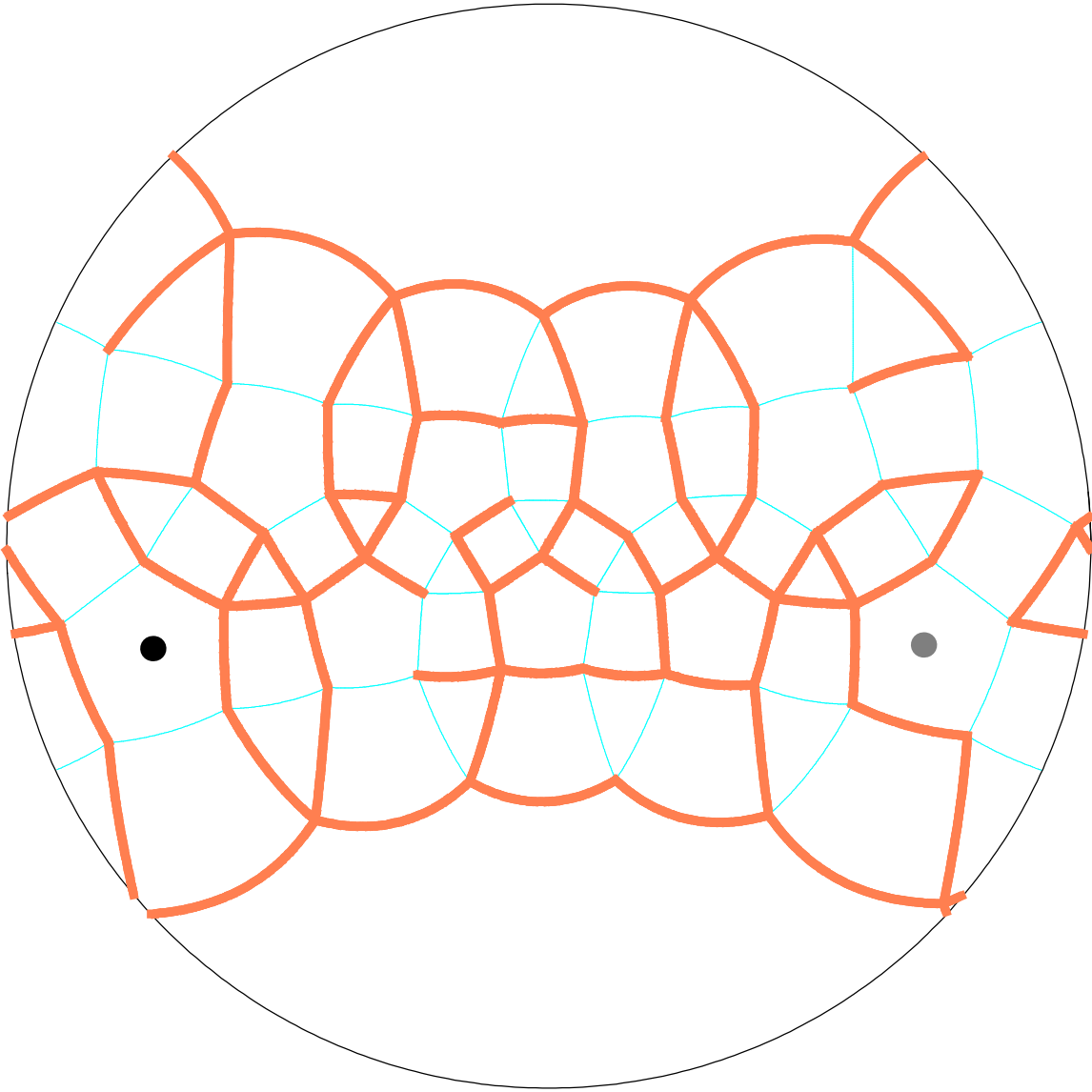


Izidor Hafner

Mazes on Uniform Polyhedra

Lagrange projection

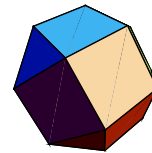
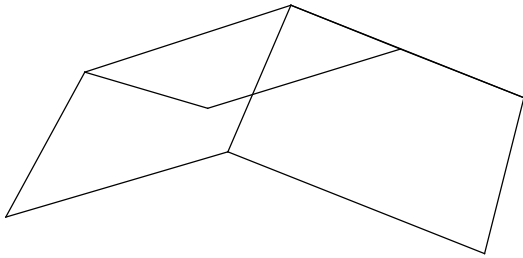


Introduction

Let us take an example. We are given a uniform polyhedron.

rhombicuboctahedron

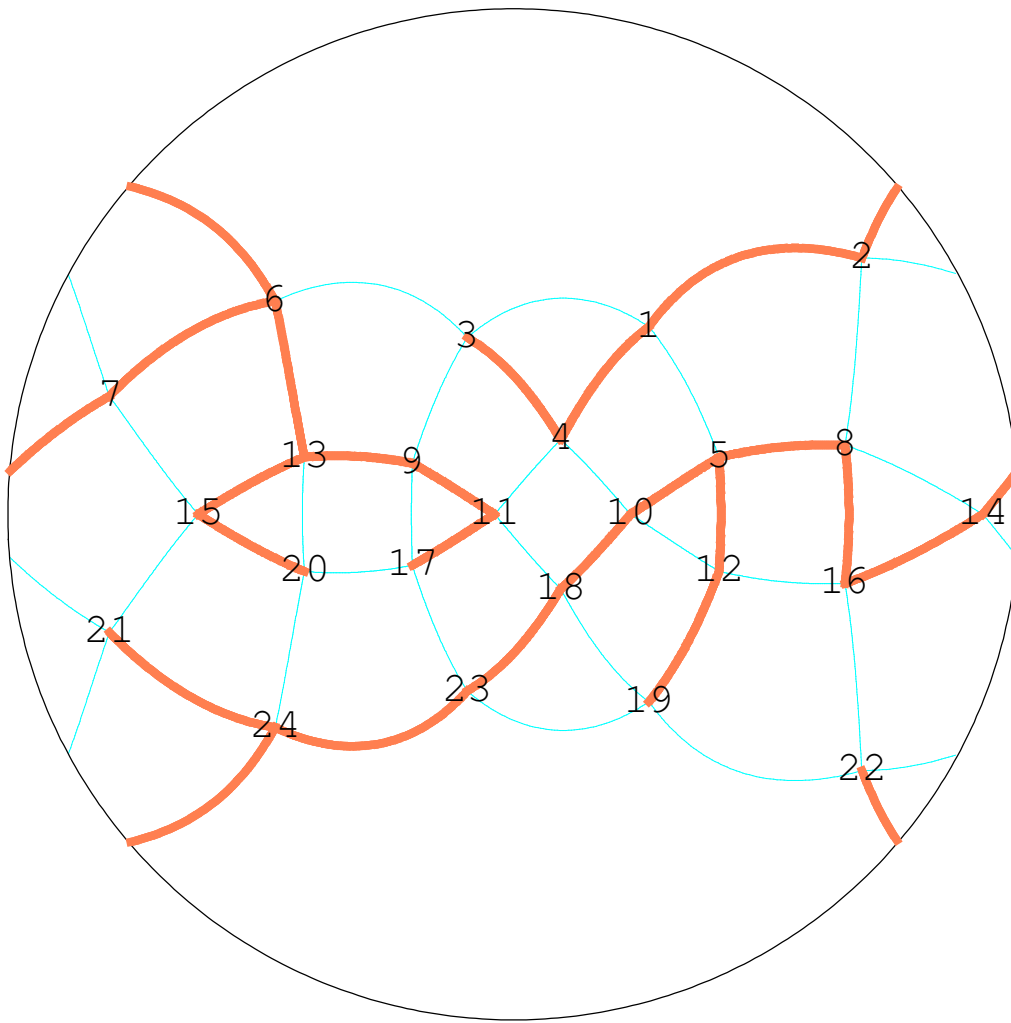
{4, 3, 4, 4}



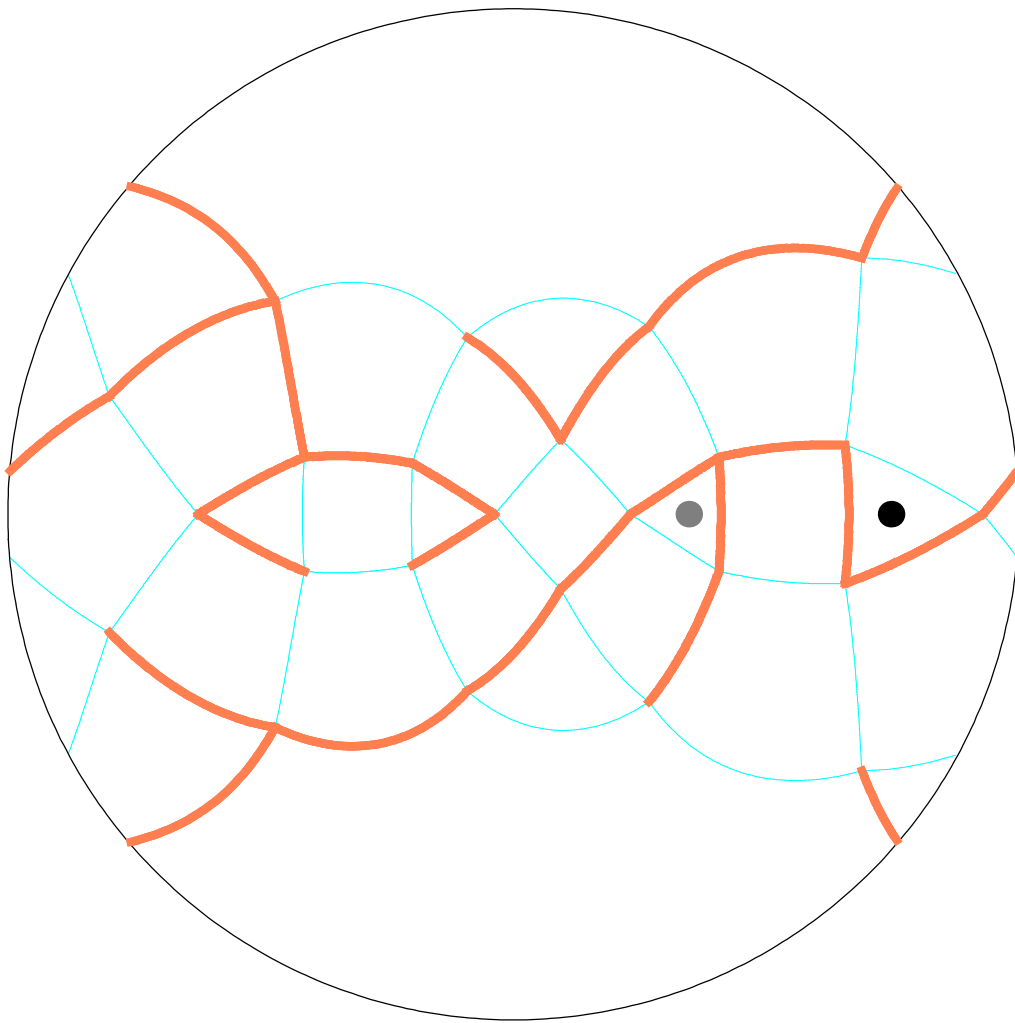
In Mathematica the polyhedron is given by a list of faces and with a list of coordinates of vertices [Roman E. Maeder, *The Mathematica Programmer II*, Academic Press 1996]. The list of faces consists of a list of lists, where a face is represented by a list of vertices, which is given by a matrix. Let us show the first five faces:

$$\begin{pmatrix} \{1, 2, 6, 3\} \\ \{1, 3, 4\} \\ \{1, 4, 10, 5\} \\ \{1, 5, 8, 2\} \\ \{2, 8, 14, 7\} \end{pmatrix}$$

