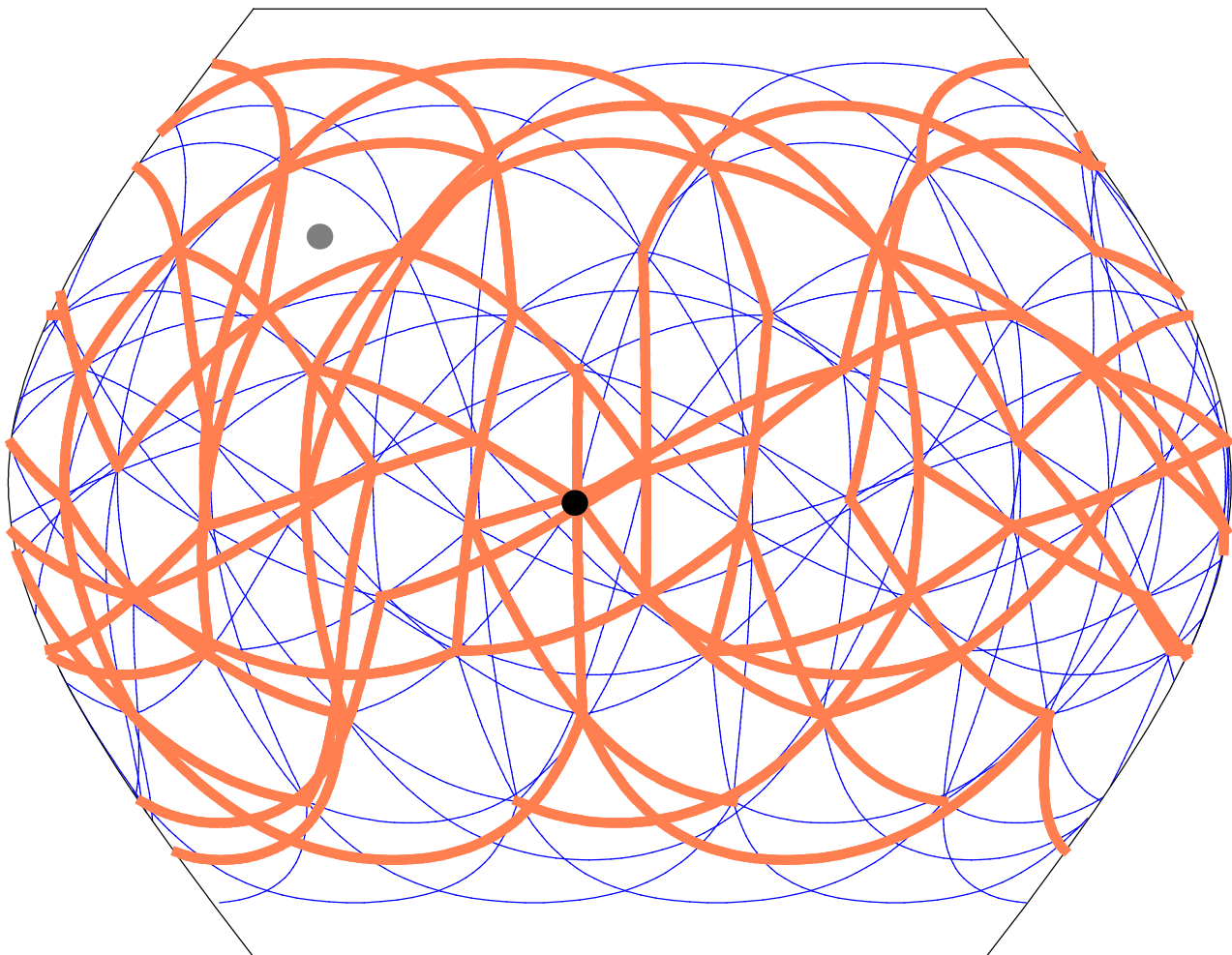
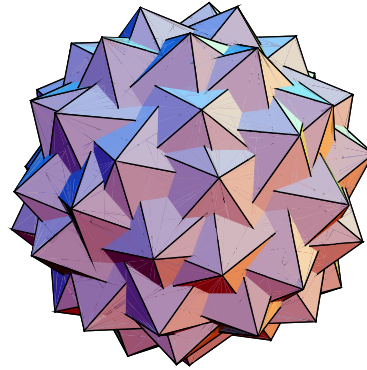
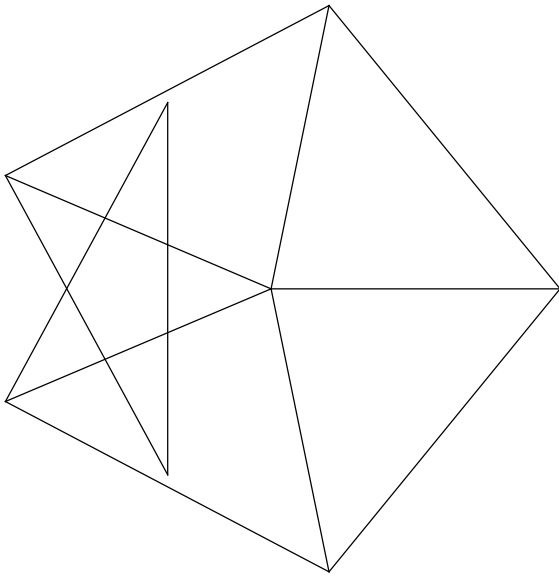
$$\{6, 4, \frac{6}{5}, \frac{4}{3}\}$$



17.

great snub icosidodecahedron

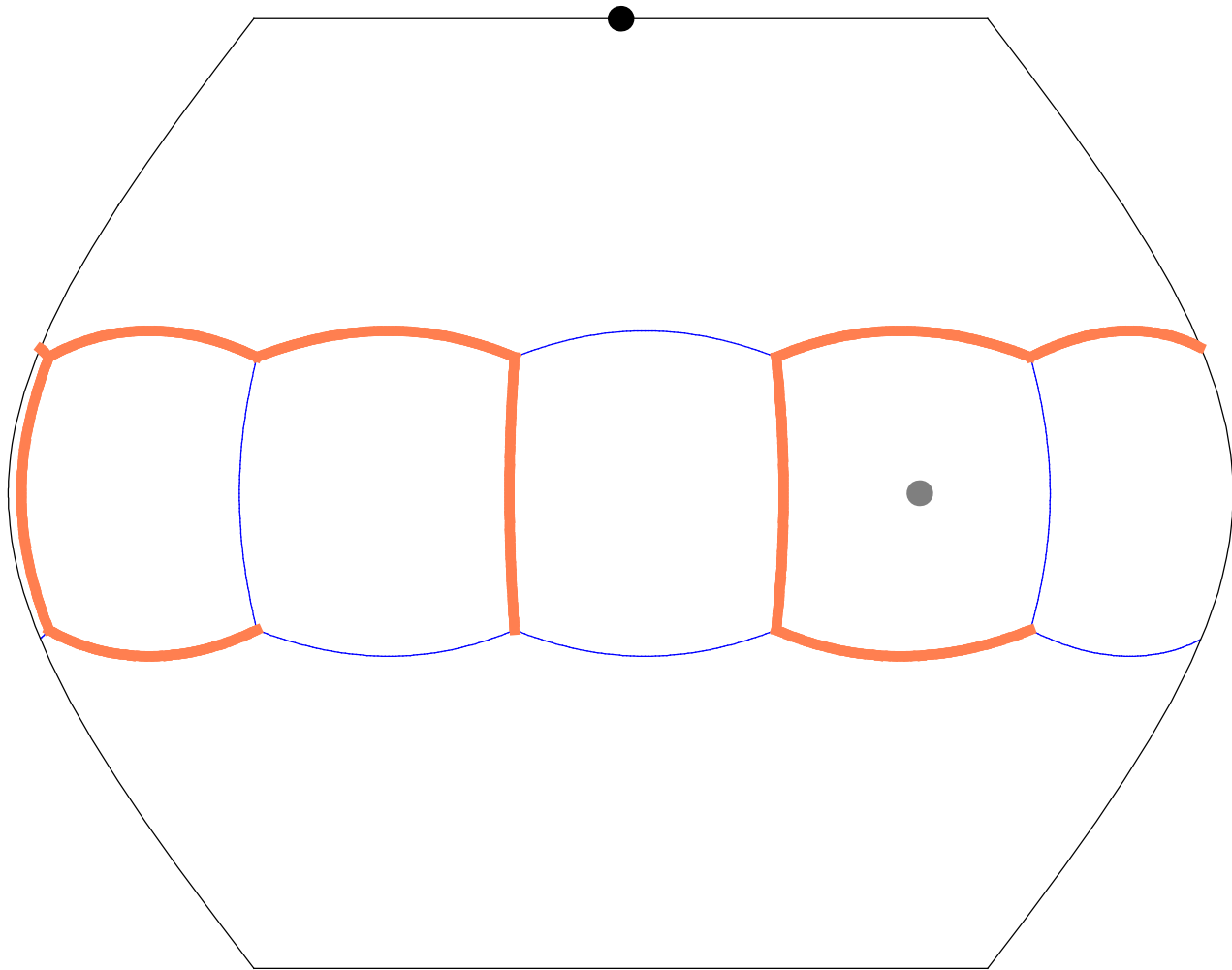
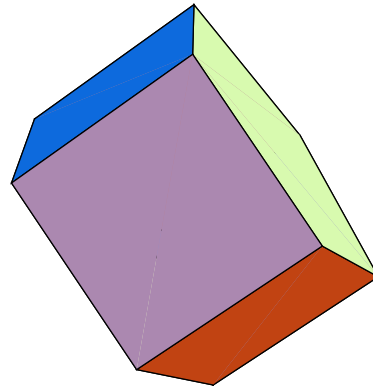
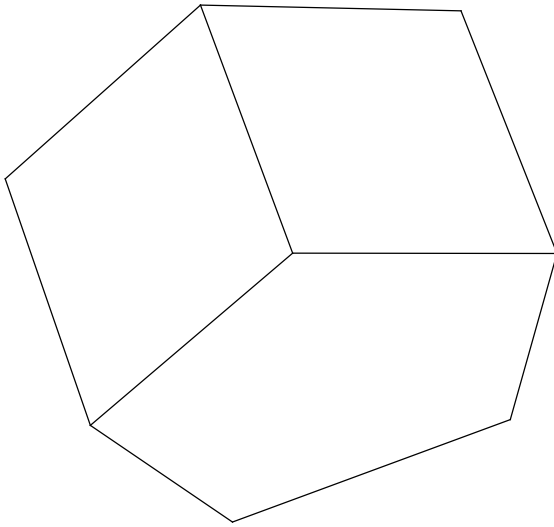
$$\{3, 3, \frac{5}{2}, 3, 3\}$$



18.

pentagonal prism

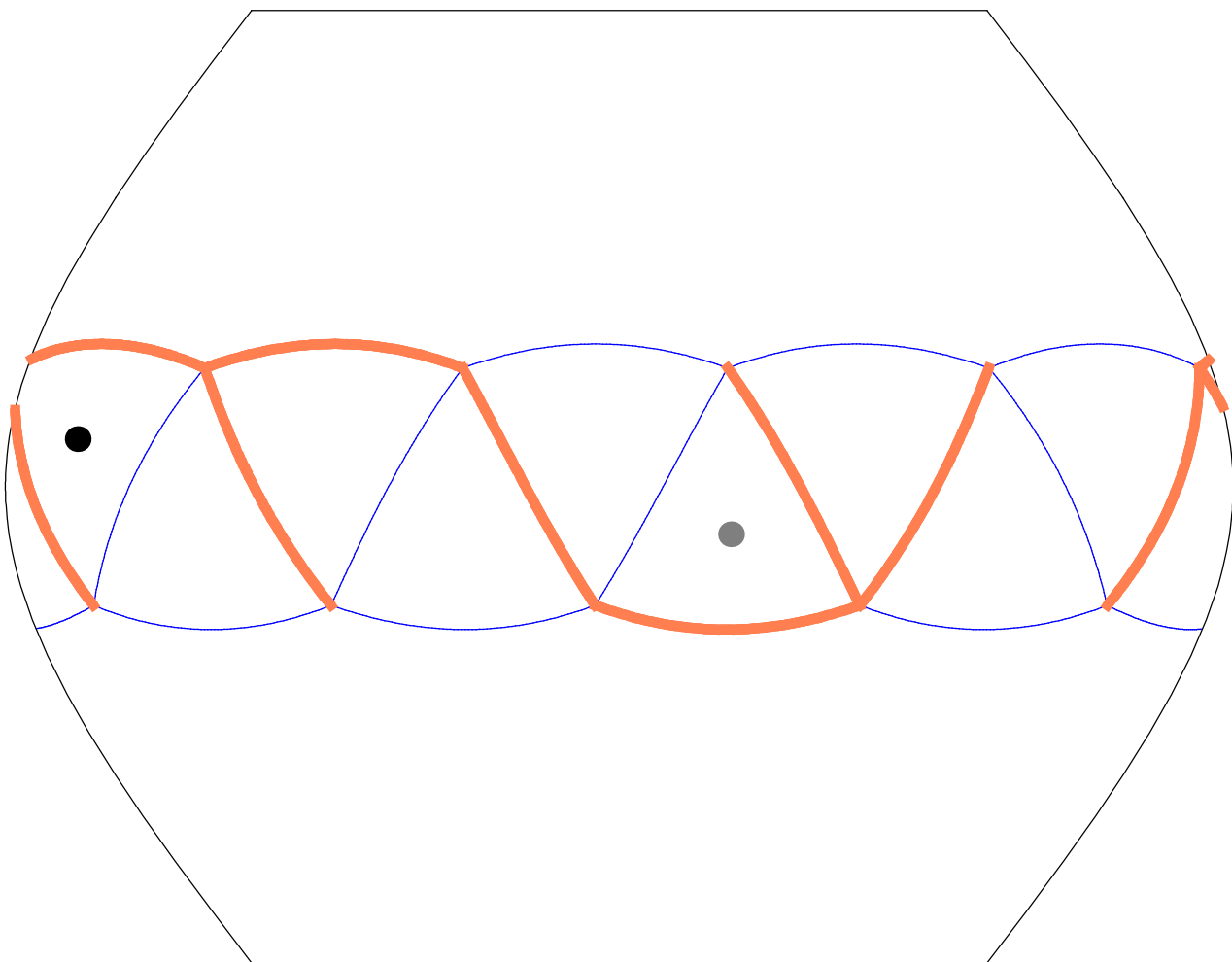
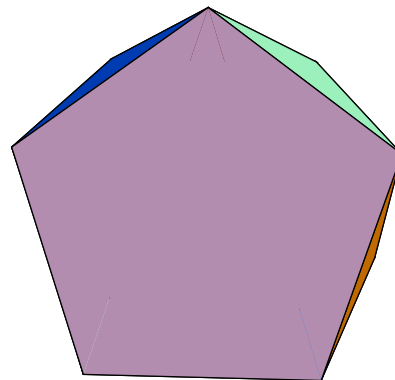
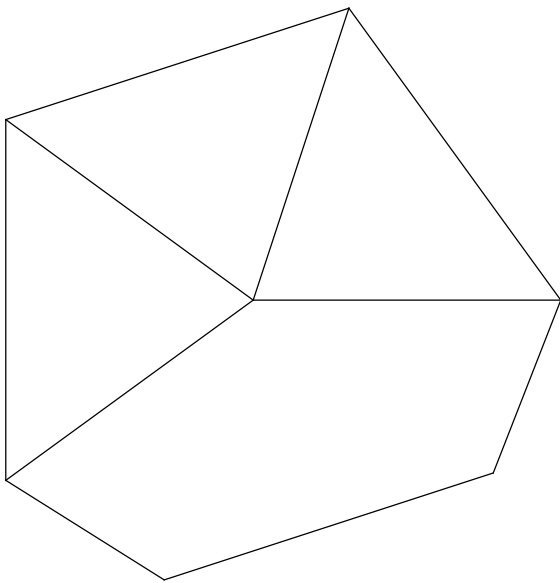
{4, 4, 5}



19.