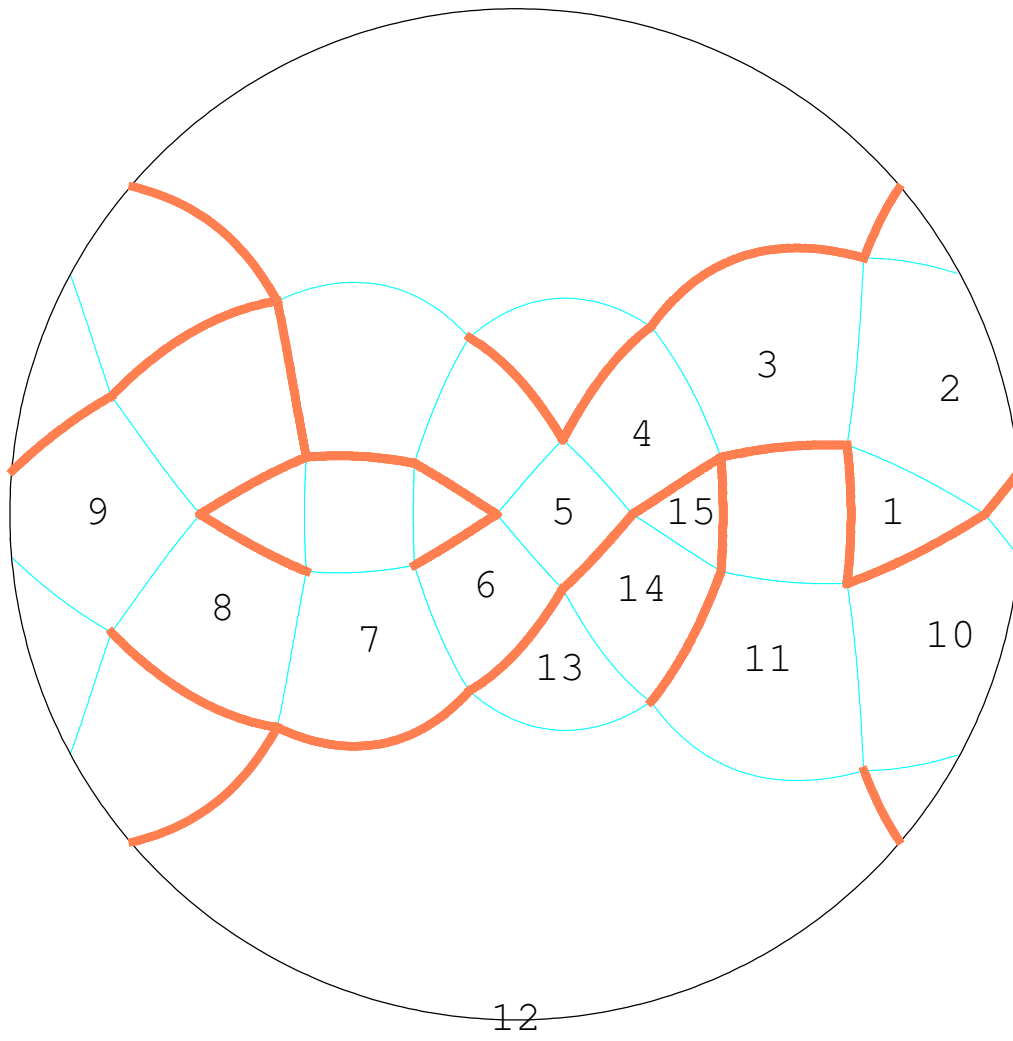
The next two figures represent faces and vertices. The polyhedron is projected onto a sphere and the sphere is projected by a cartographic projection.



The problem is to find the path from the black dot to gray dot, where thick lines represent walls of a maze.



The solution is given by a list of faces passed from the black to gray dot.

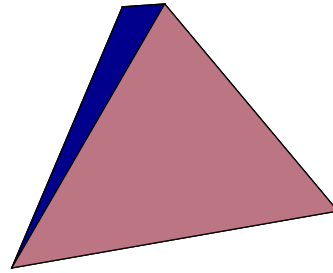
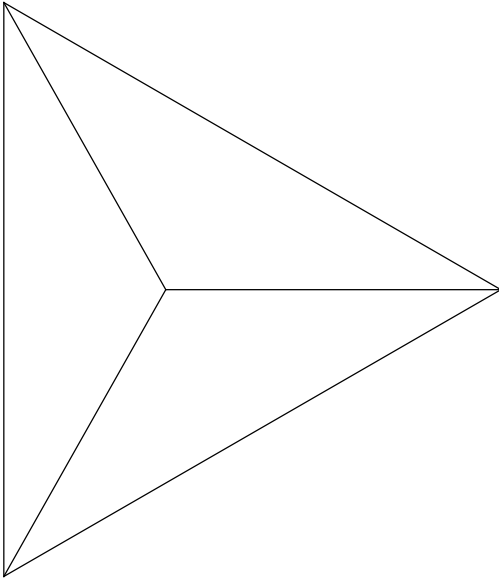


Problems

1.

tetrahedron

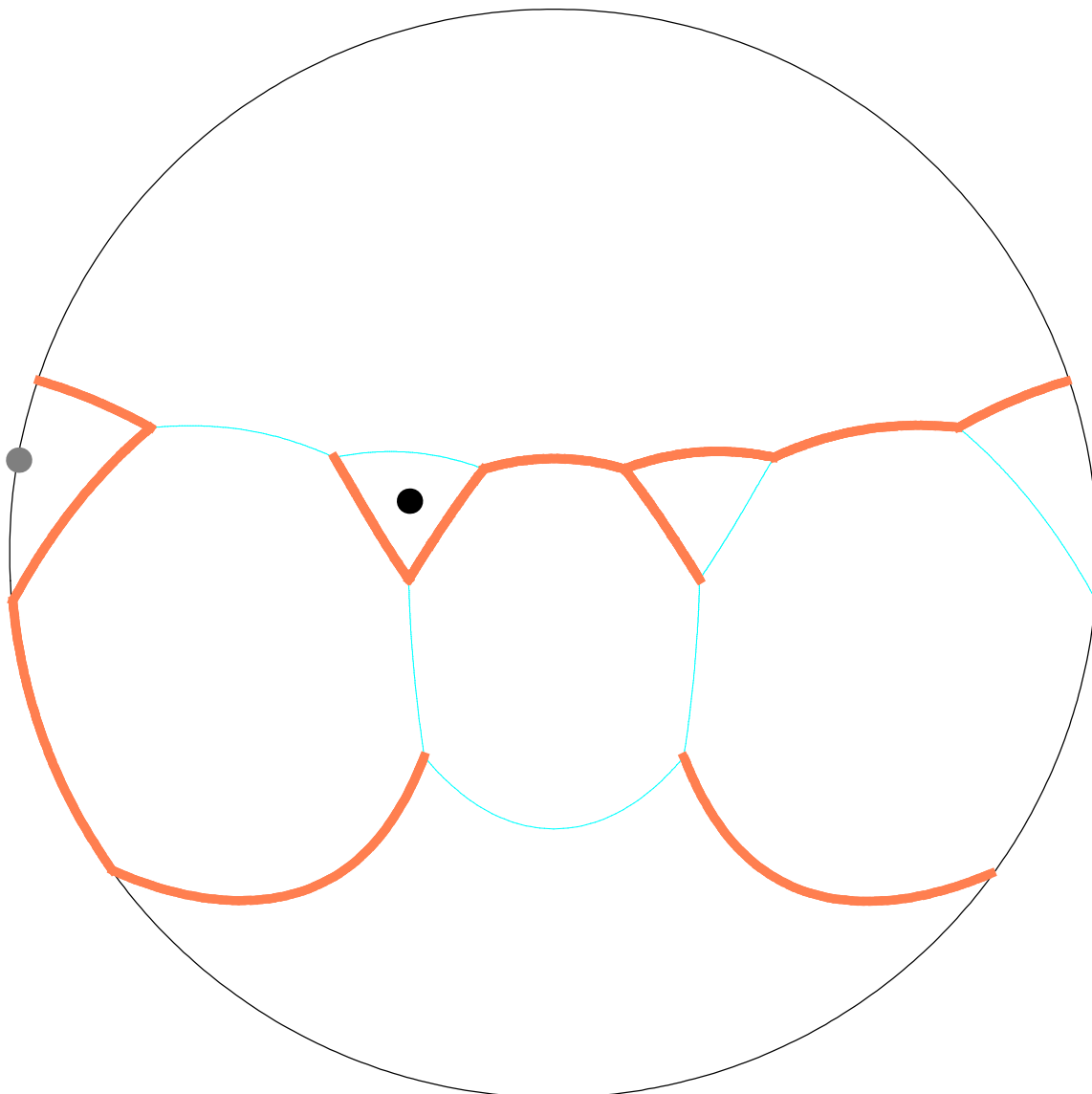
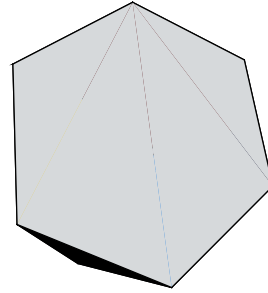
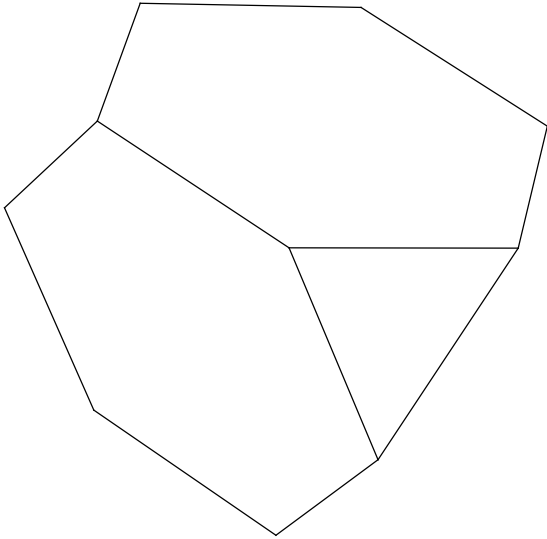
{3, 3, 3}