pentagonal antiprism

{3, 3, 3, 5}

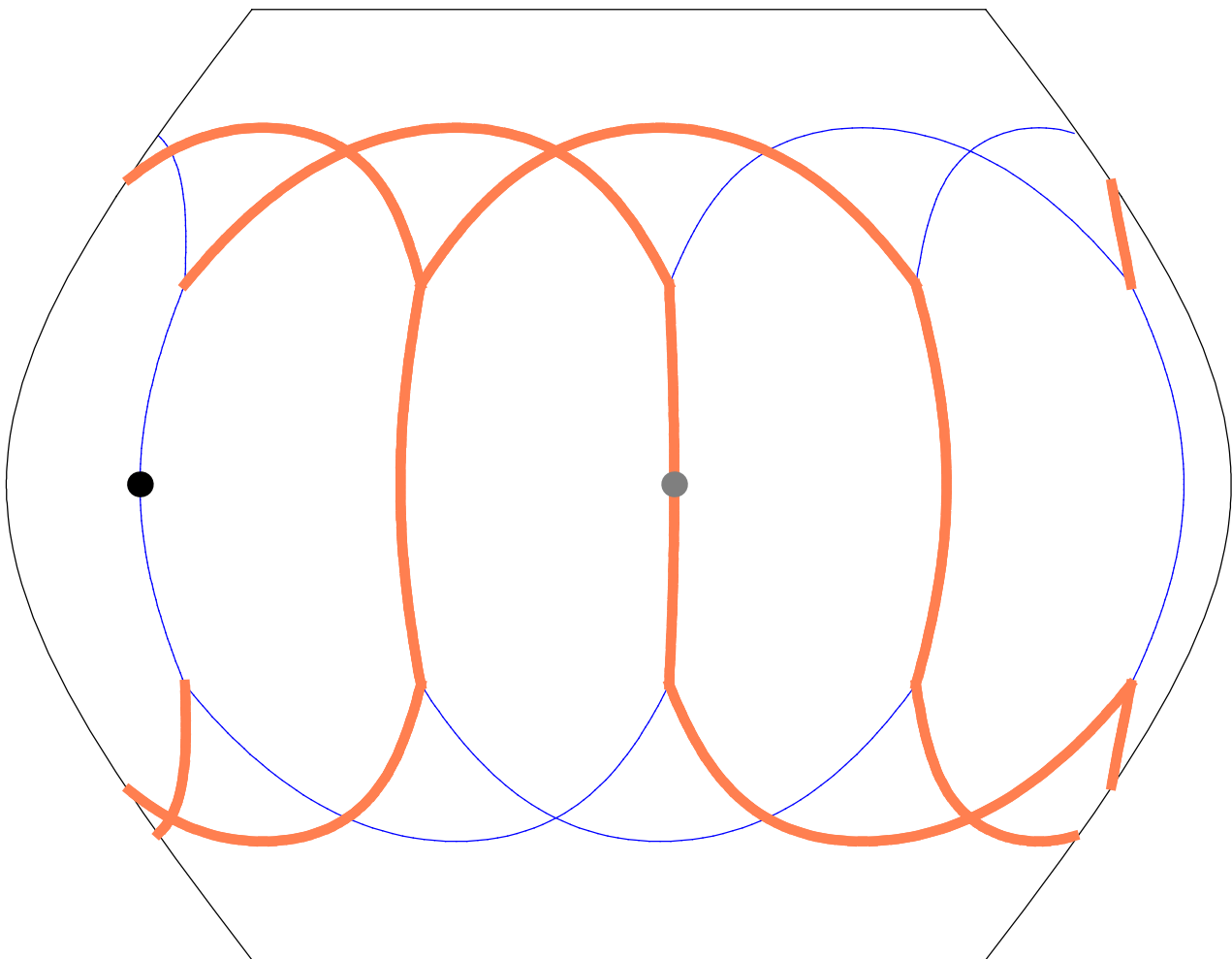
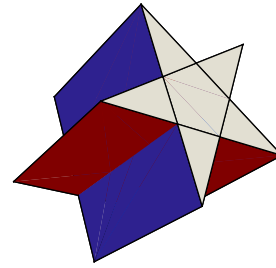
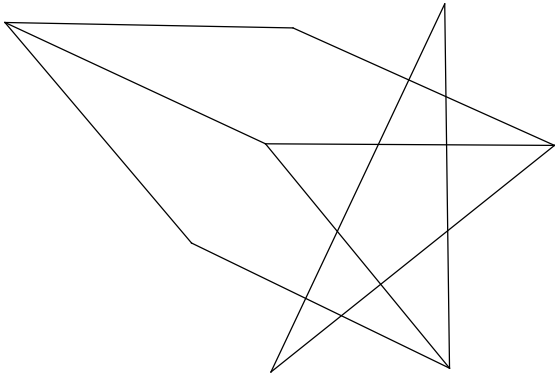




20.

pentagrammic prism

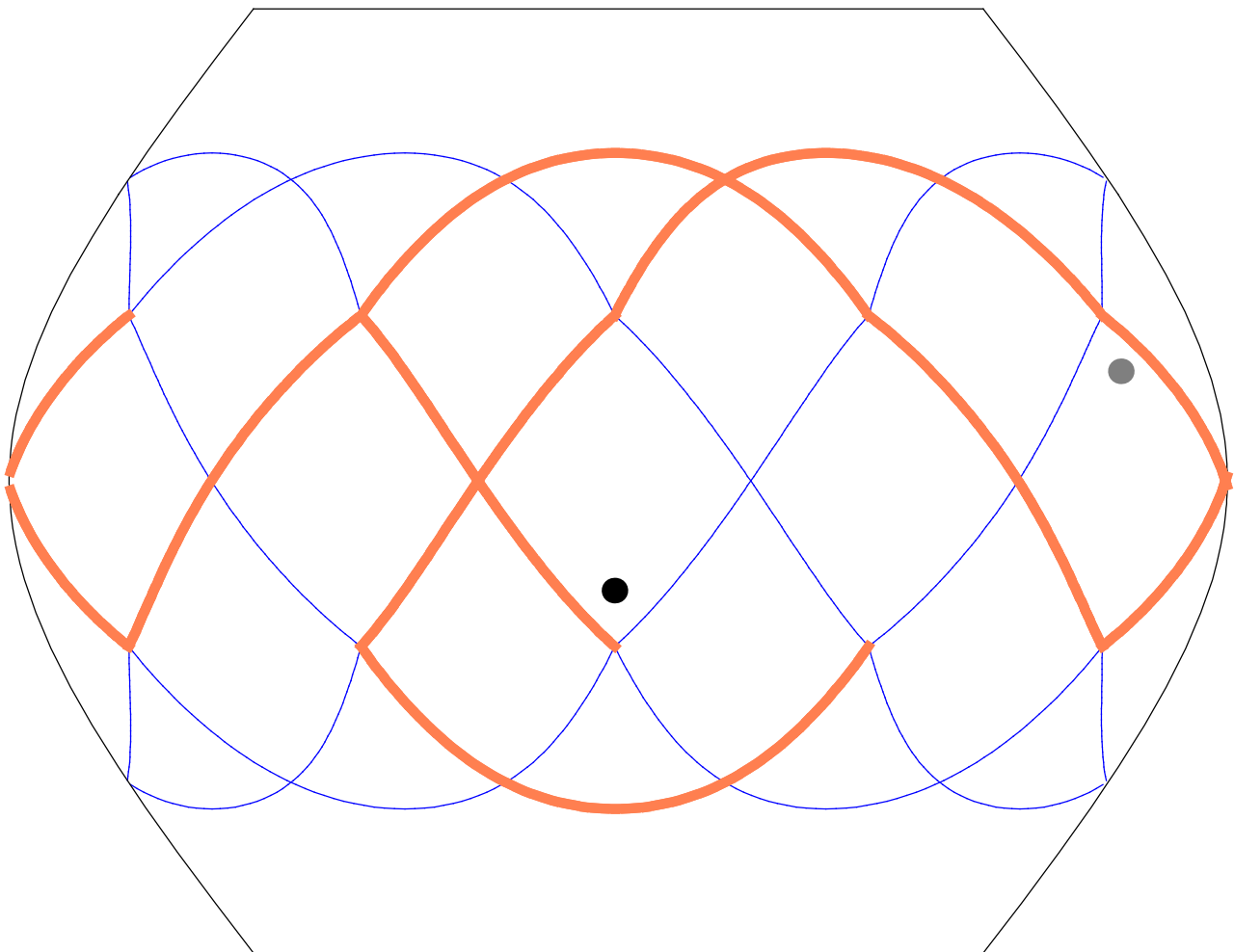
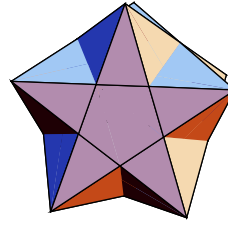
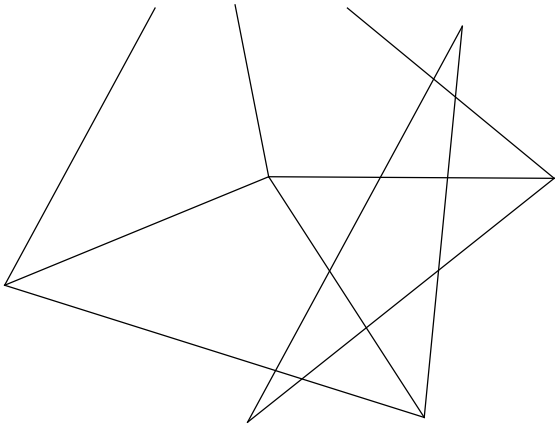
$$\{4, 4, \frac{5}{2}\}$$



21.

pentagrammic antiprism

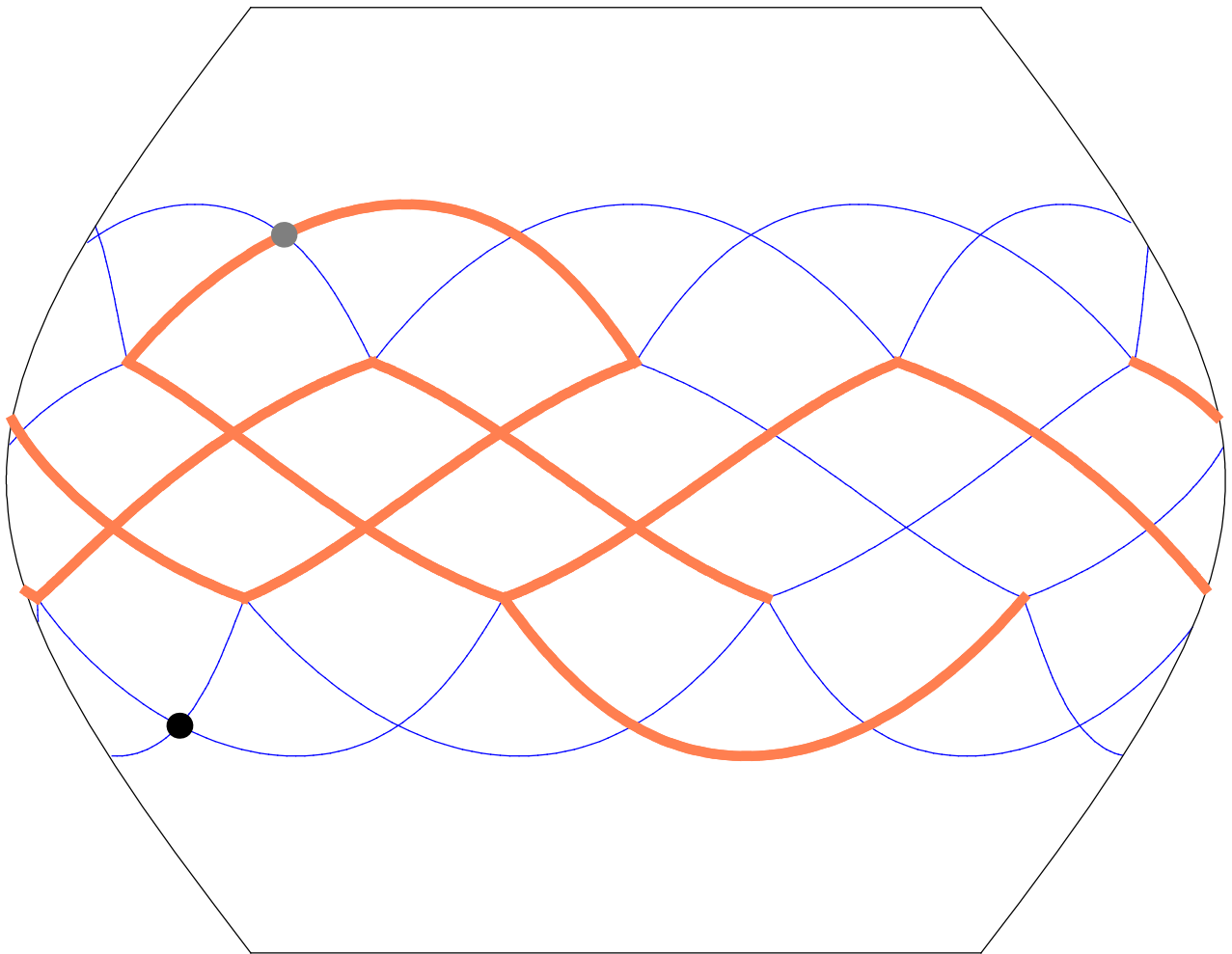
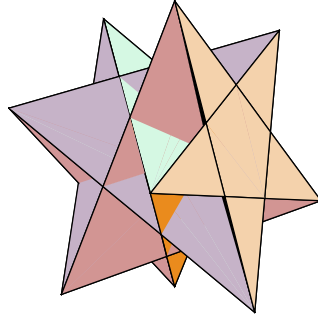
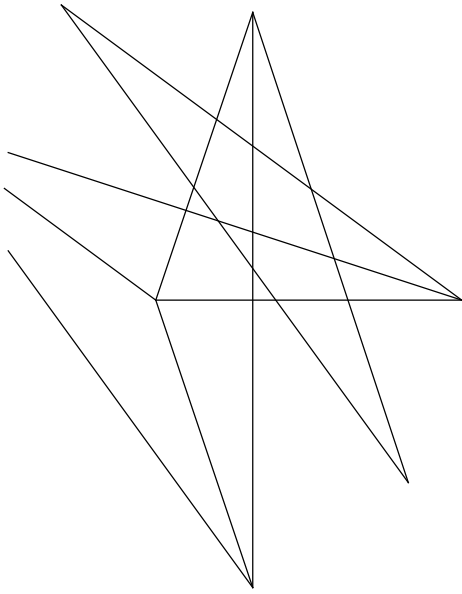
$$\{3, 3, 3, \frac{5}{2}\}$$



22.

pentagrammic crossed antiprism

$$\{3, 3, 3, \frac{5}{3}\}$$

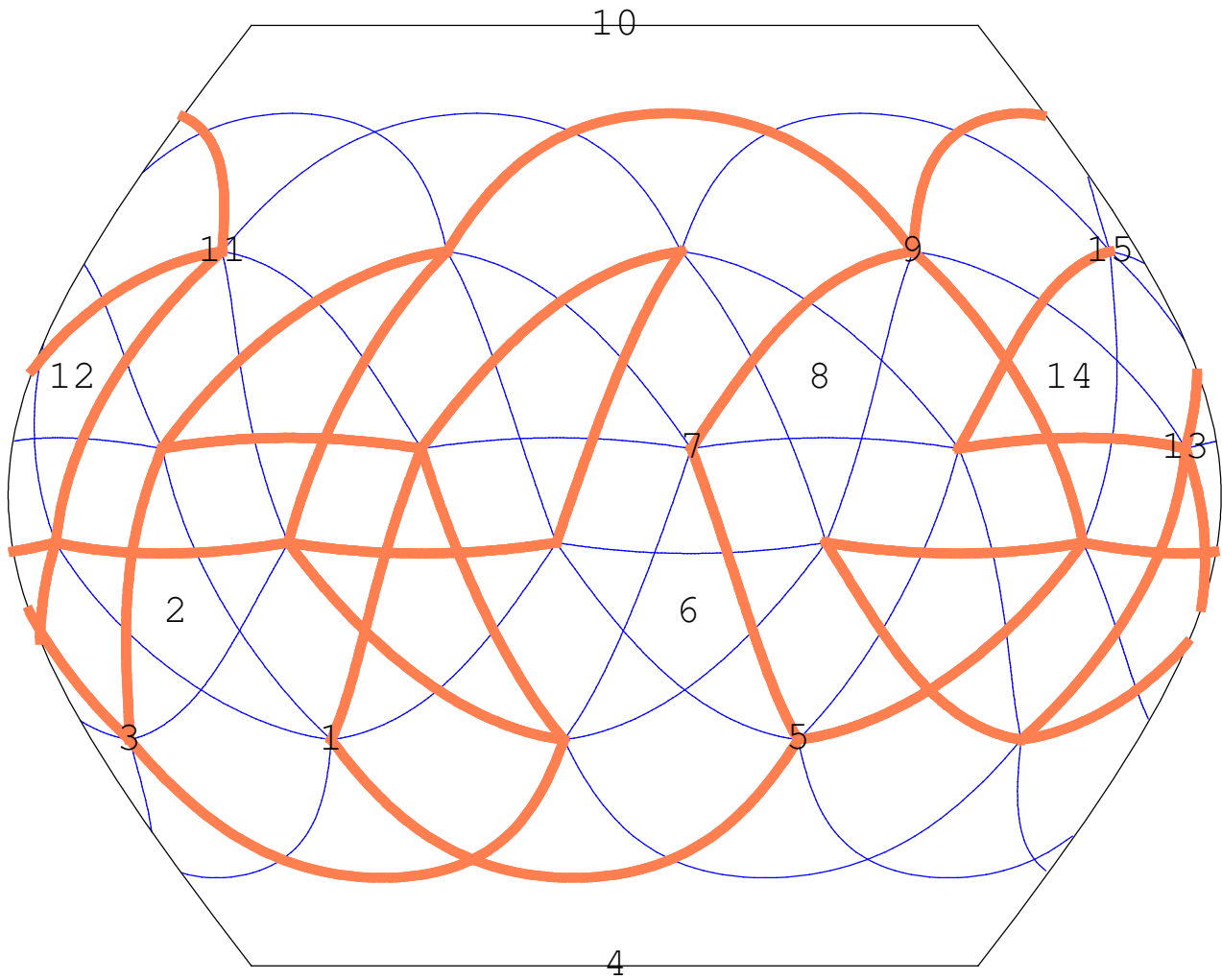
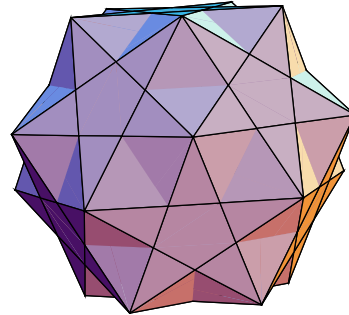
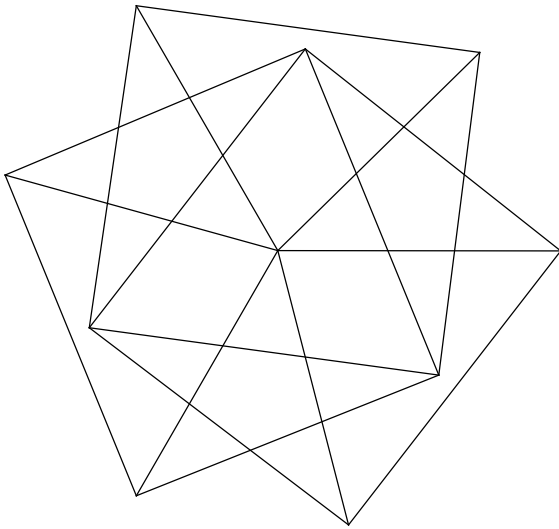


# Solutions

1.

small ditrigonal icosidodecahedron

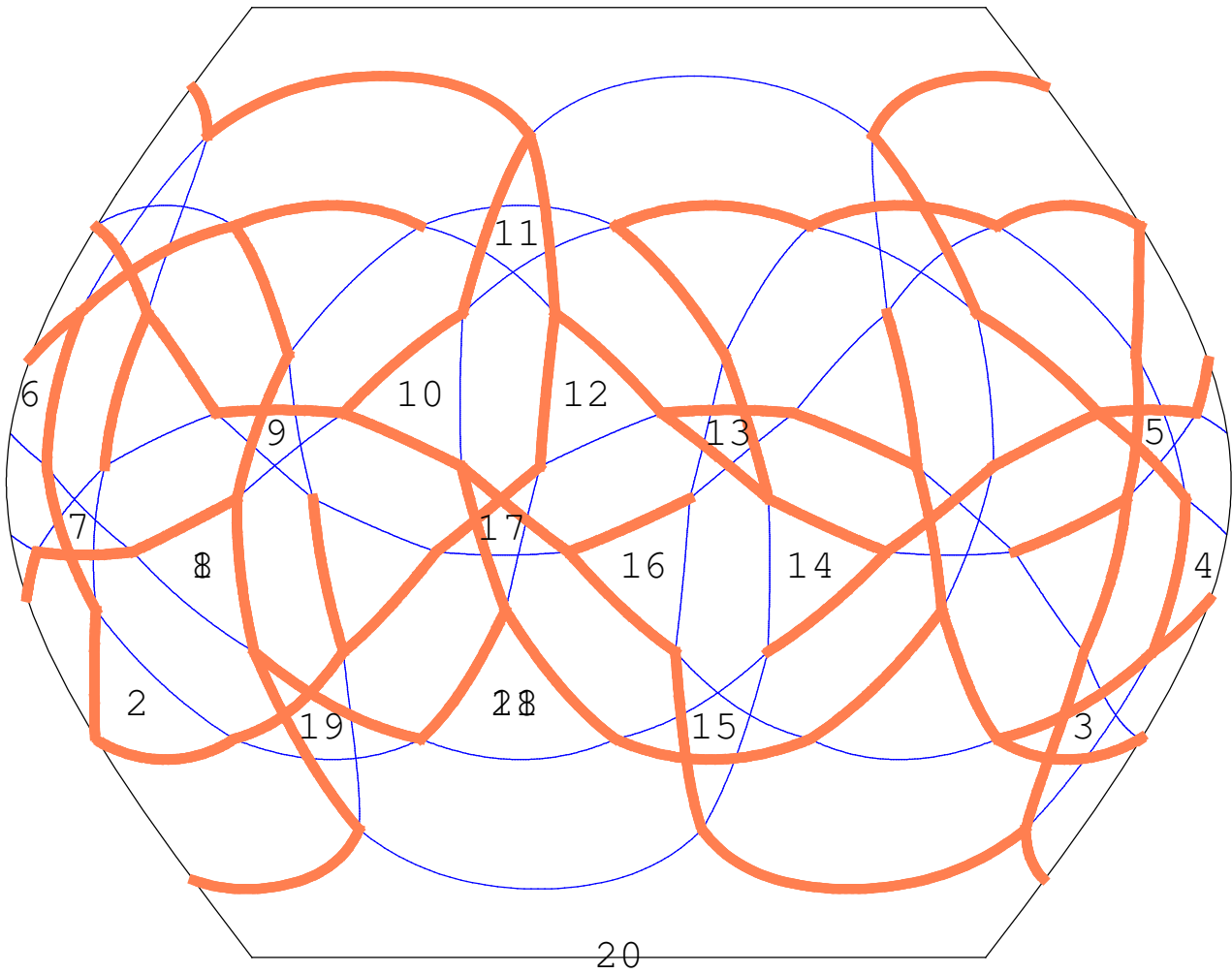
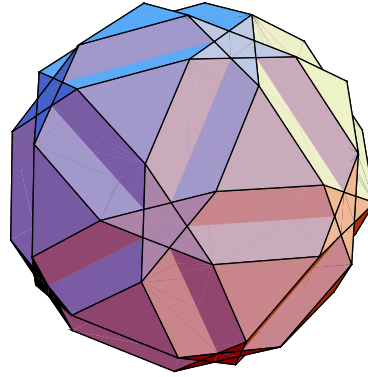
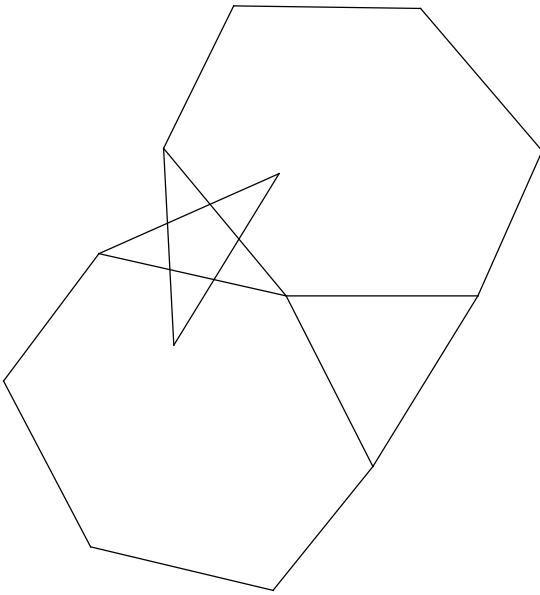
$$\left\{ \frac{5}{2}, 3, \frac{5}{2}, 3, \frac{5}{2}, 3 \right\}$$



2.

small icosicosidodecahedron

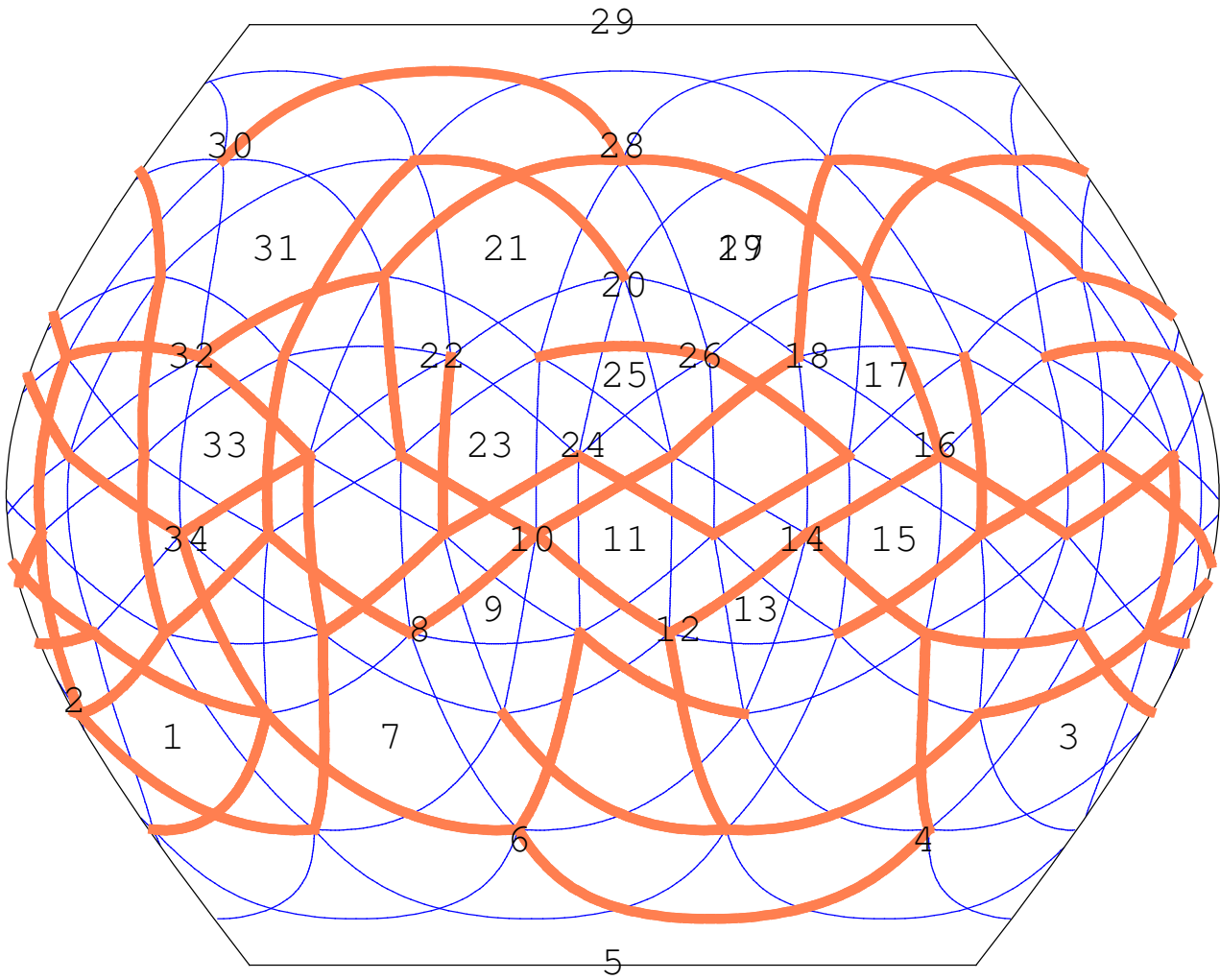
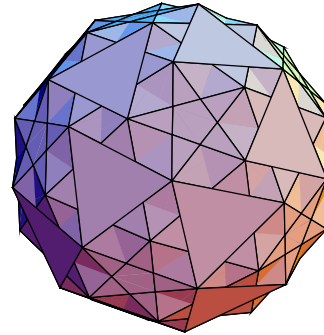
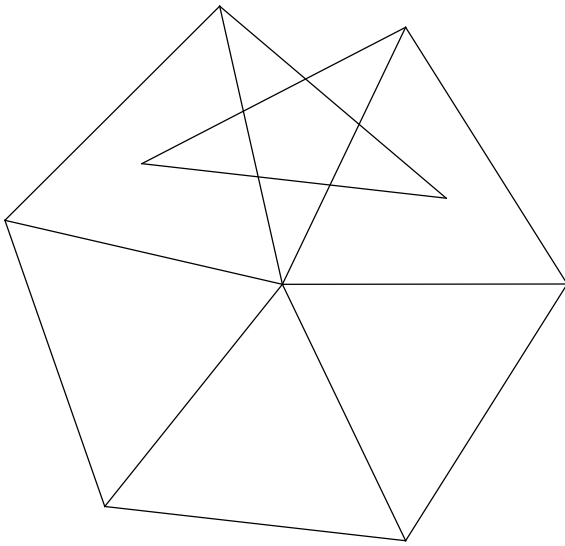
$$\{6, \frac{5}{2}, 6, 3\}$$



3.

small snub icosicosidodecahedron

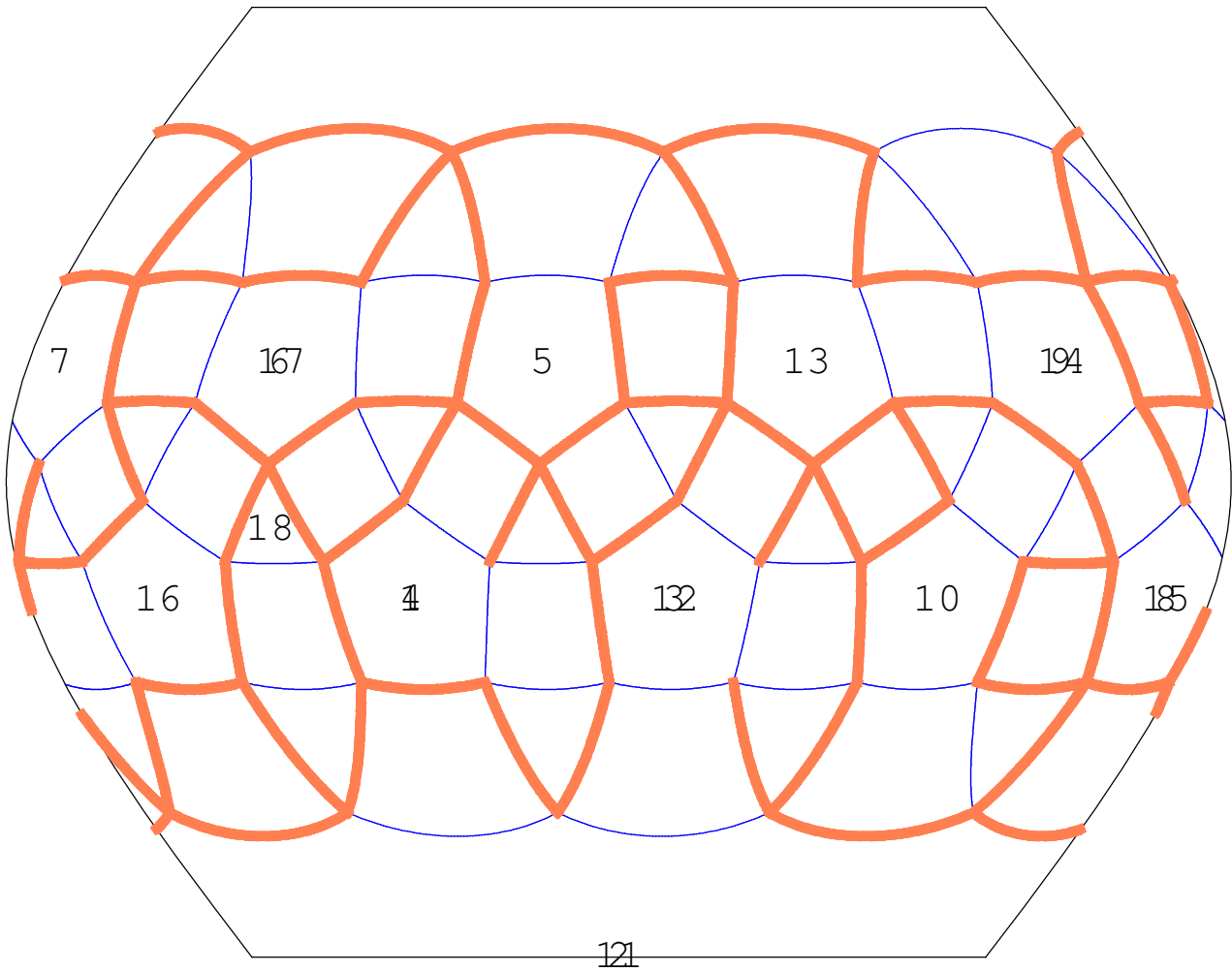
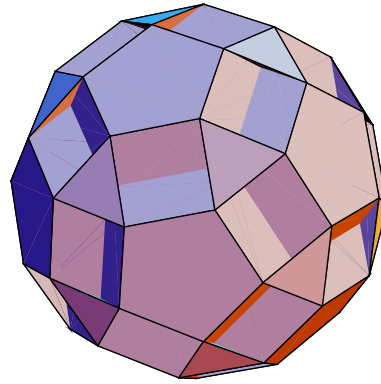
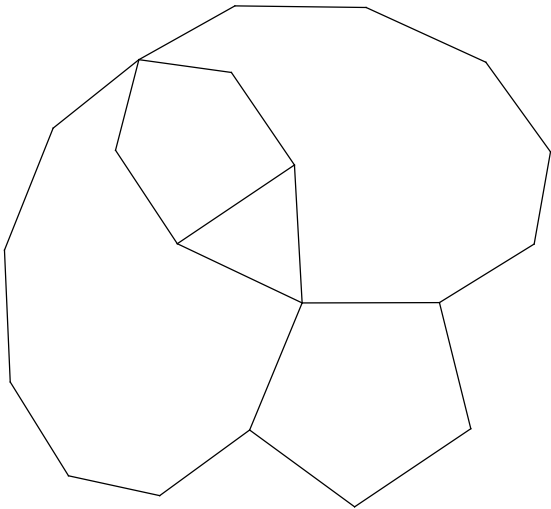
$$\left\{3, \frac{5}{2}, 3, 3, 3, 3\right\}$$



4.