2.

truncated tetrahedron

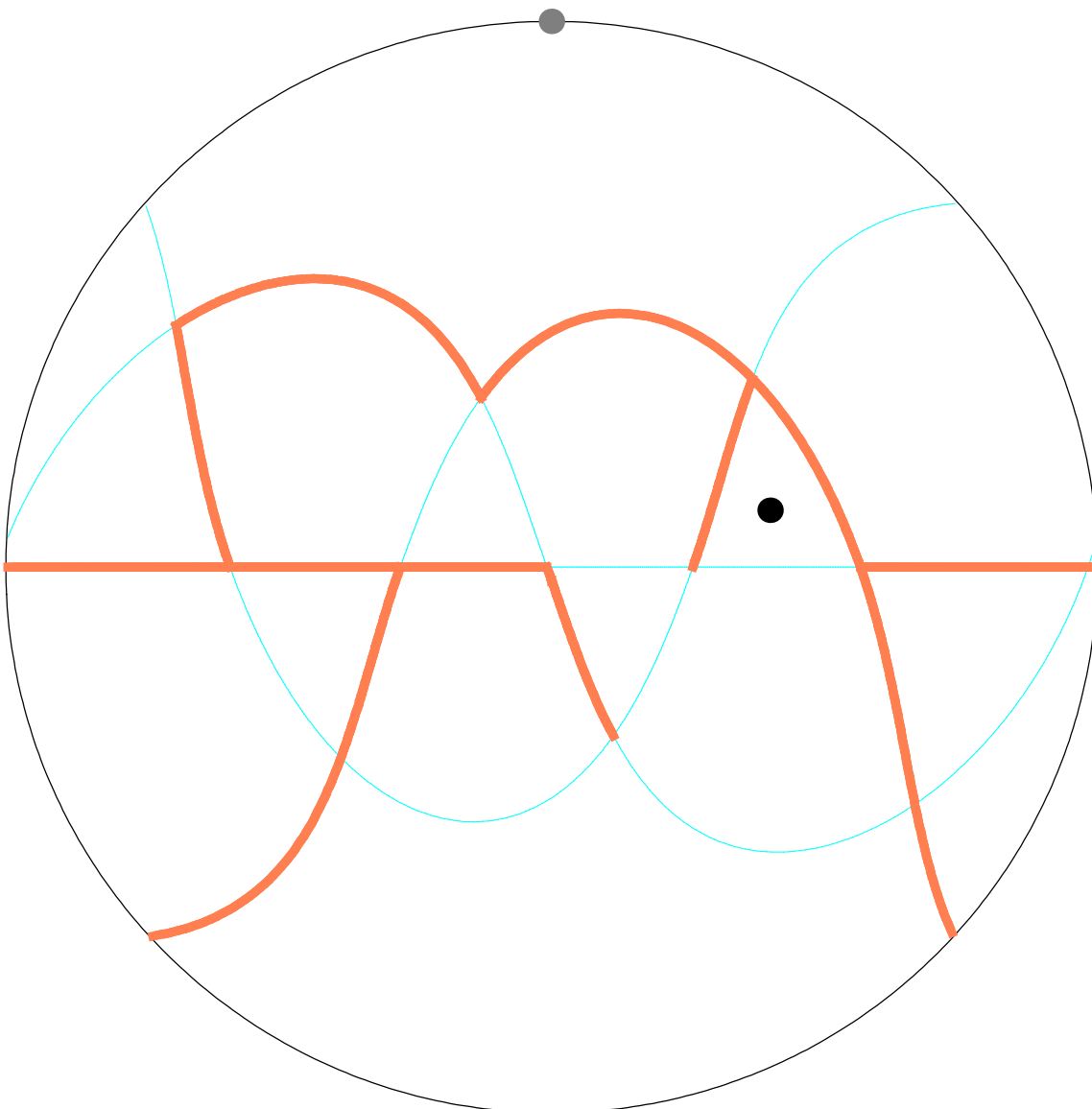
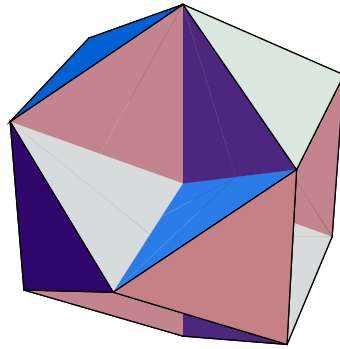
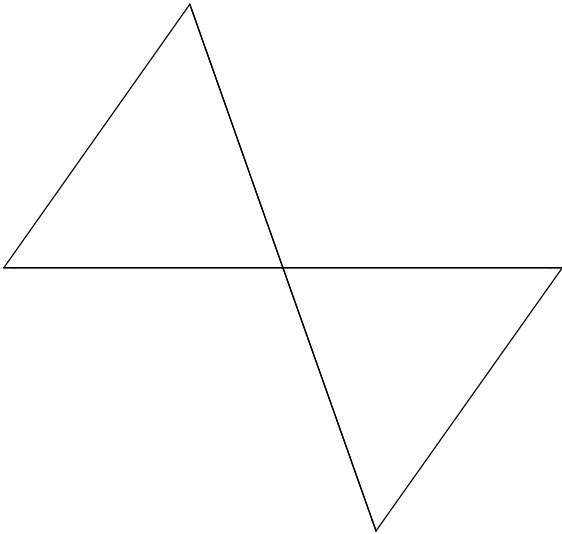
{6, 6, 3}



3.

octahemioctahedron

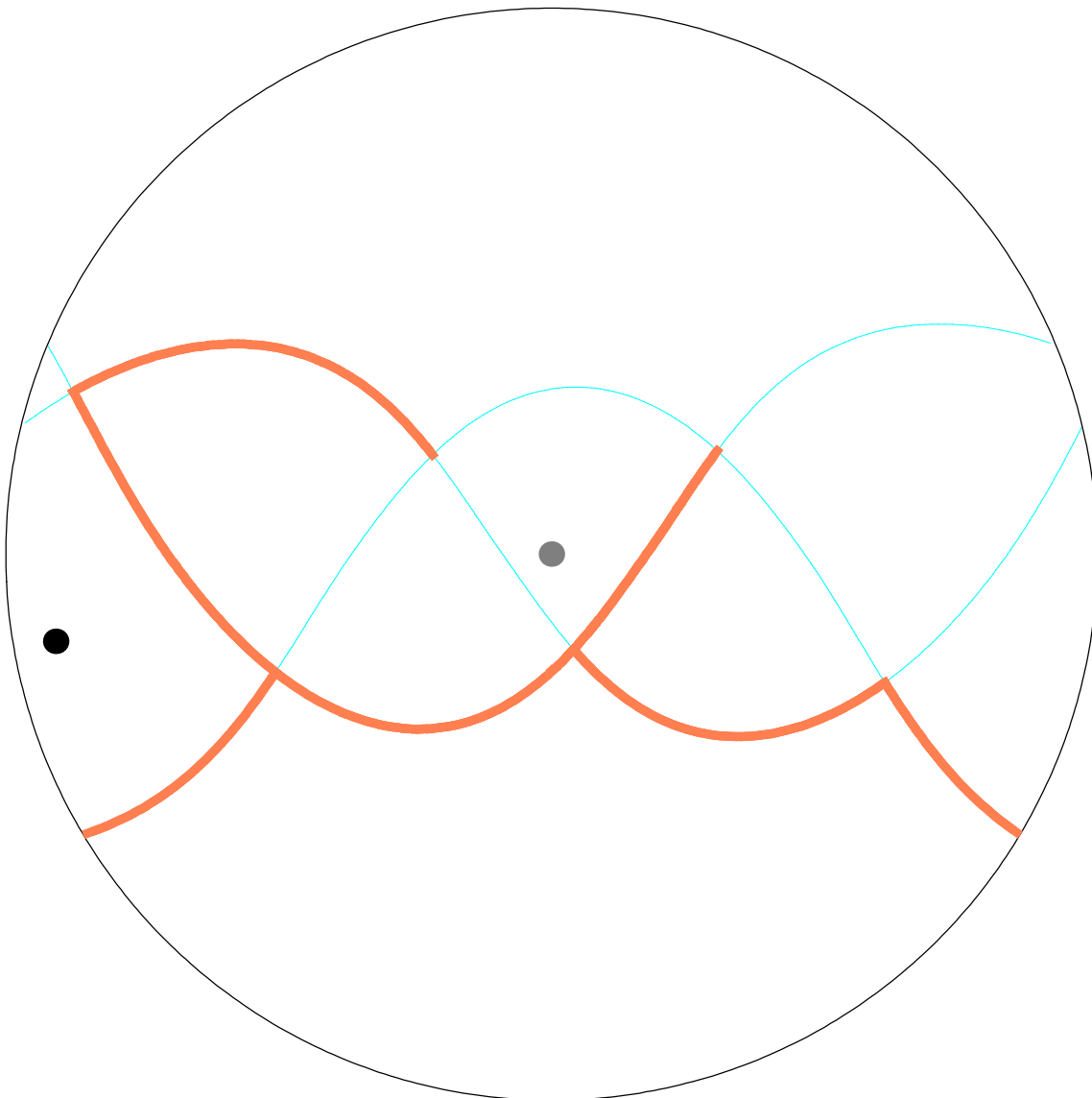
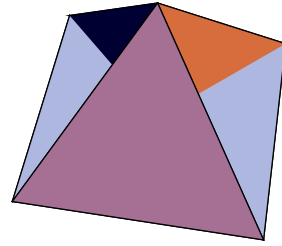
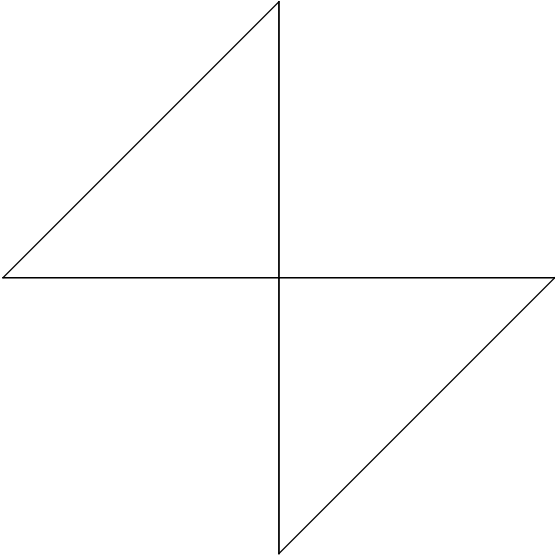
$$\left\{6, \frac{3}{2}, 6, 3\right\}$$