small dodecicosidodecahedron

$$\left\{10, \frac{3}{2}, 10, 5\right\}$$

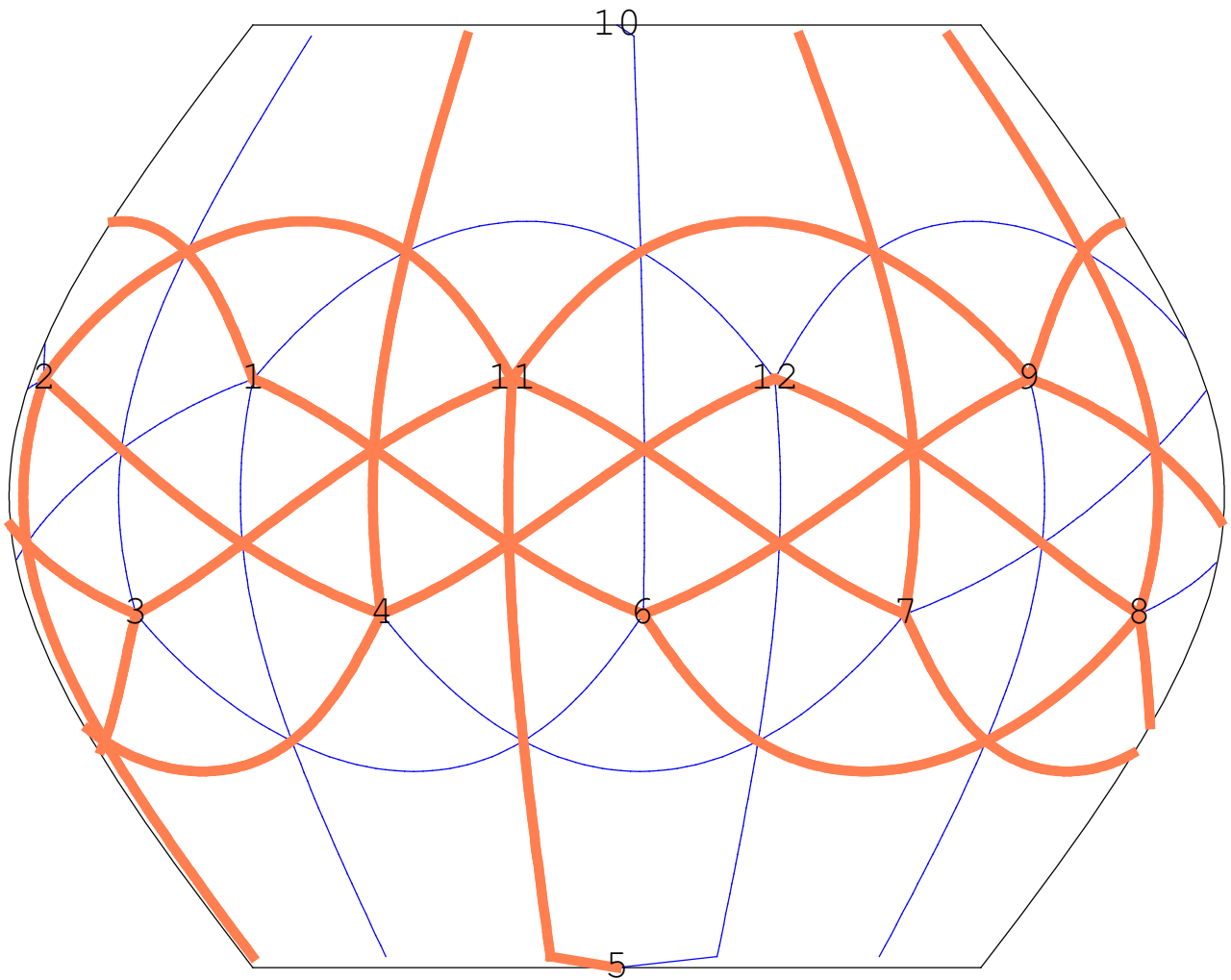
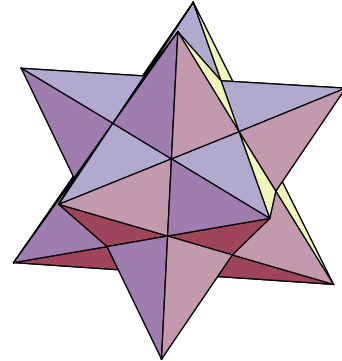
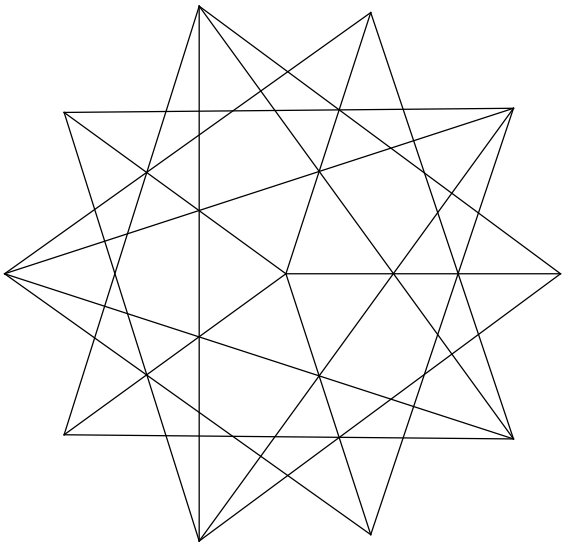




5.

small stellated dodecahedron

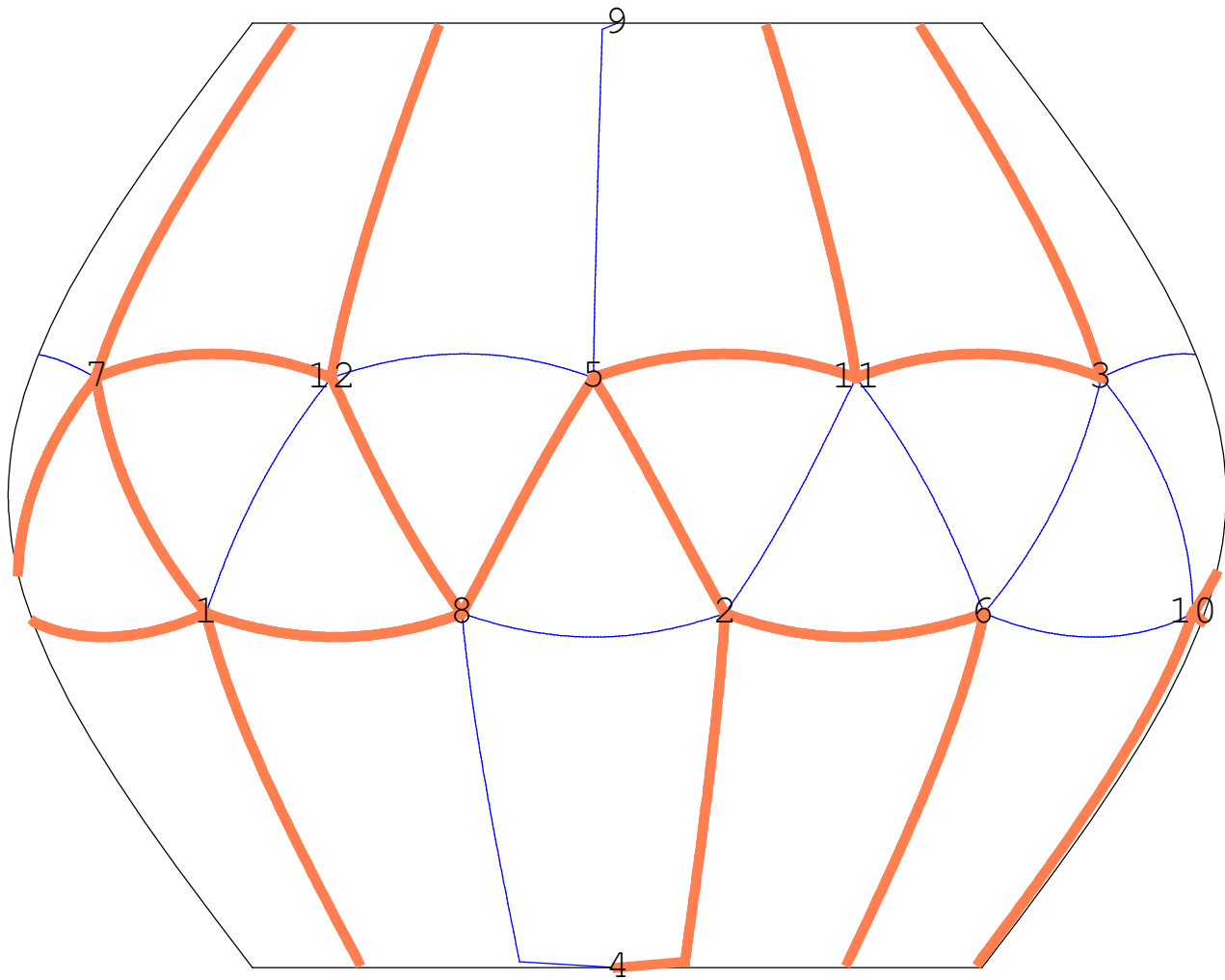
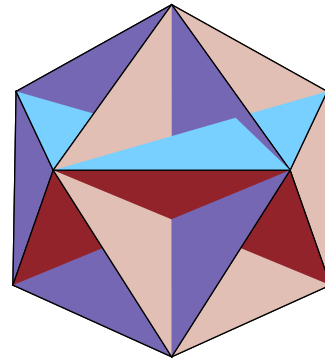
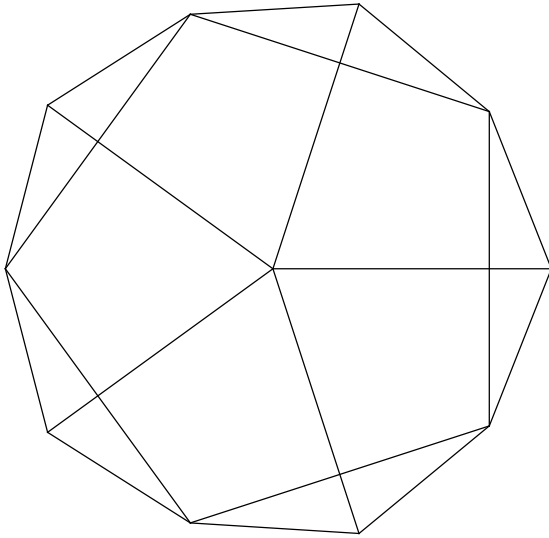
$$\left\{ \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2} \right\}$$



6.

great dodecahedron

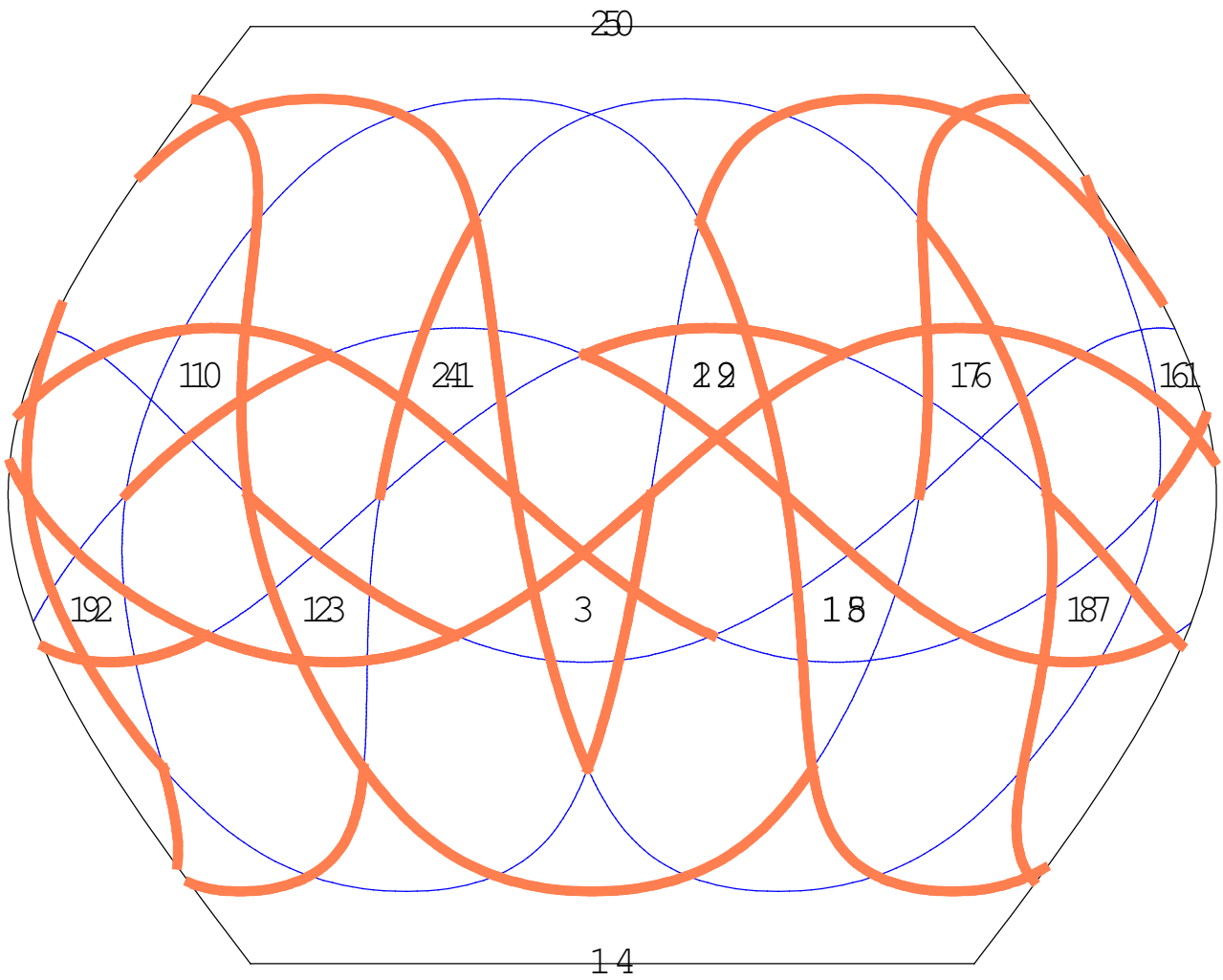
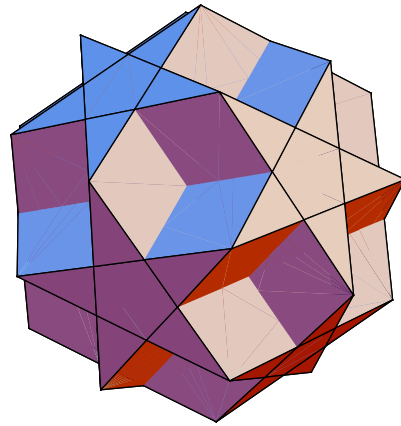
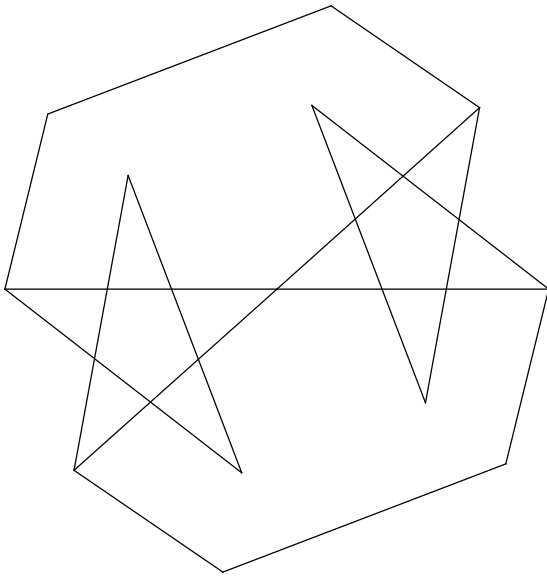
$$\frac{1}{2} \{5, 5, 5, 5, 5\}$$



7.

dodecadodecahedron

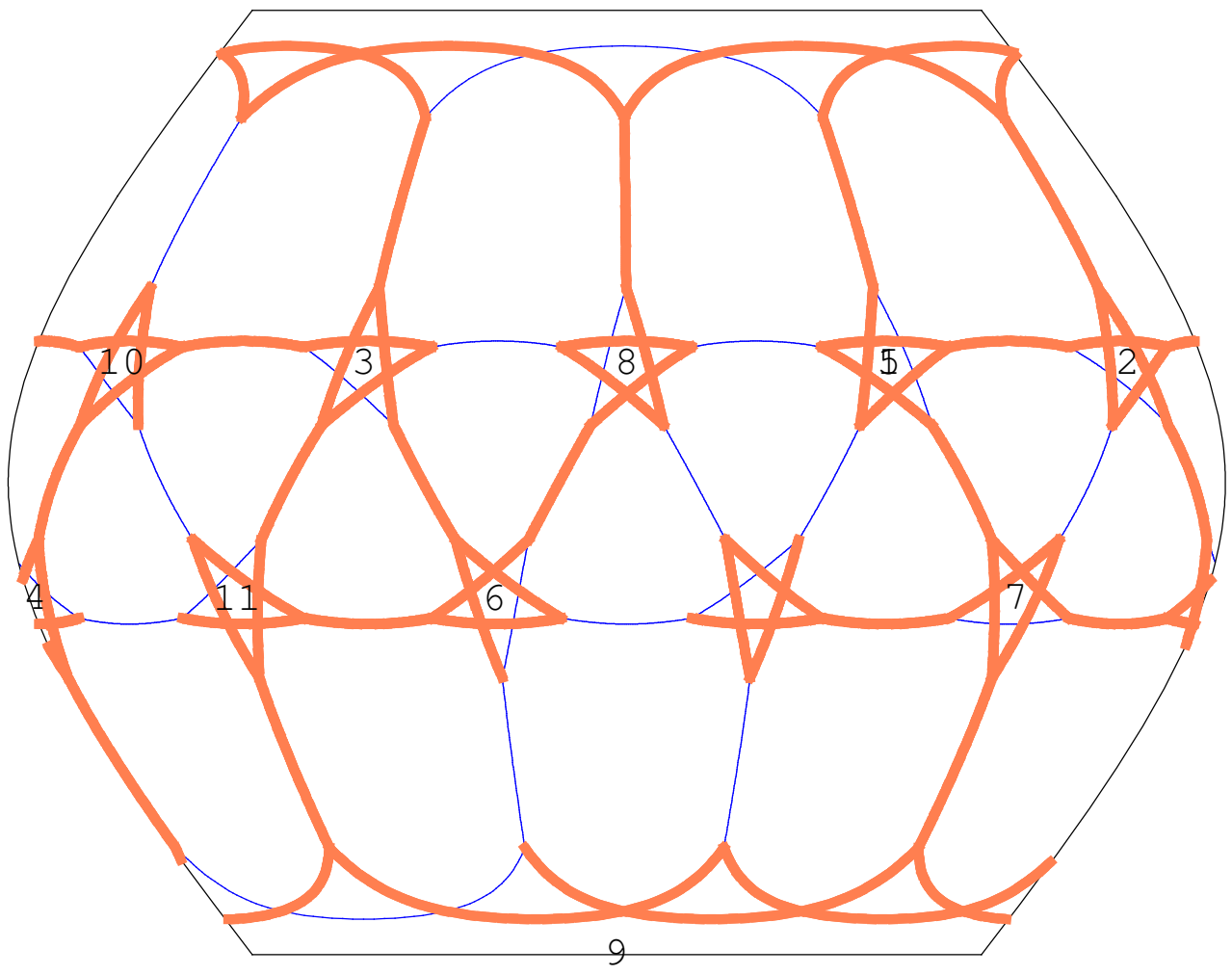
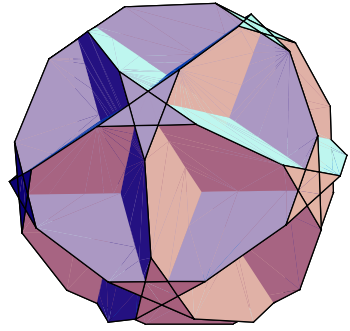
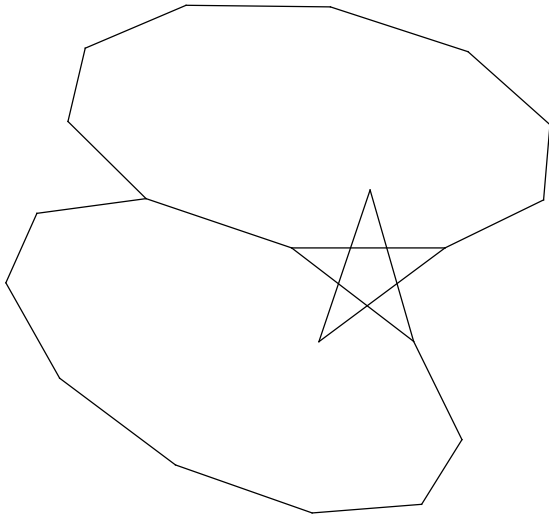
$$\left\{ \frac{5}{2}, 5, \frac{5}{2}, 5 \right\}$$



8.

truncated great dodecahedron

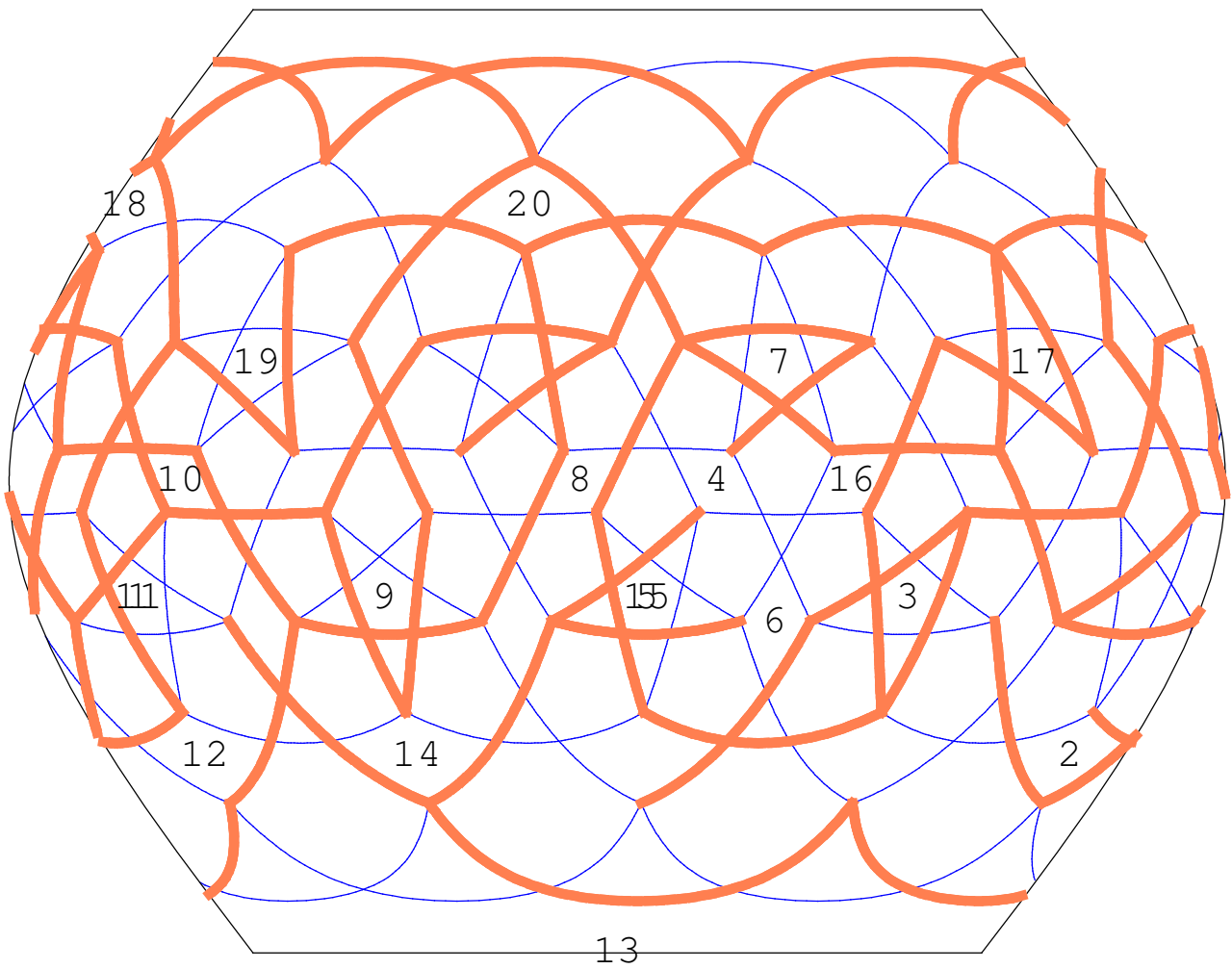
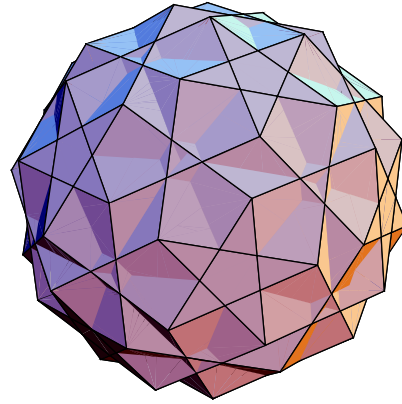
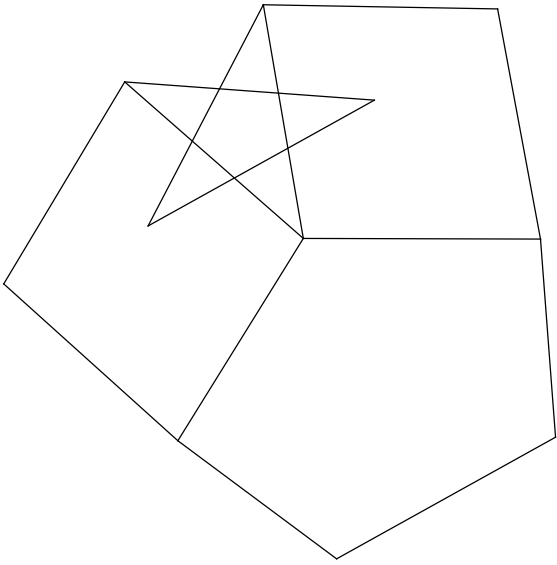
$$\{10, 10, \frac{5}{2}\}$$



9.

rhombidodecadodecahedron

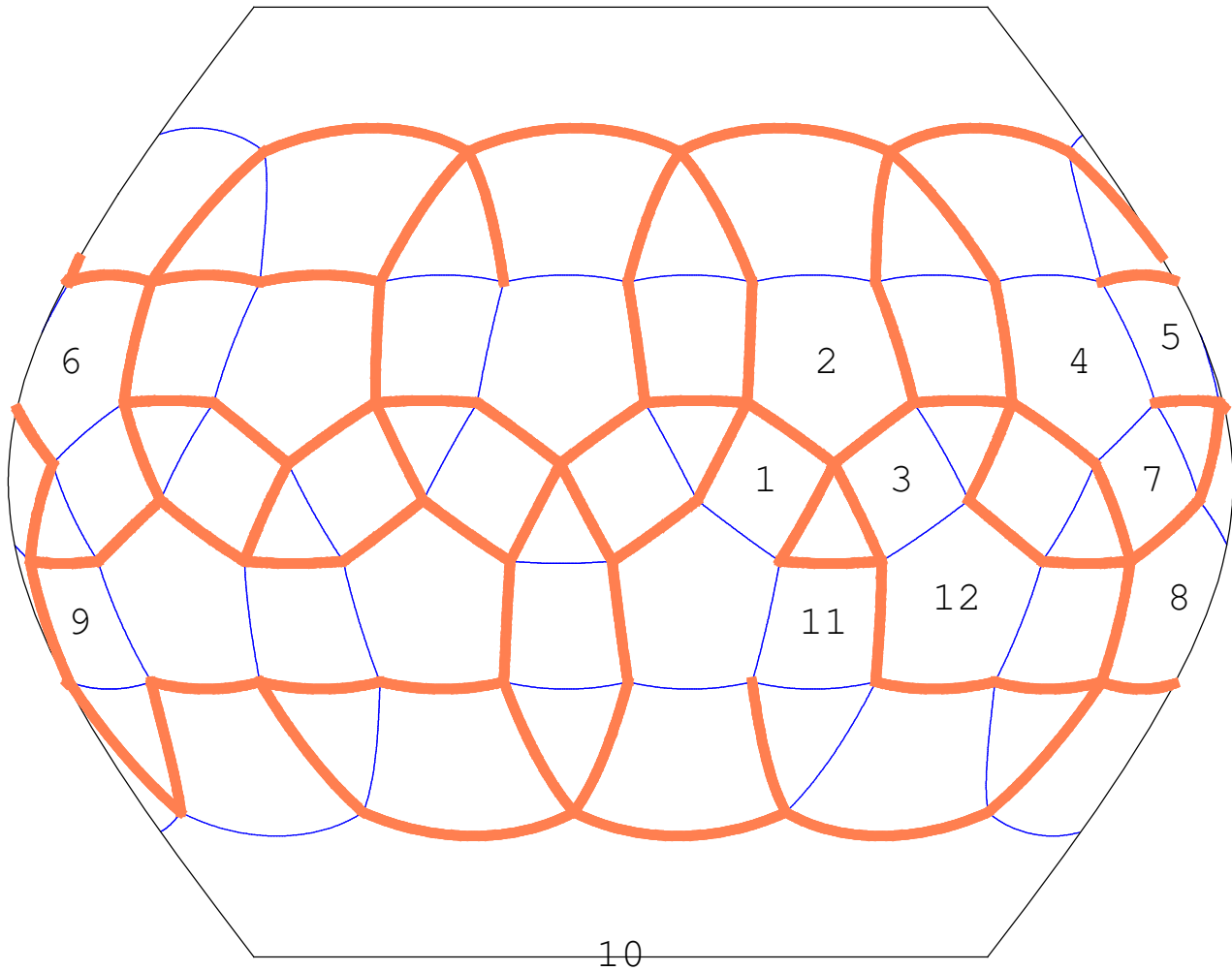
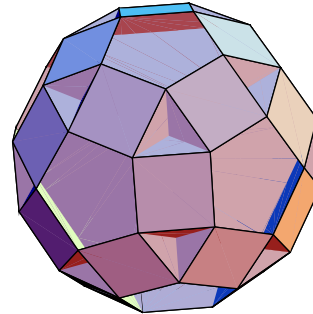
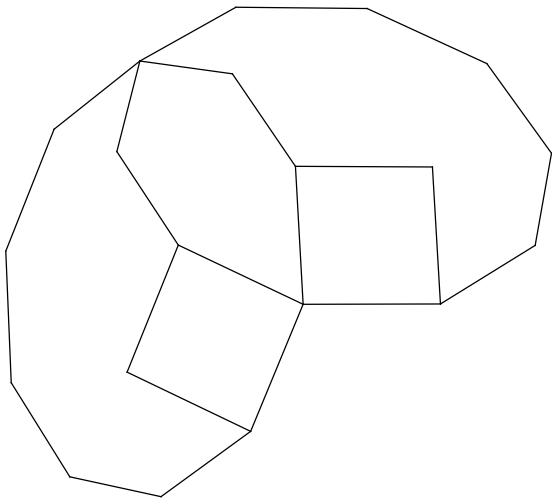
$$\{4, \frac{5}{2}, 4, 5\}$$



10.

small rhombidodecahedron

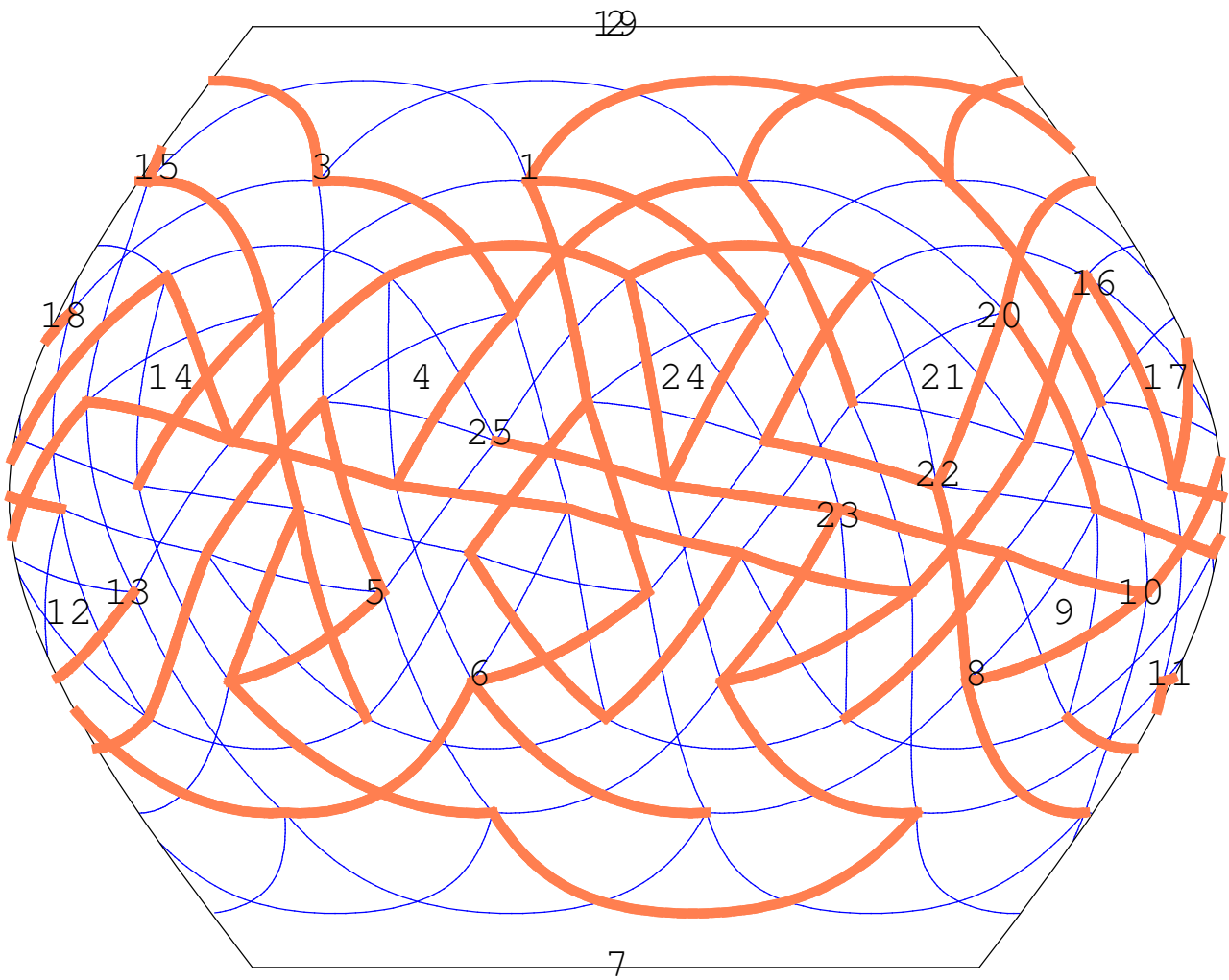
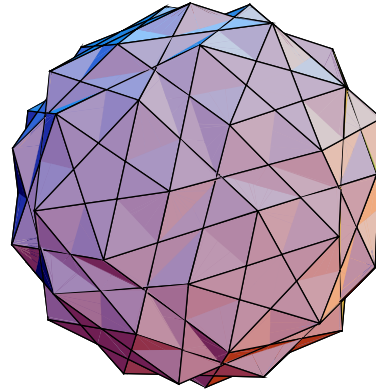
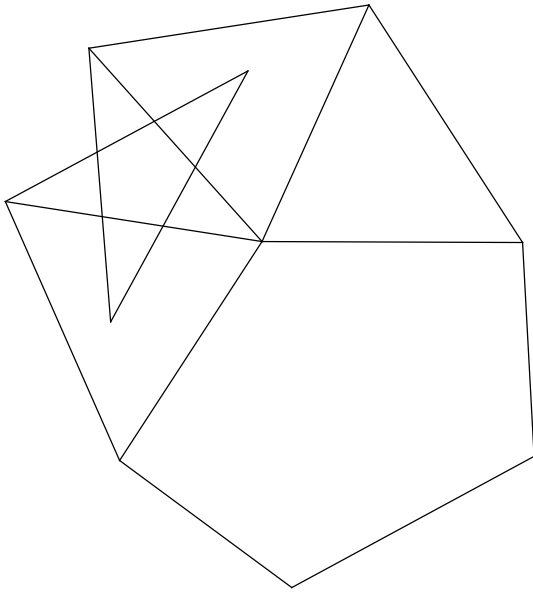
$$\left\{10, 4, \frac{10}{9}, \frac{4}{3}\right\}$$