4.

tetrahemihexahedron

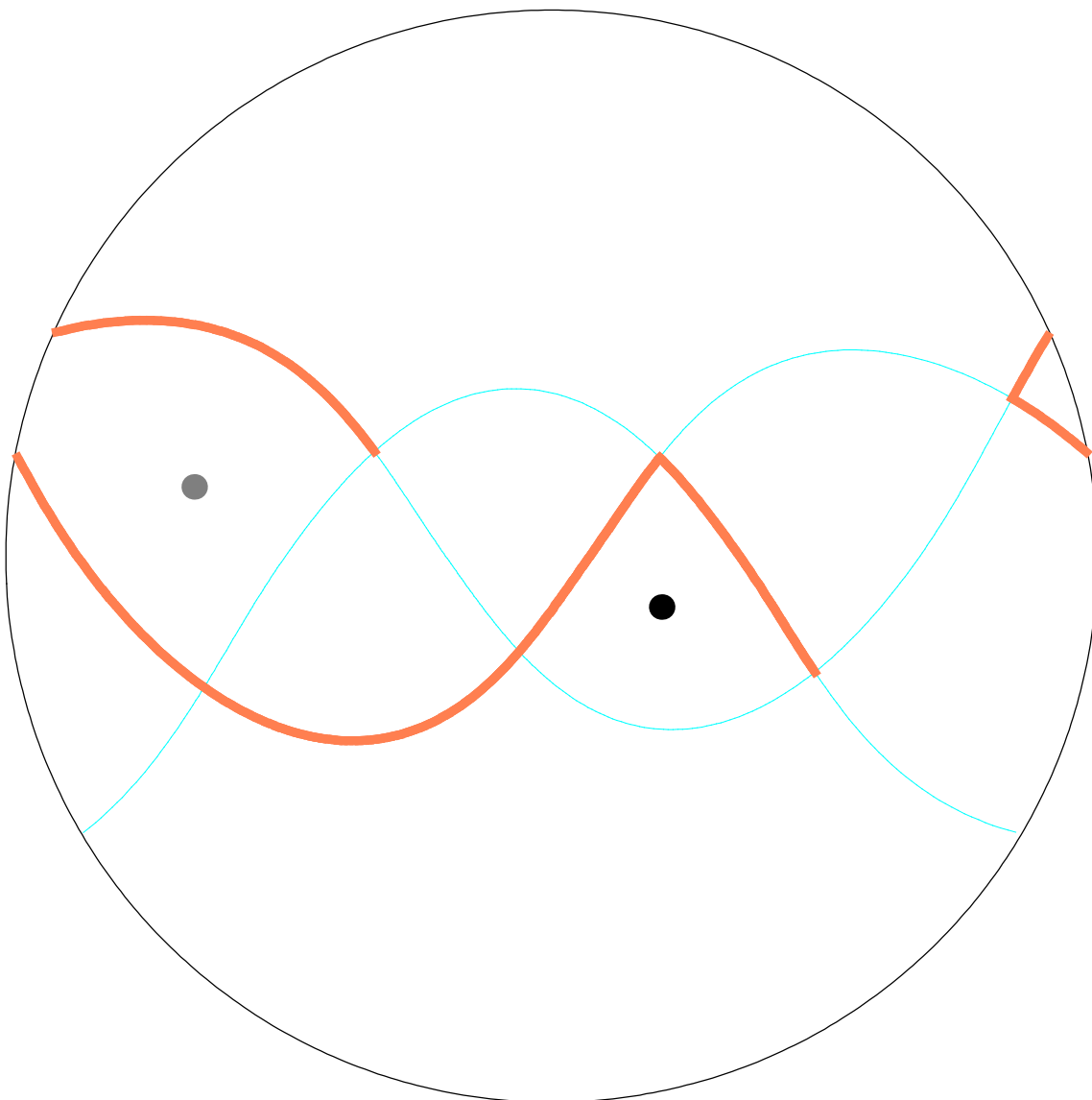
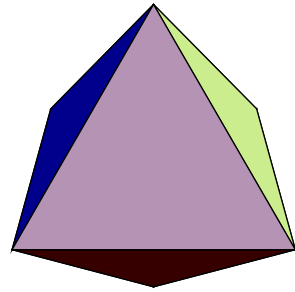
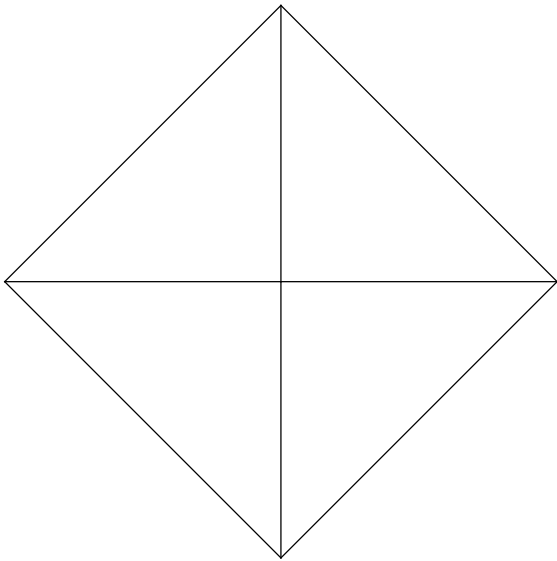
$$\left\{4, \frac{3}{2}, 4, 3\right\}$$



5.

octahedron

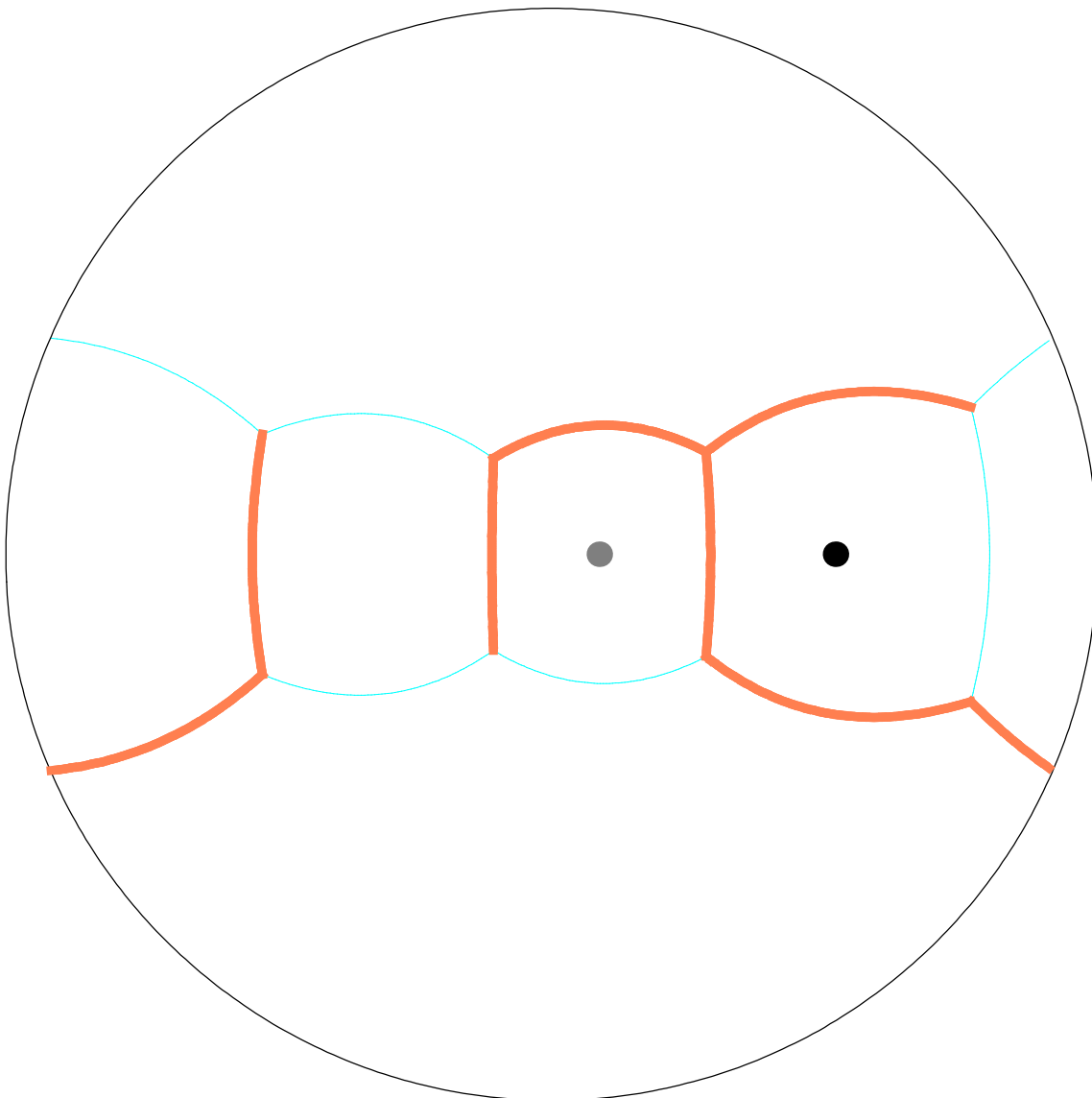
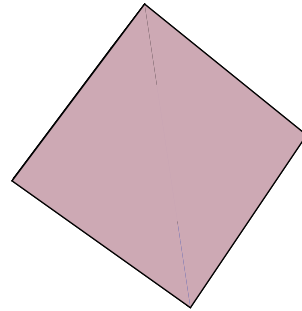
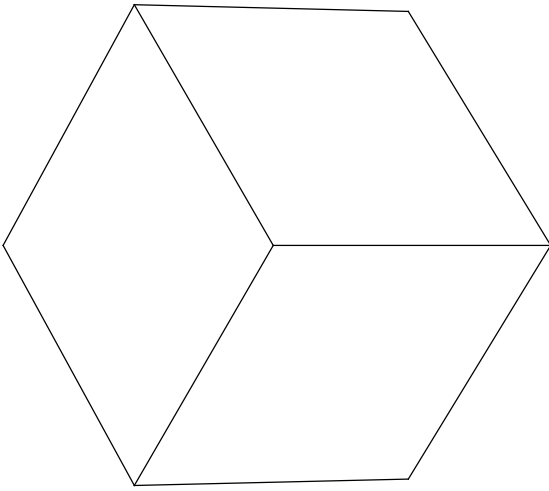
{3, 3, 3, 3}