11.

snub dodecadodecahedron

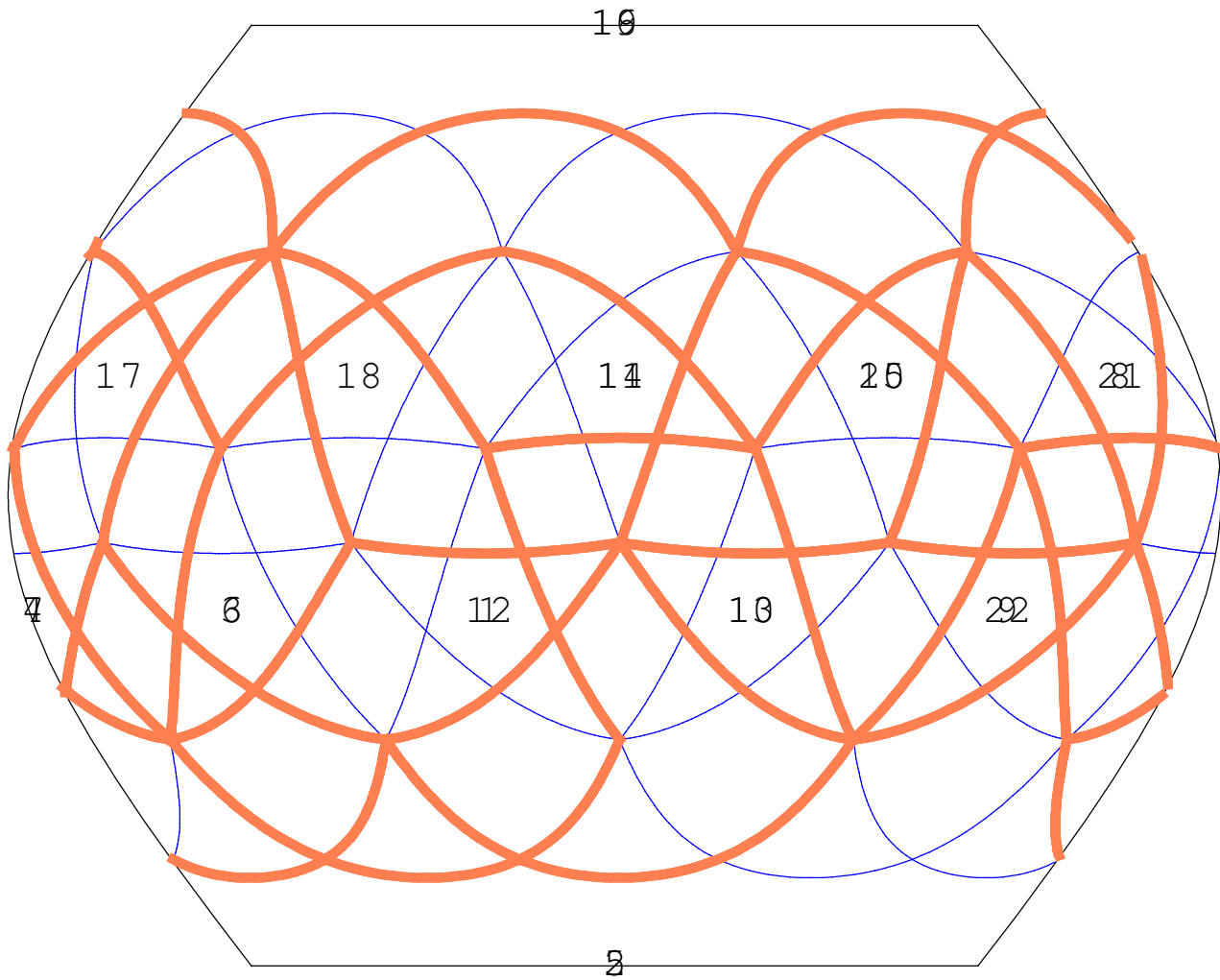
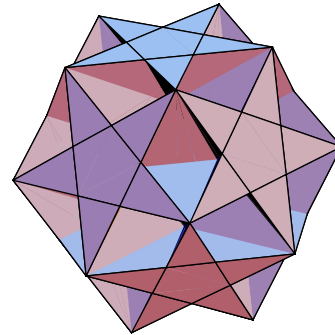
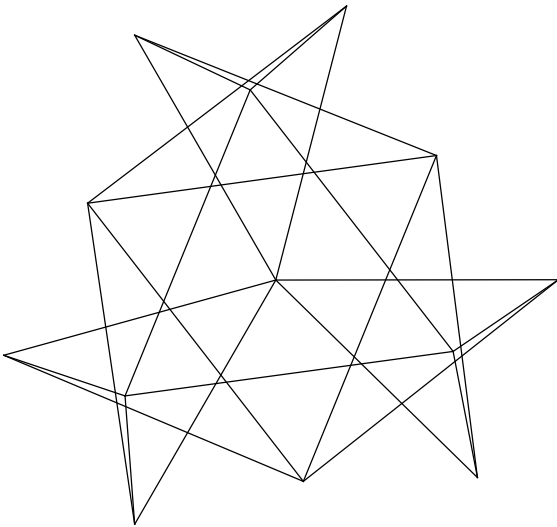
$$\{3, 3, \frac{5}{2}, 3, 5\}$$



12.

ditrigrinal dodecadodecahedron

$$\left\{ \frac{5}{3}, 5, \frac{5}{3}, 5, \frac{5}{3}, 5 \right\}$$

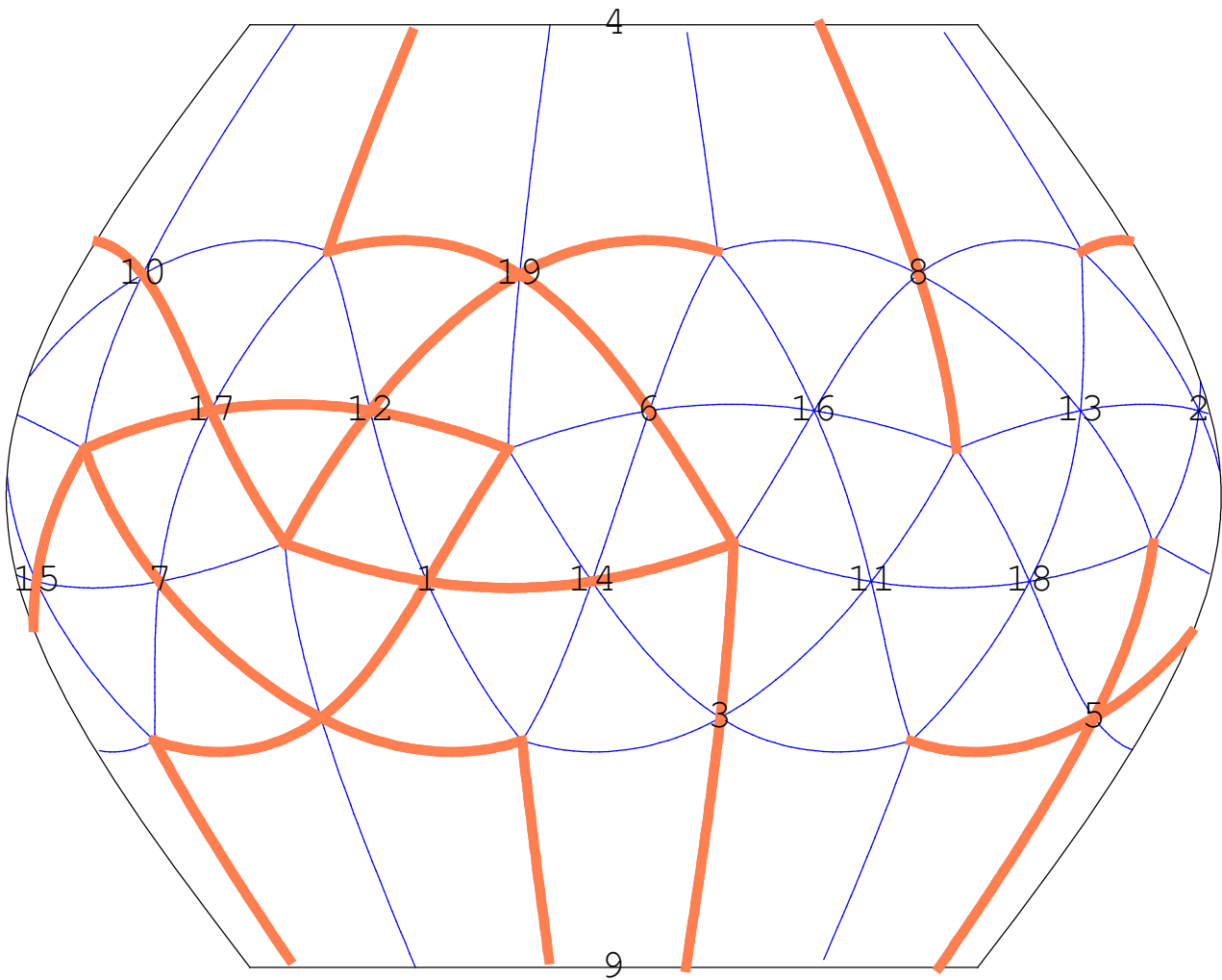
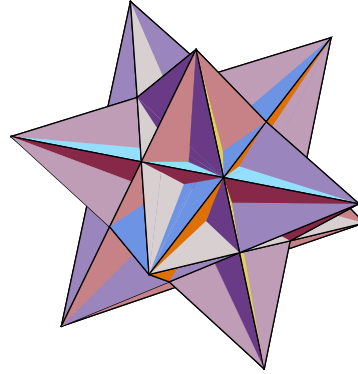
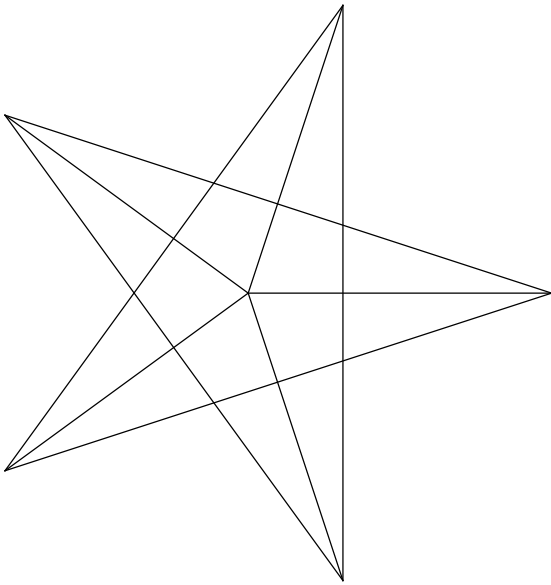




13.

great icosahedron

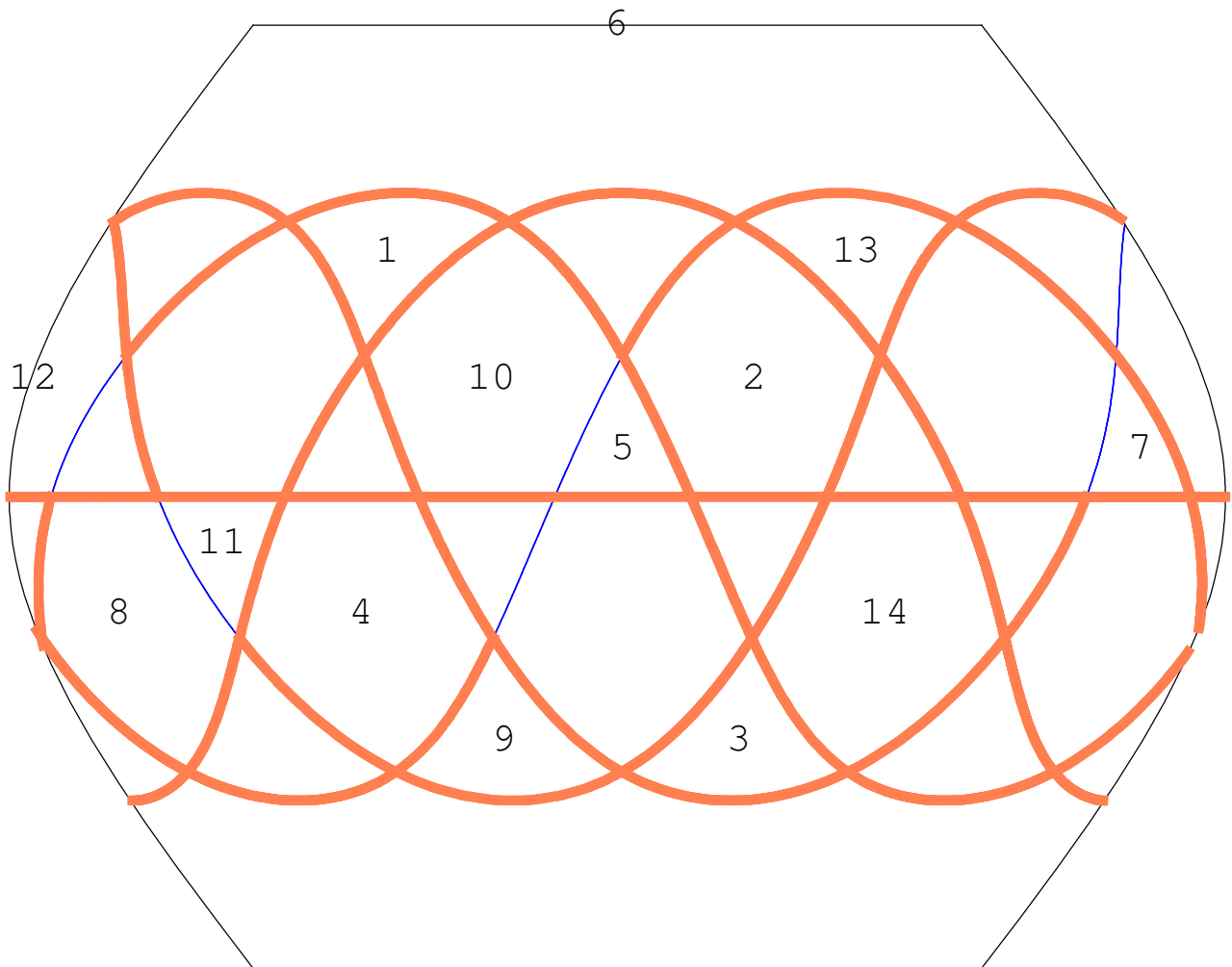
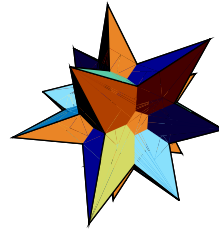
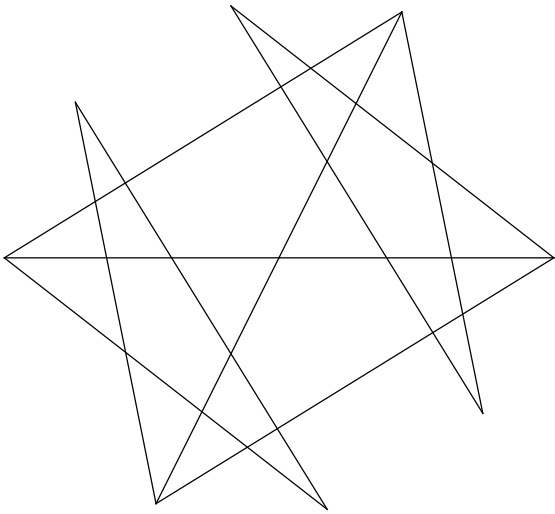
$$\frac{1}{2} \{3, 3, 3, 3, 3\}$$



14.