6.

cube

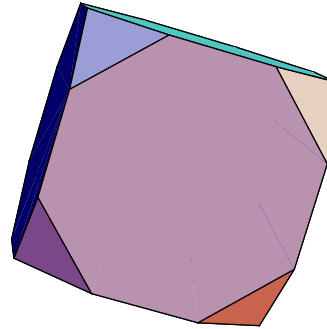
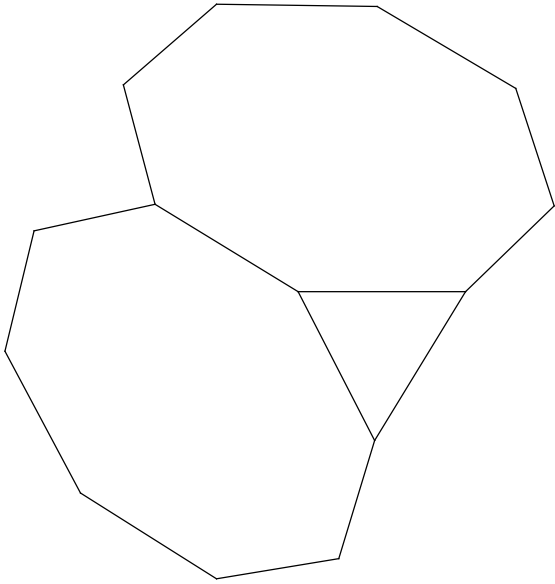
{4, 4, 4}



9.

truncated cube

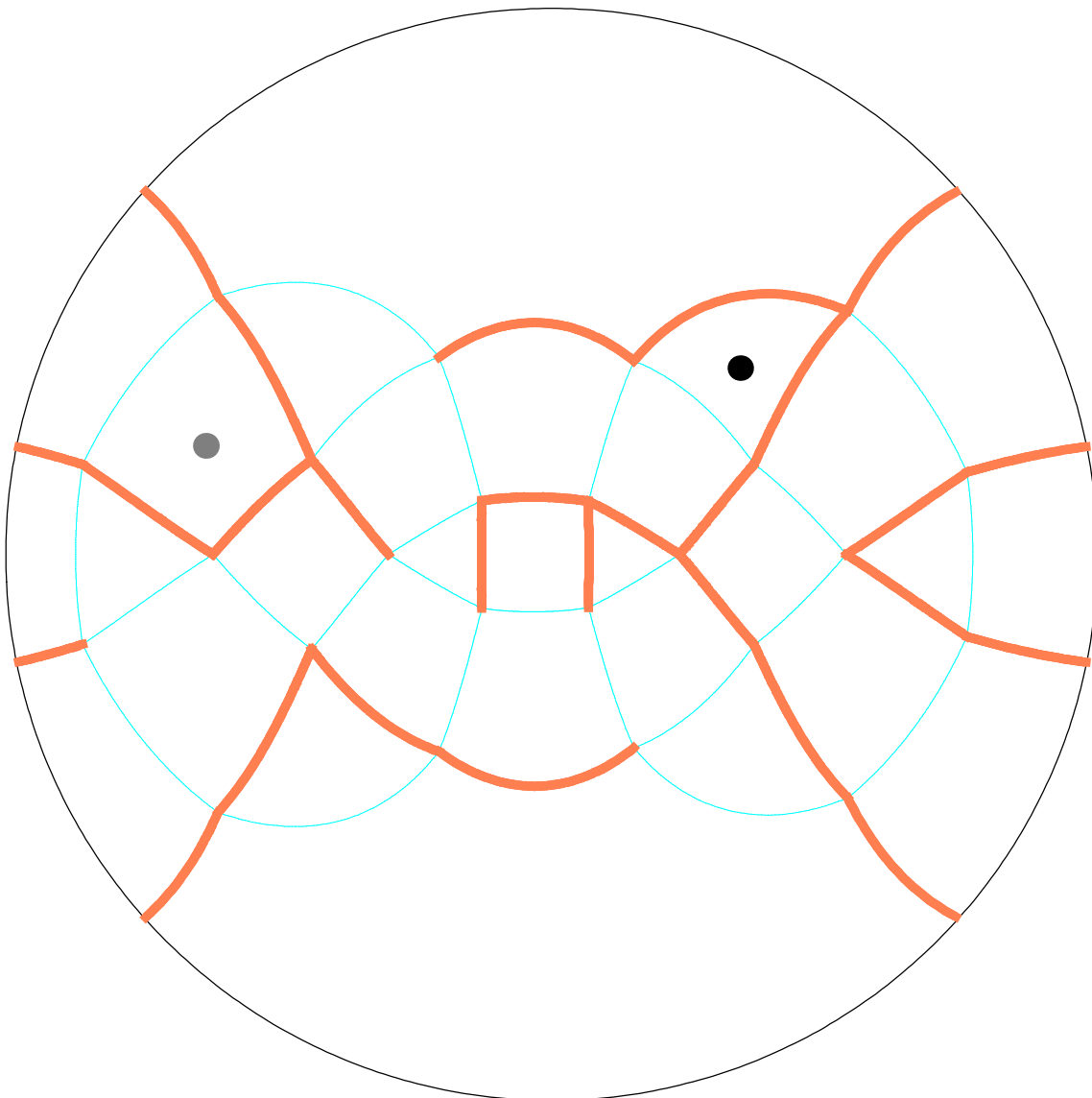
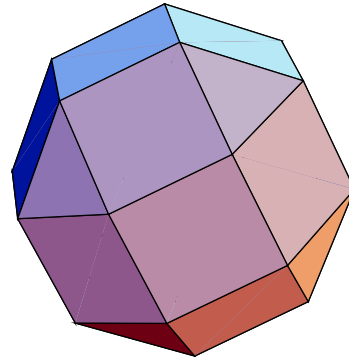
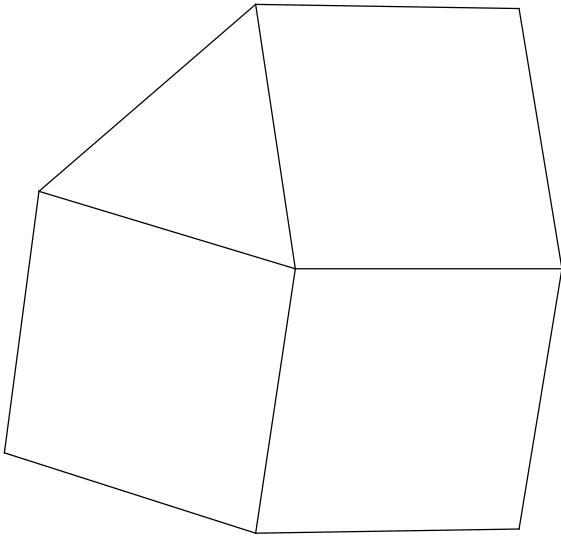
{8, 8, 3}



10.

rhombicuboctahedron

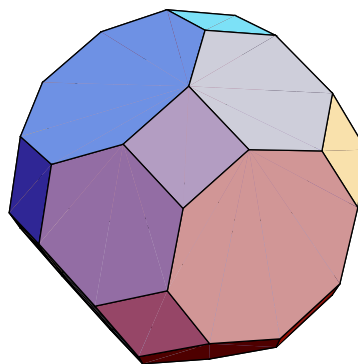
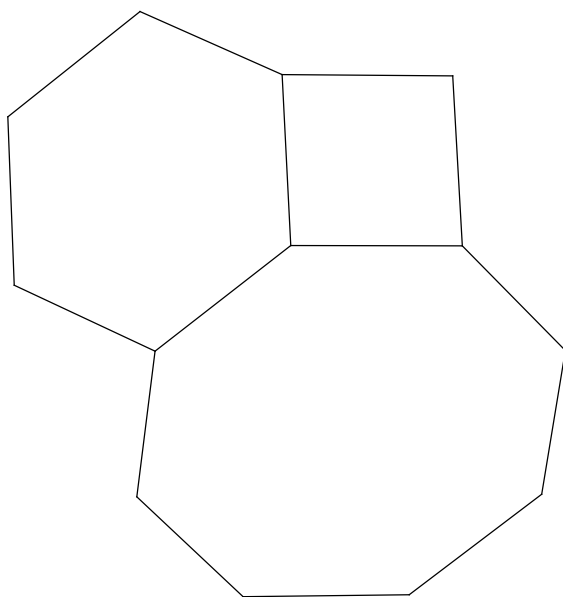
{4, 3, 4, 4}

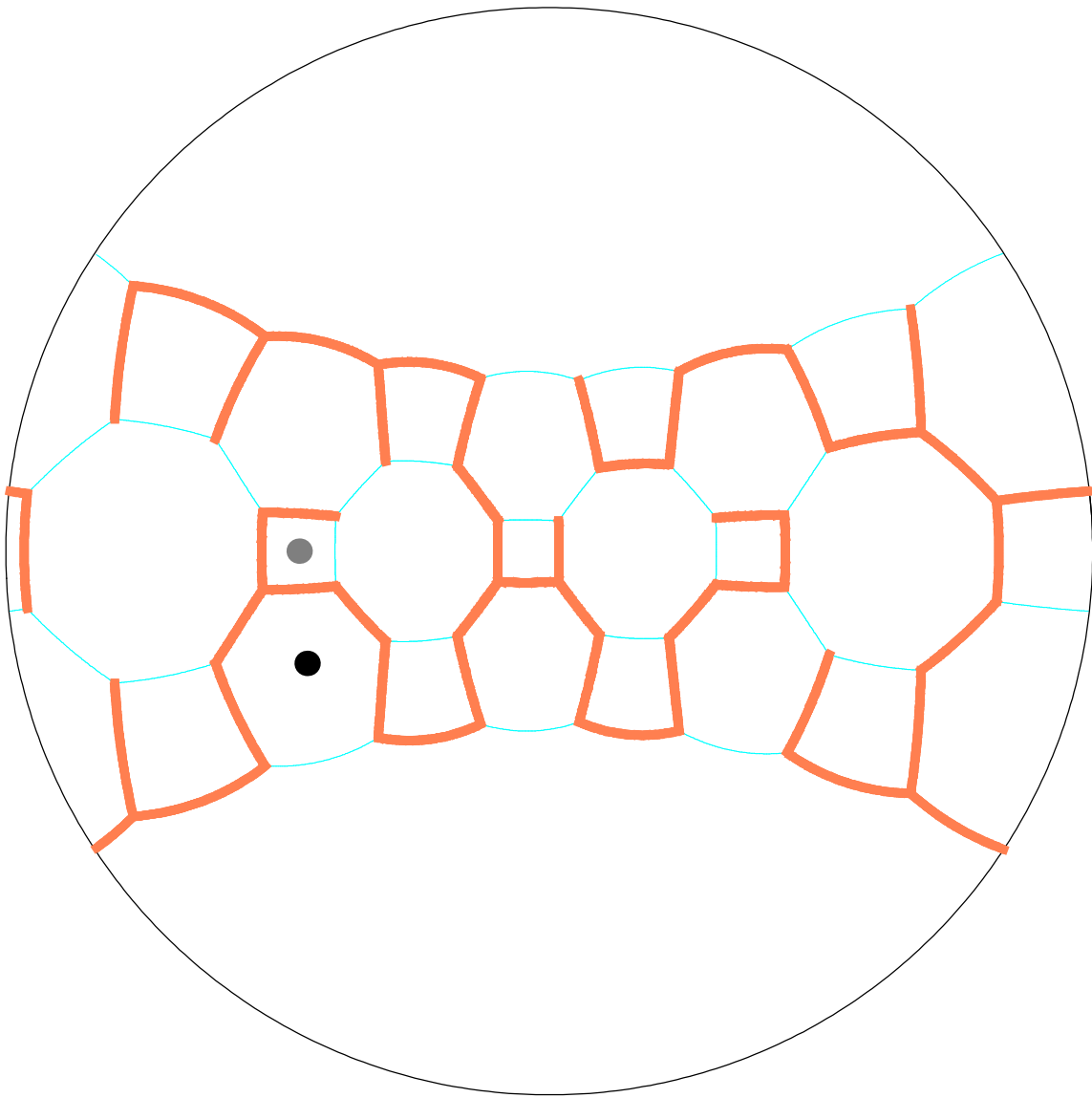


11.

truncated cuboctahedron

{4, 6, 8}

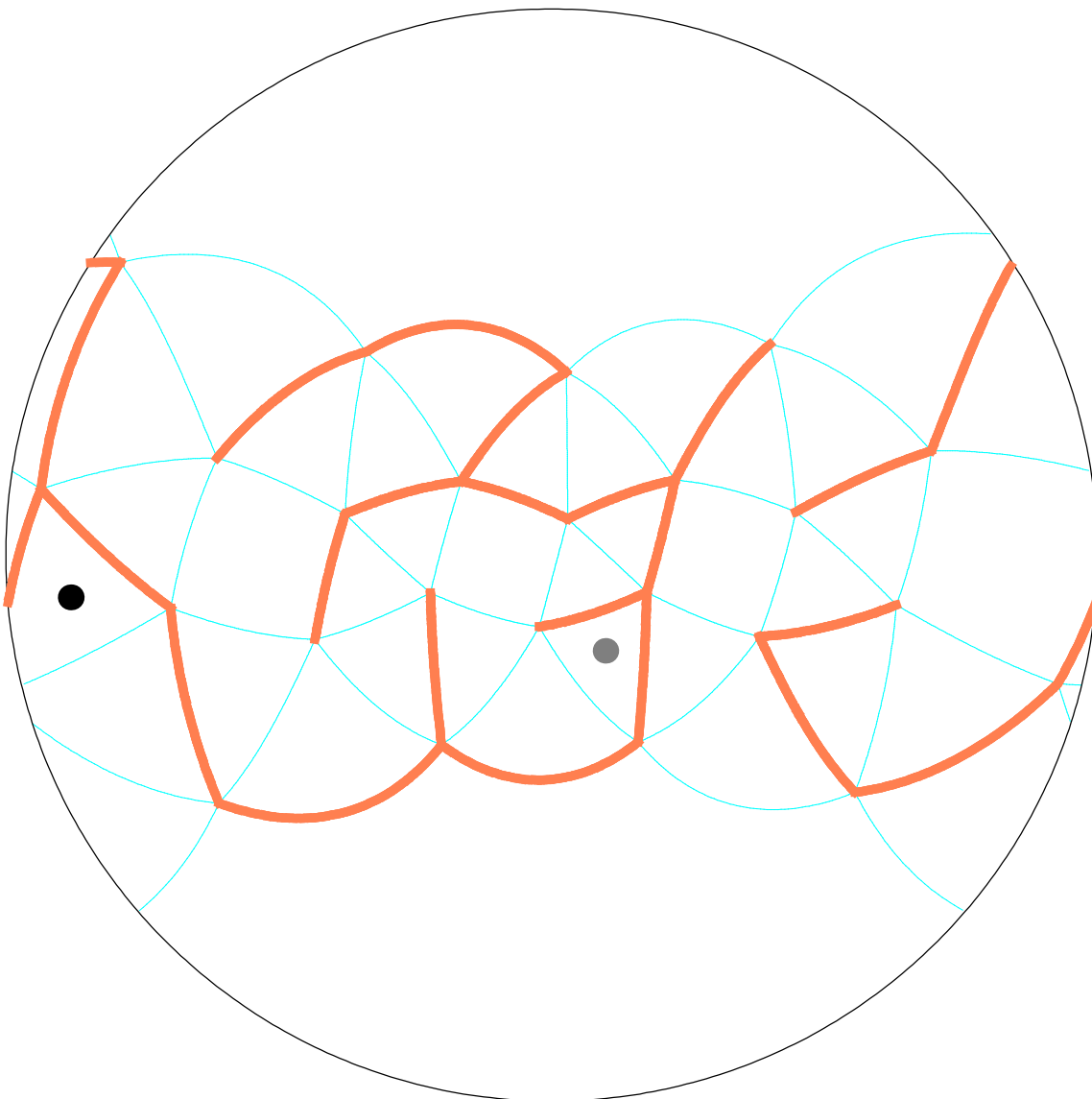
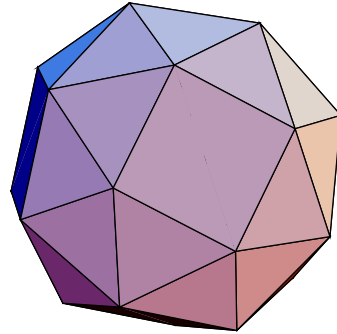
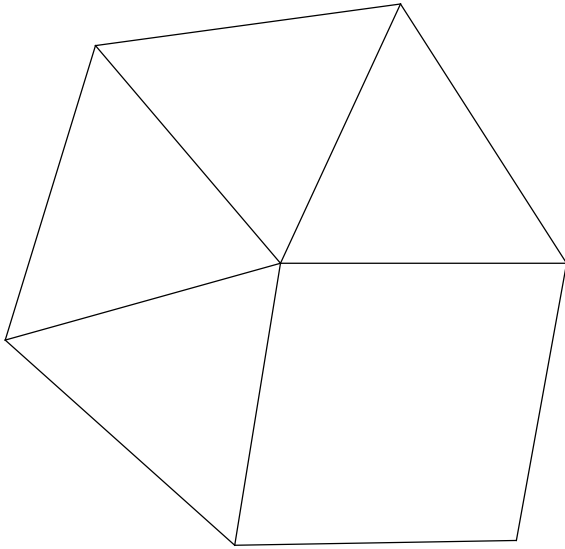




12.

snub cube

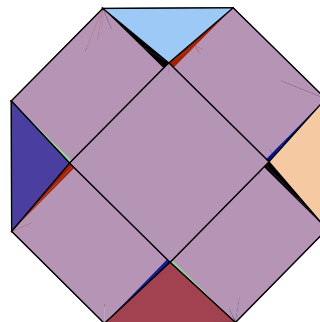
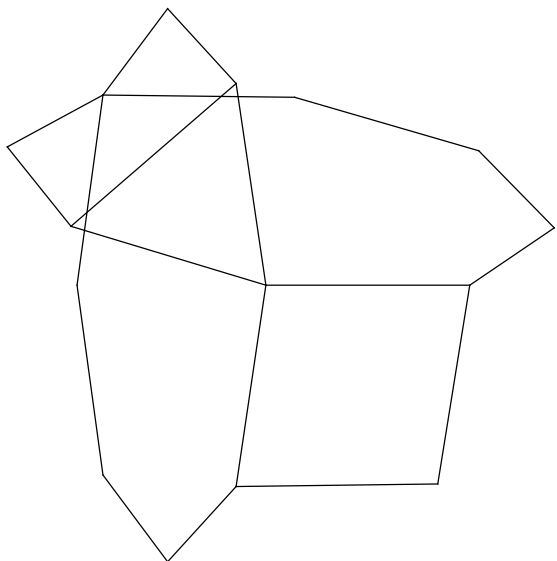
{3, 3, 3, 3, 4}