great icosidodecahedron

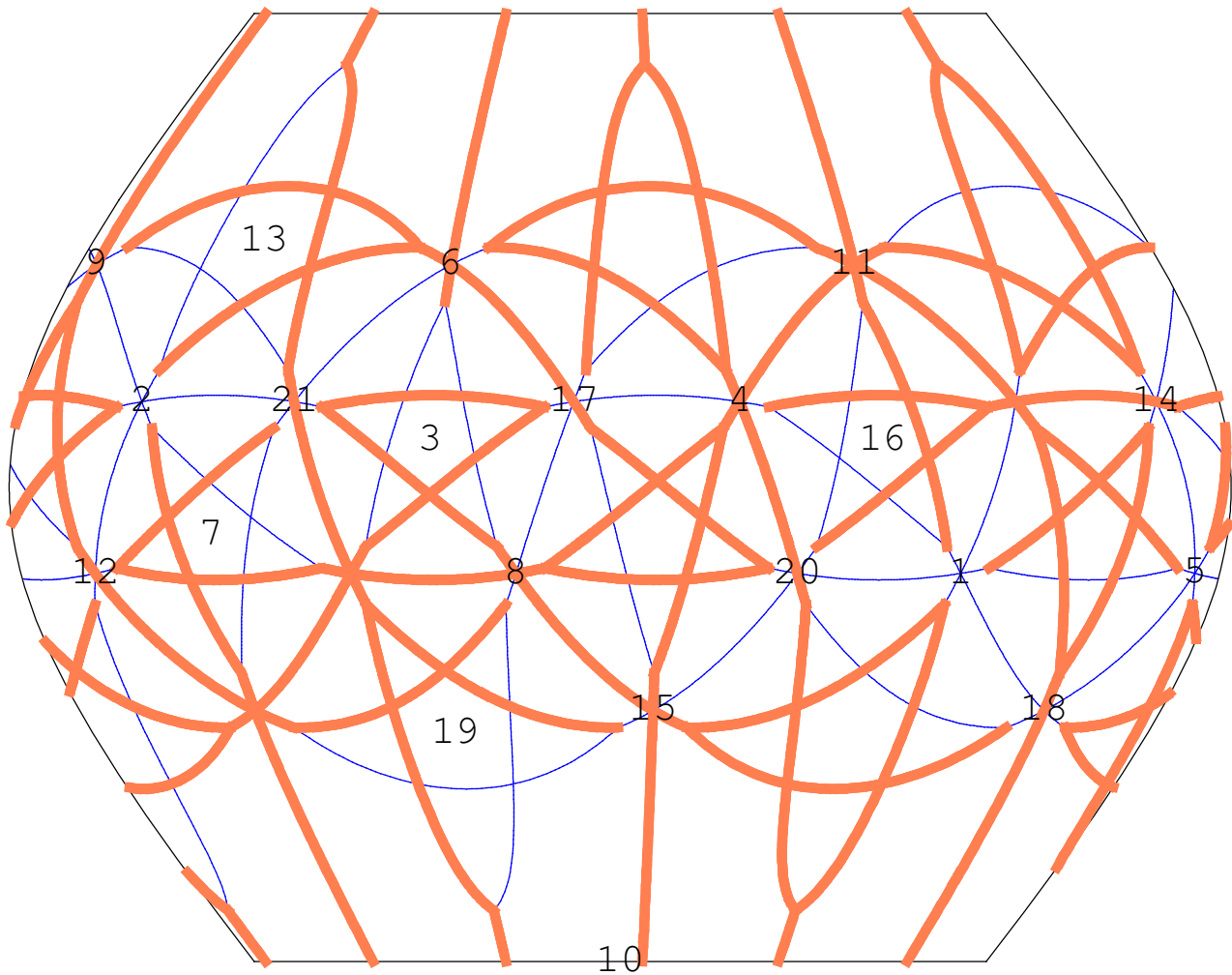
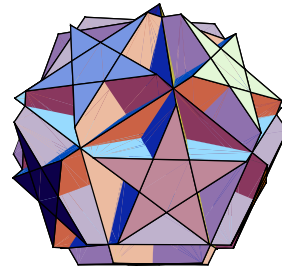
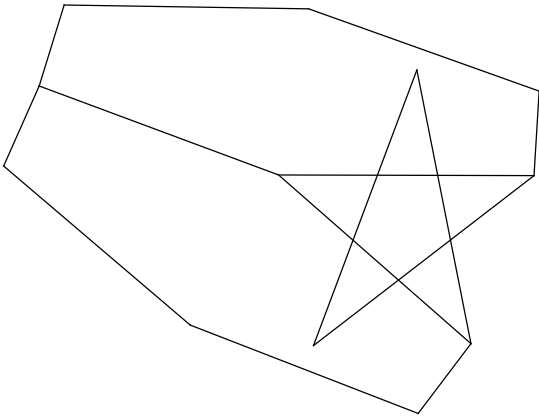
$$\left\{ \frac{5}{2}, 3, \frac{5}{2}, 3 \right\}$$



15.

great truncated icosahedron

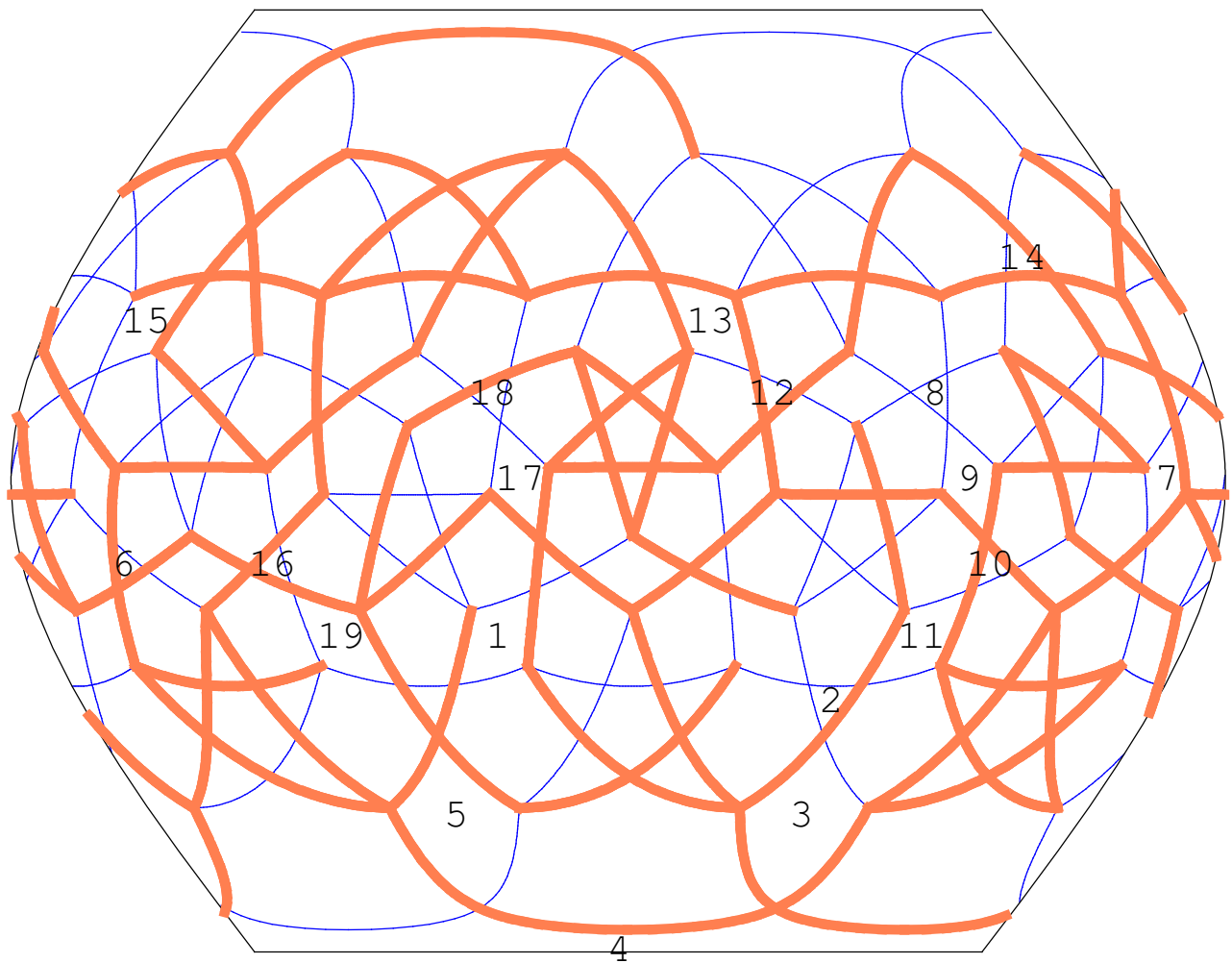
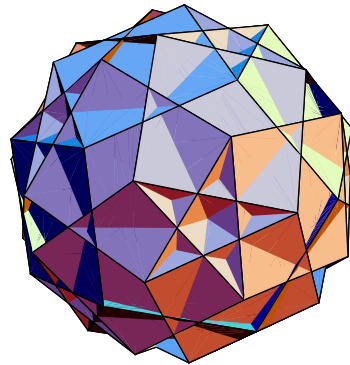
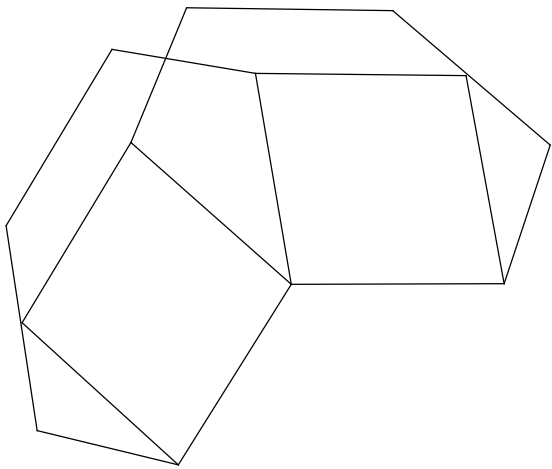
$$\{6, 6, \frac{5}{2}\}$$



16.

rhombicosahedron

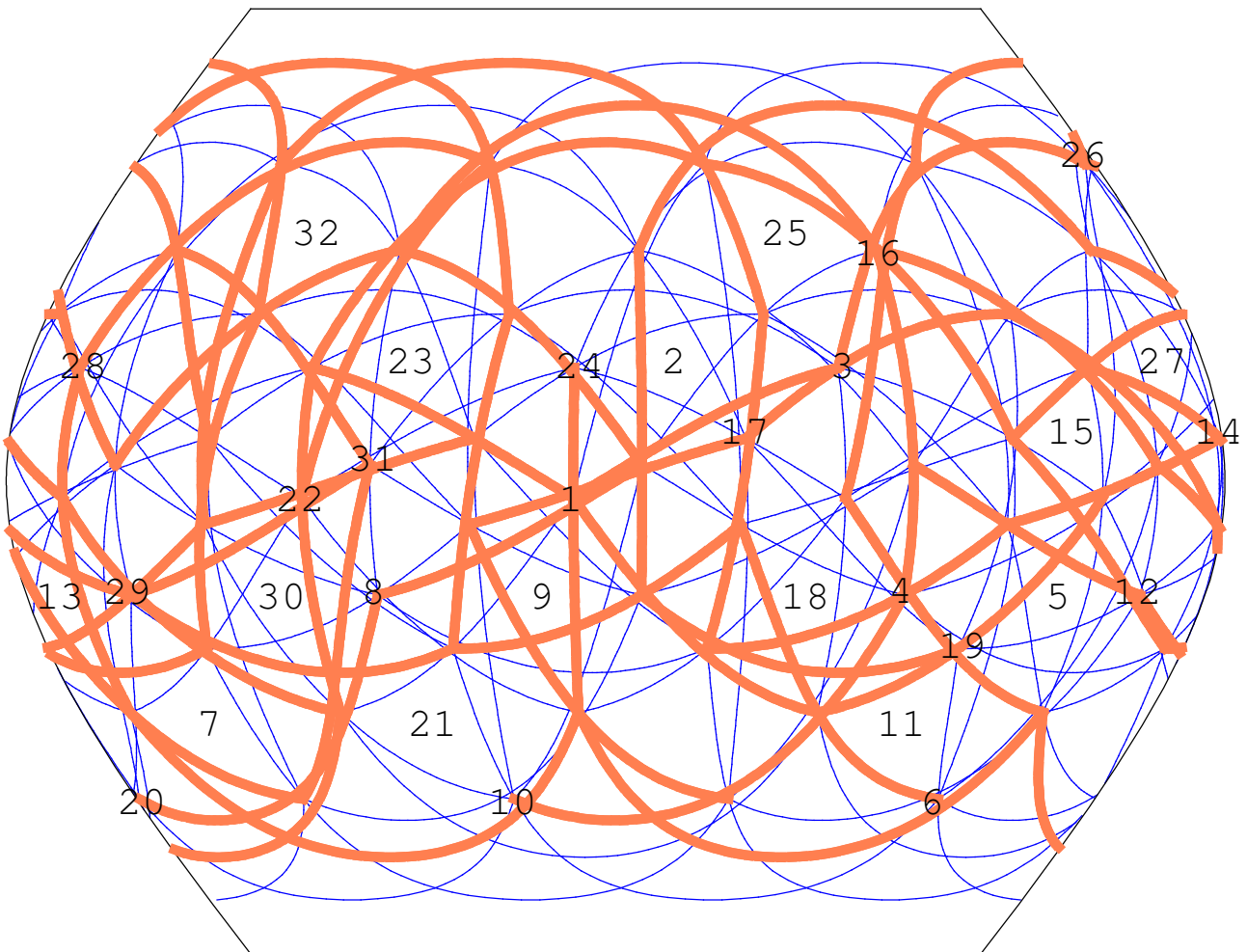
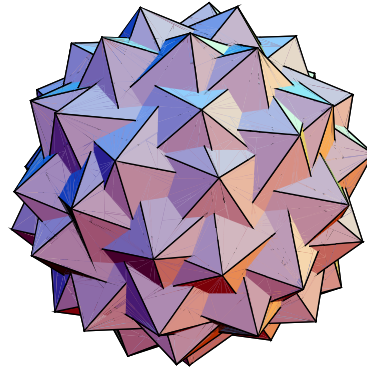
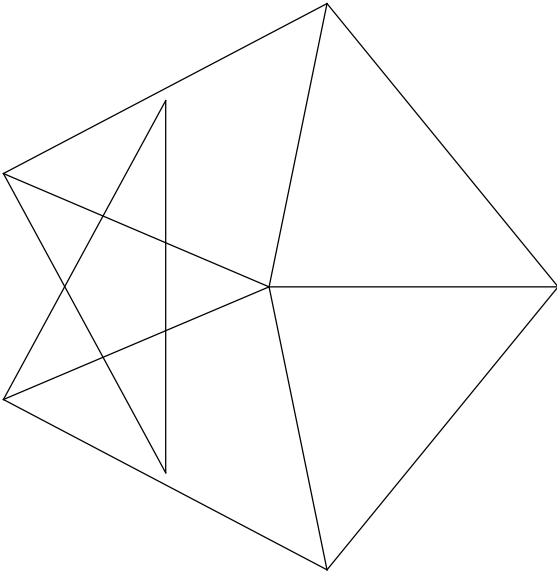
$$\{6, 4, \frac{6}{5}, \frac{4}{3}\}$$



17.

great snub icosidodecahedron

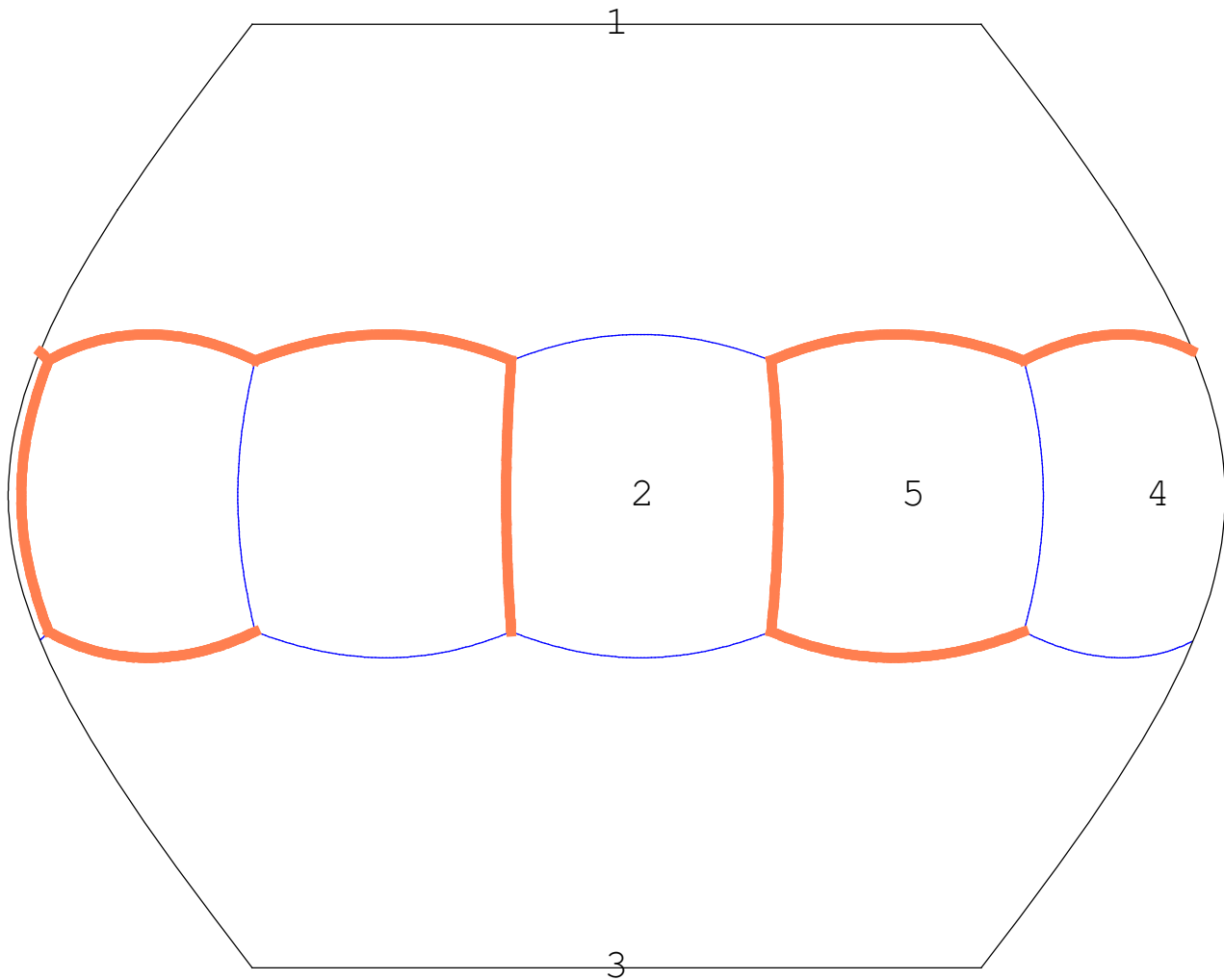
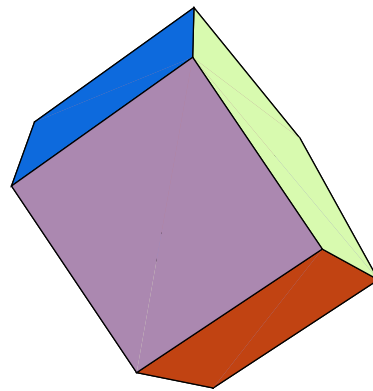
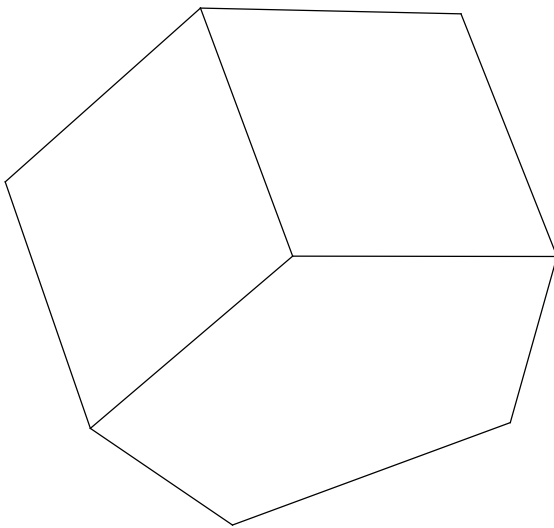
$$\{3, 3, \frac{5}{2}, 3, 3\}$$