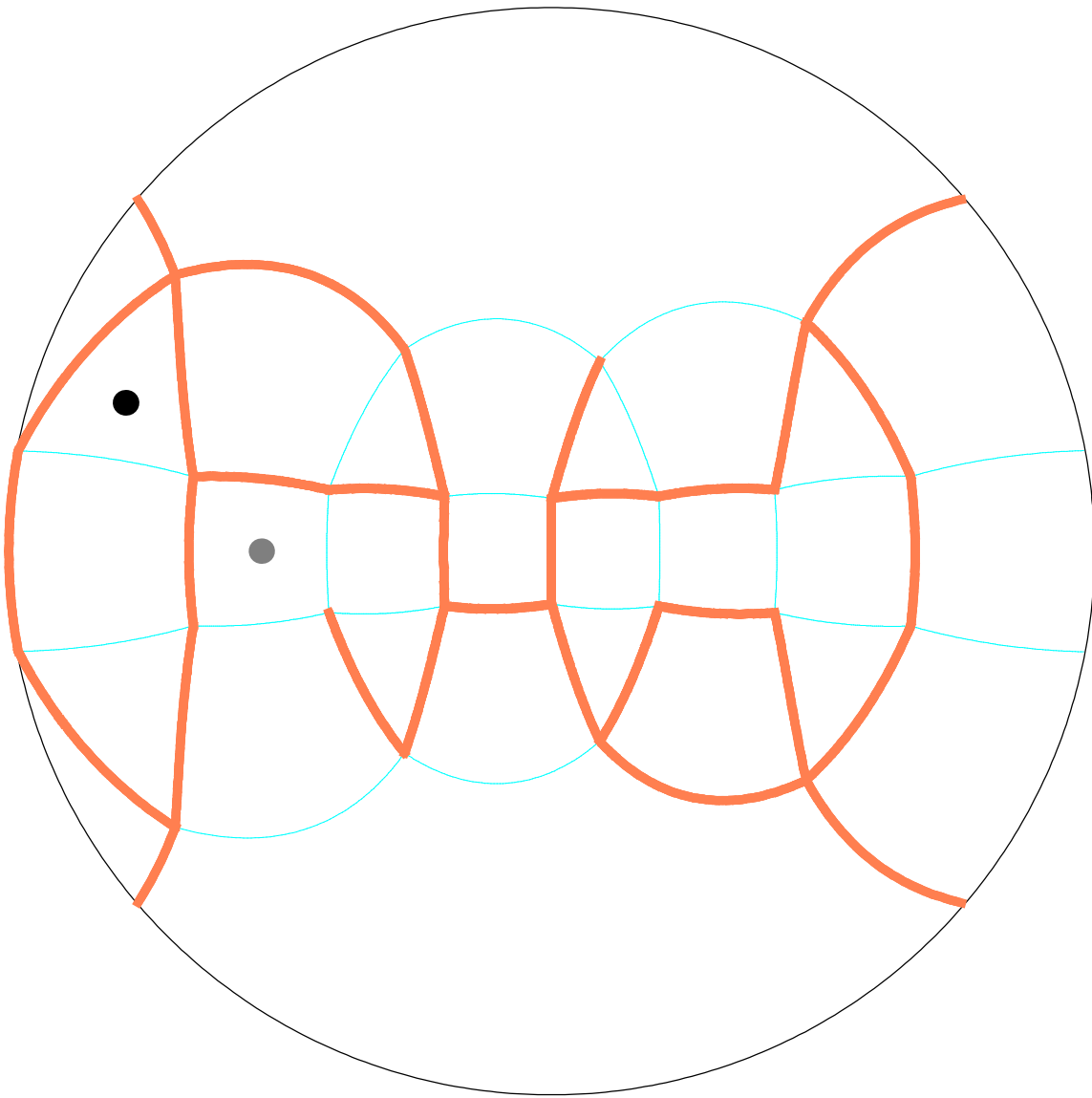


13.

small cubicuboctahedron

$$\left\{8, \frac{3}{2}, 8, 4\right\}$$

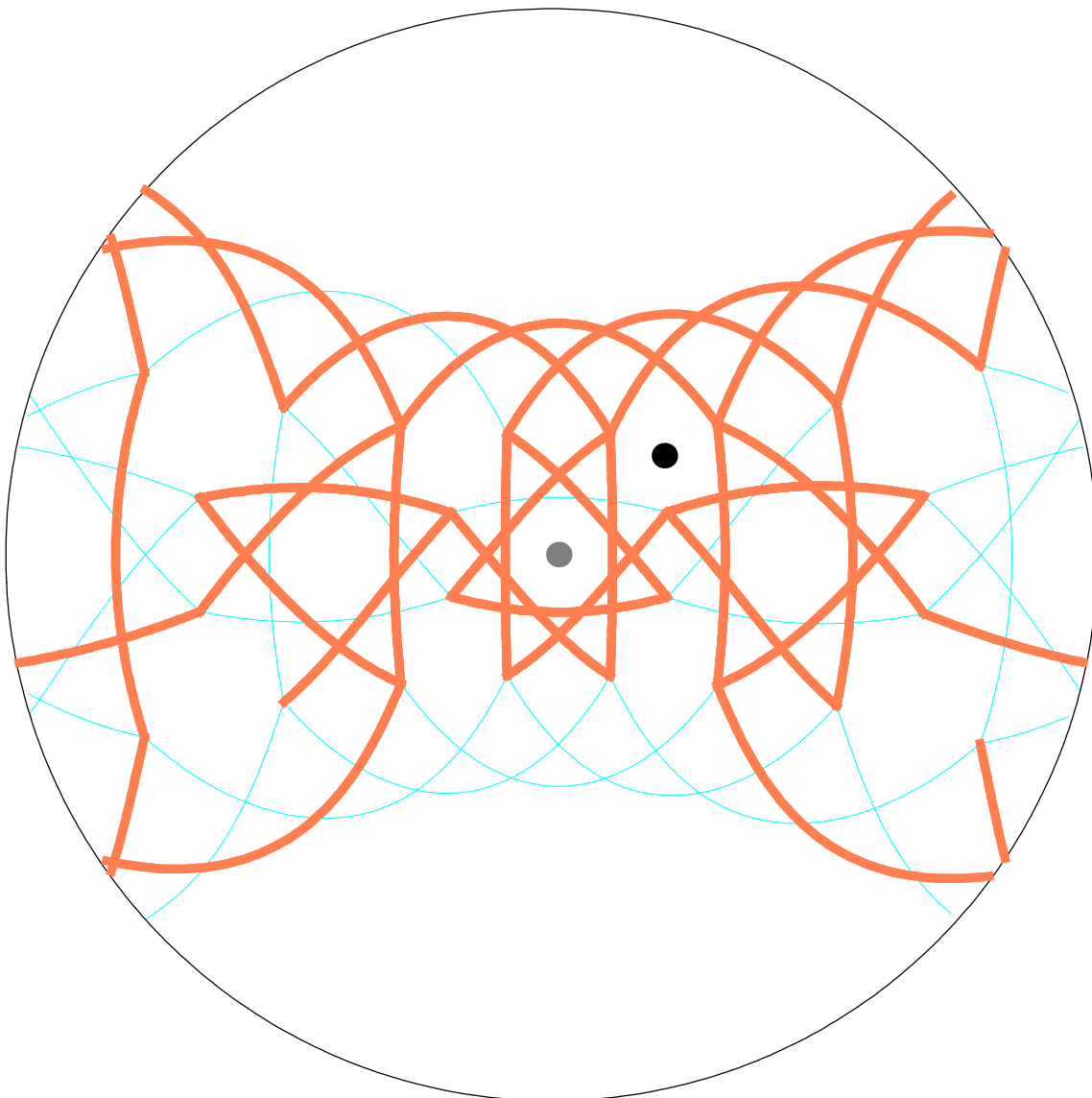
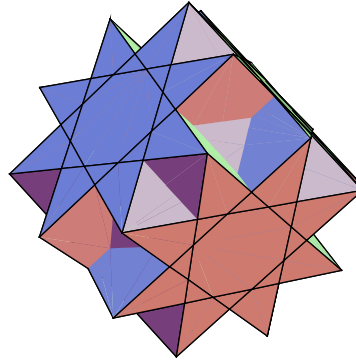
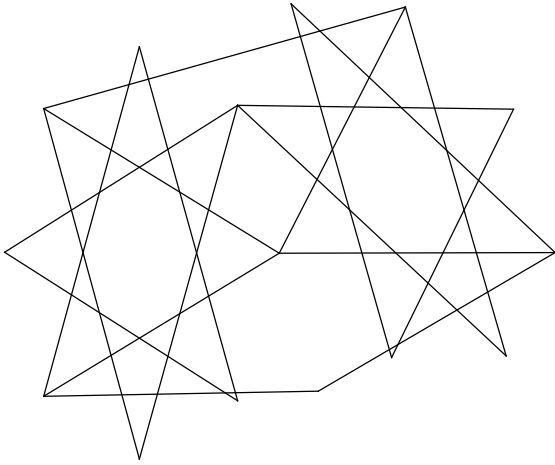




14.

great cubicuboctahedron

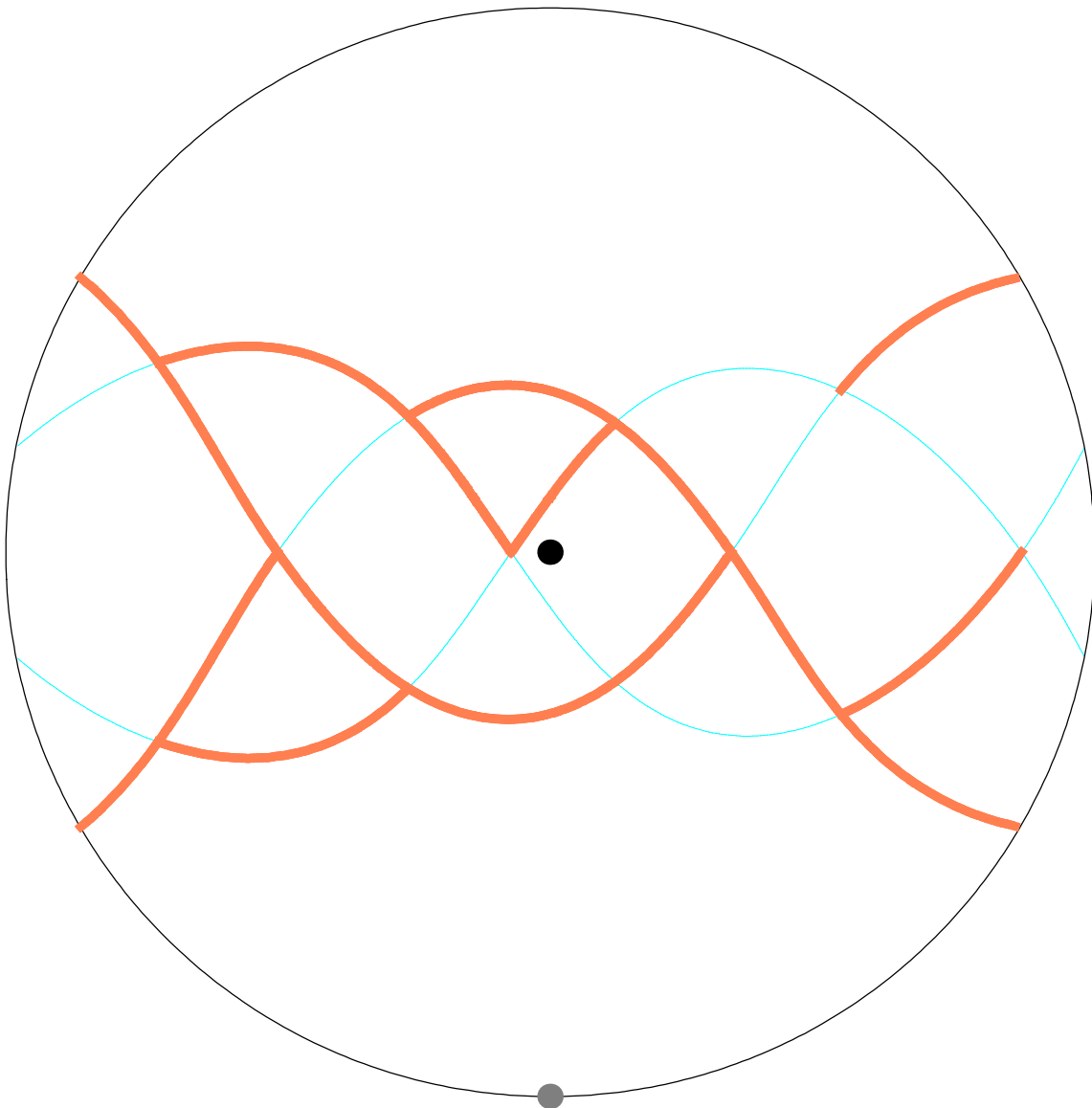
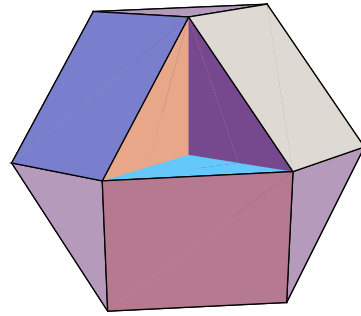
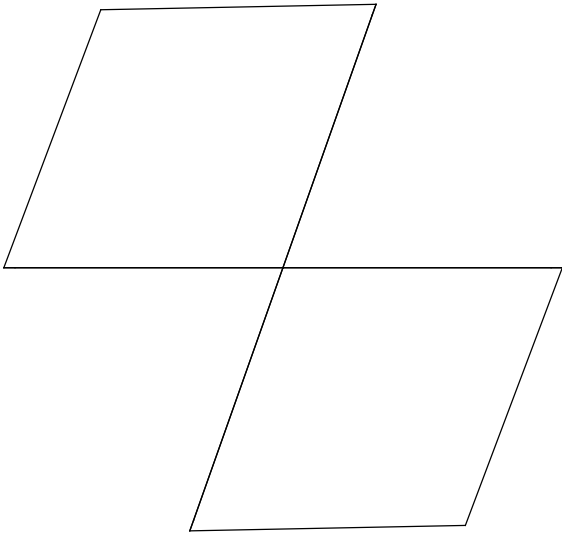
$$\left\{ \frac{8}{3}, 3, \frac{8}{3}, 4 \right\}$$