18.

pentagonal prism

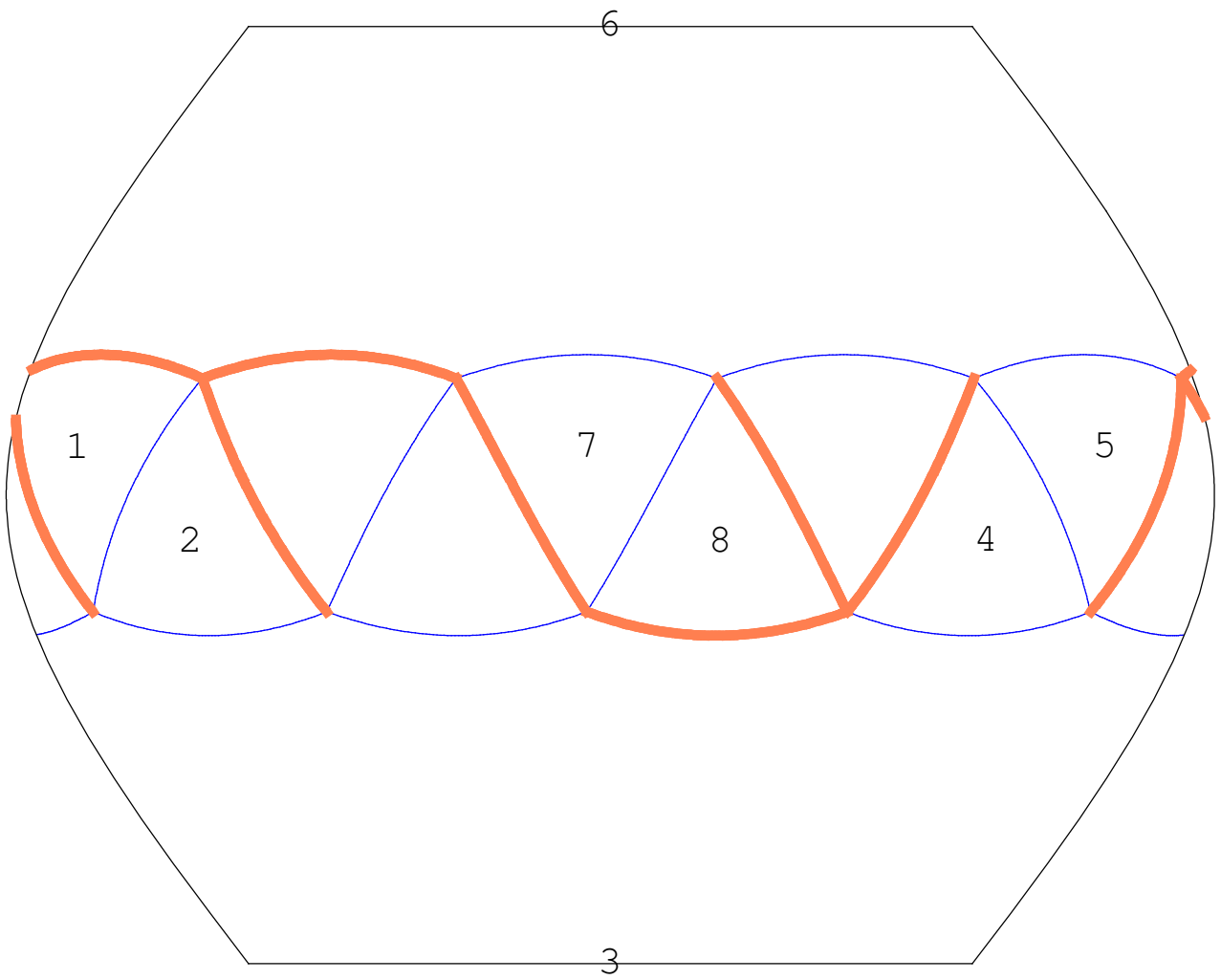
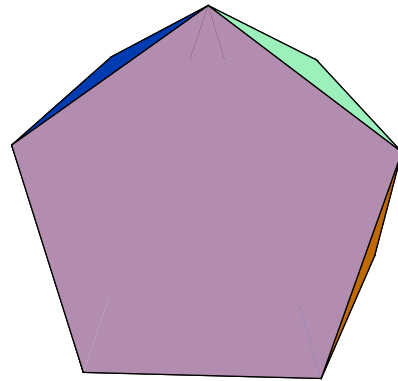
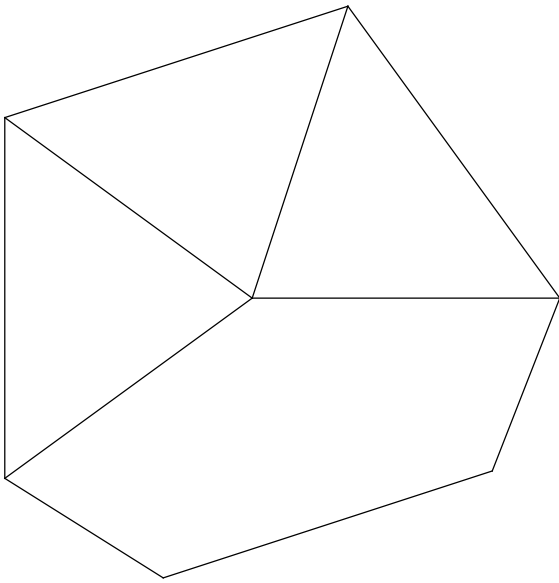
{4, 4, 5}



19.

pentagonal antiprism

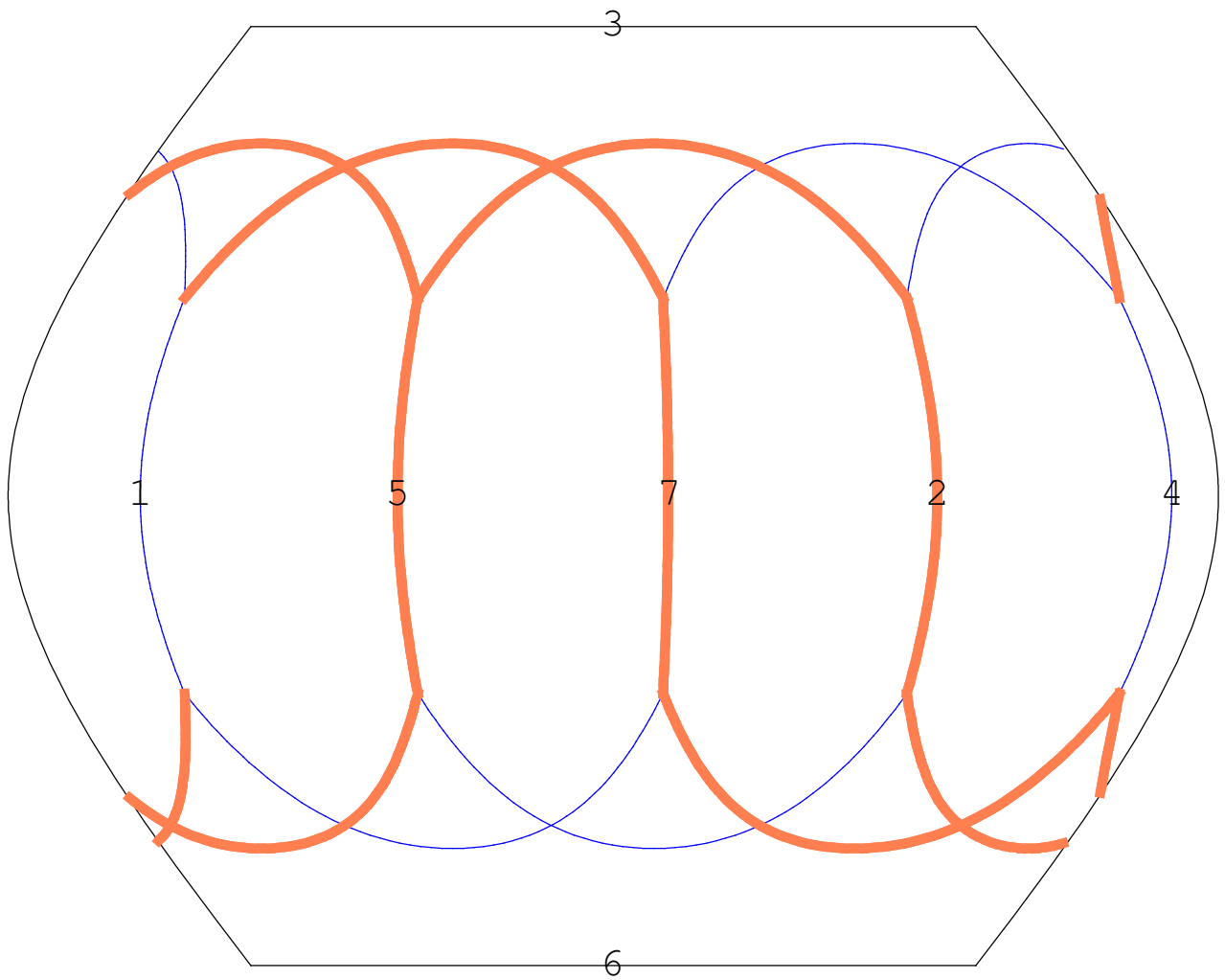
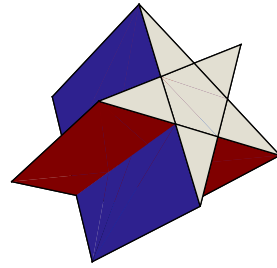
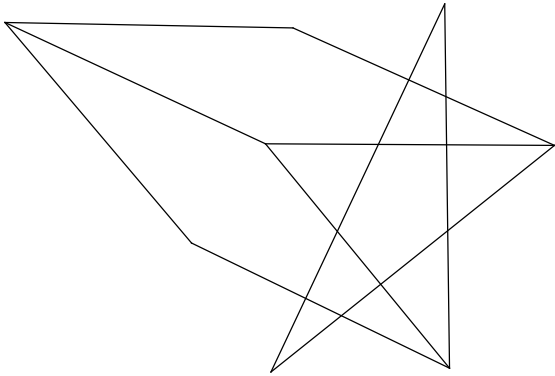
{3, 3, 3, 5}



20.

pentagrammic prism

$$\{4, 4, \frac{5}{2}\}$$

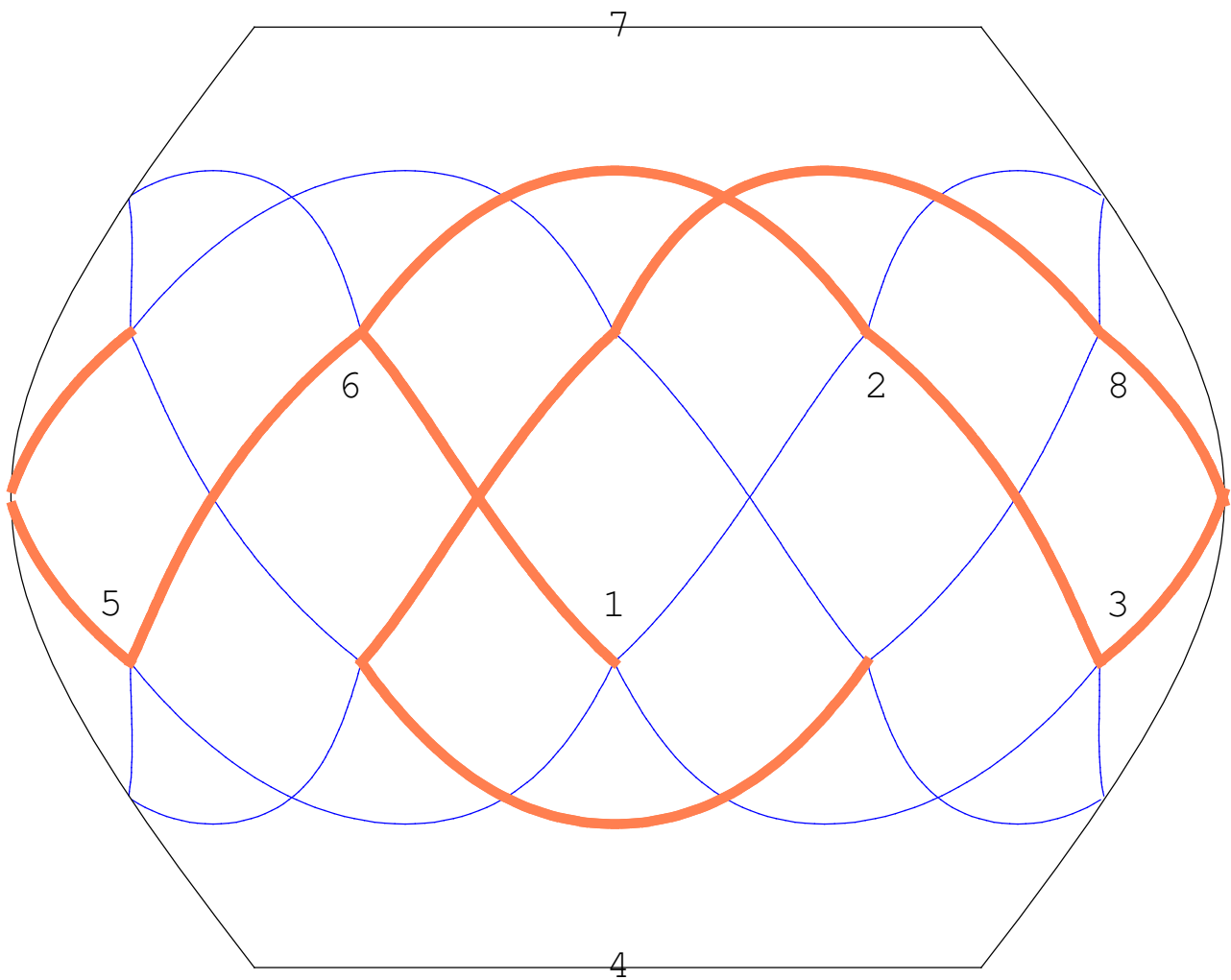
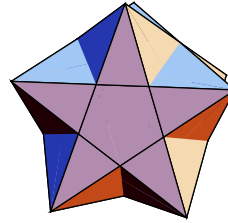
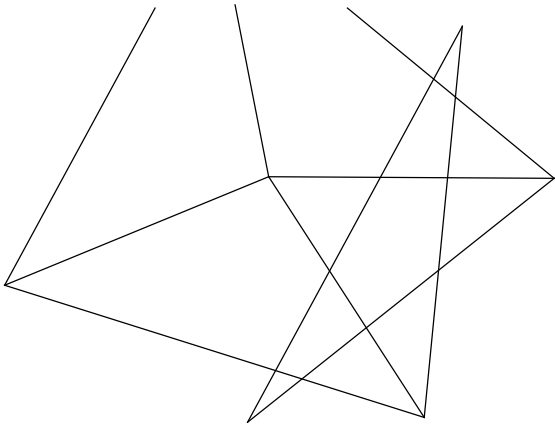




21.

pentagrammic antiprism

$$\{3, 3, 3, \frac{5}{2}\}$$



22.

pentagrammic crossed antiprism

$$\{3, 3, 3, \frac{5}{3}\}$$