15.

cubohemioctahedron

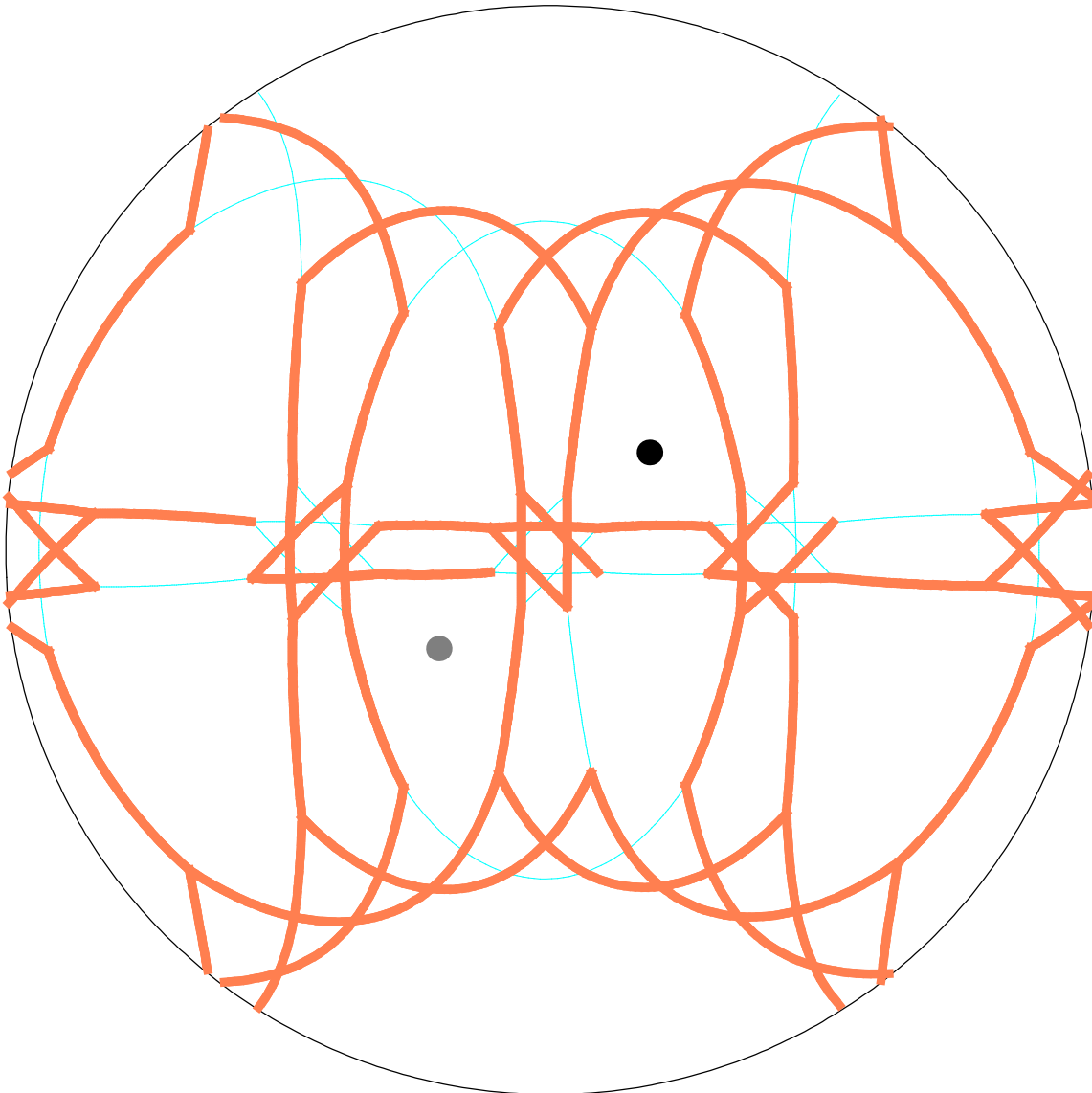
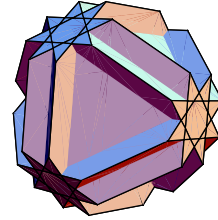
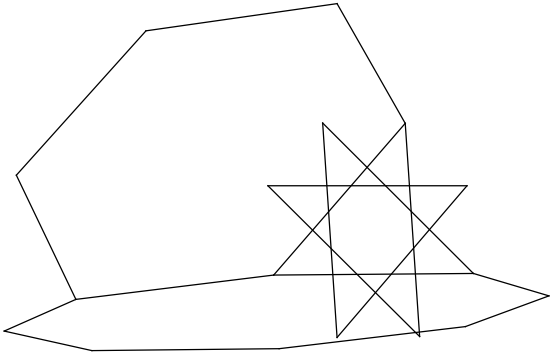
$$\left\{6, \frac{4}{3}, 6, 4\right\}$$



16.

cubitruncated cuboctahedron

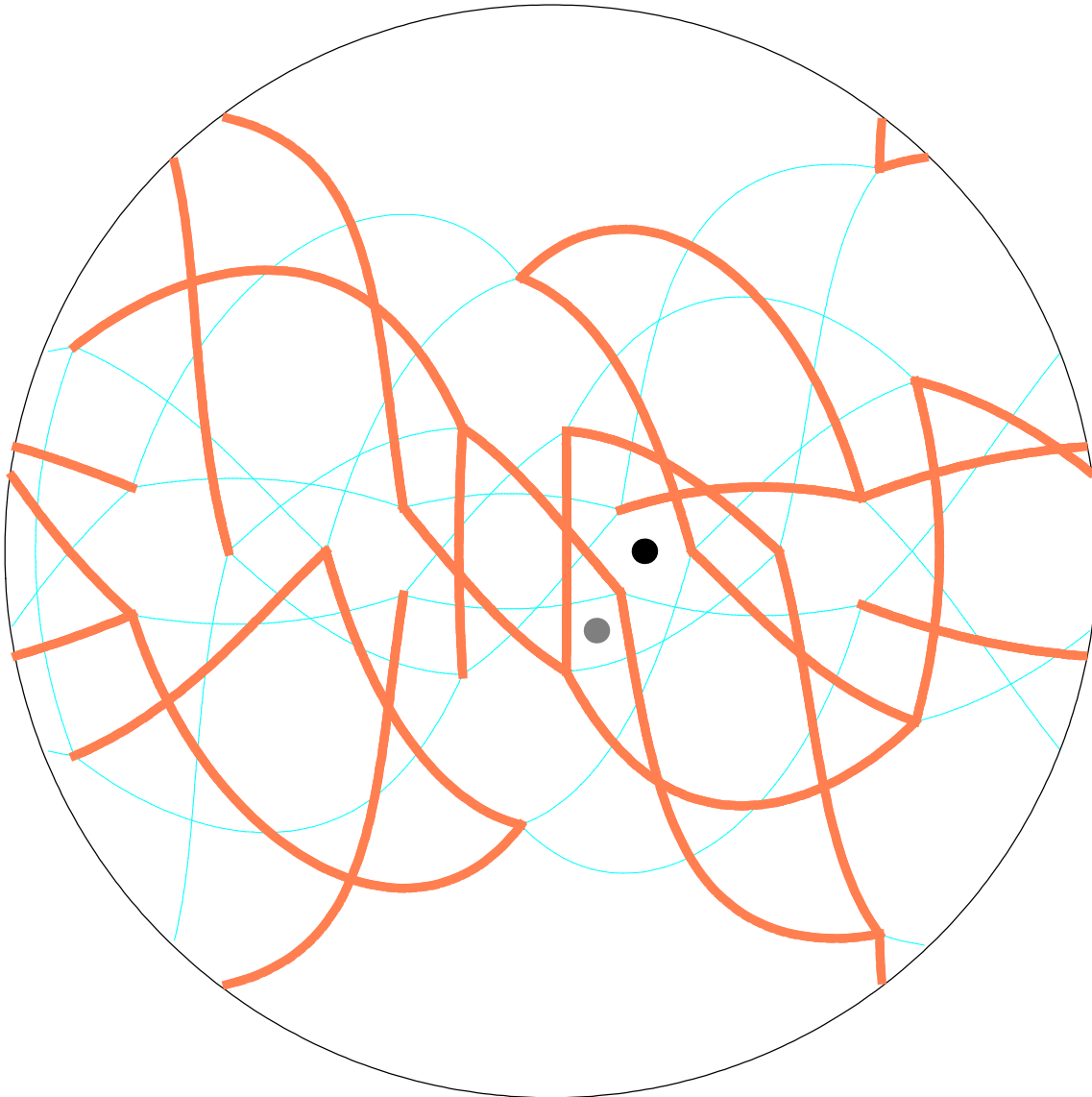
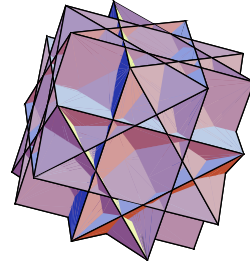
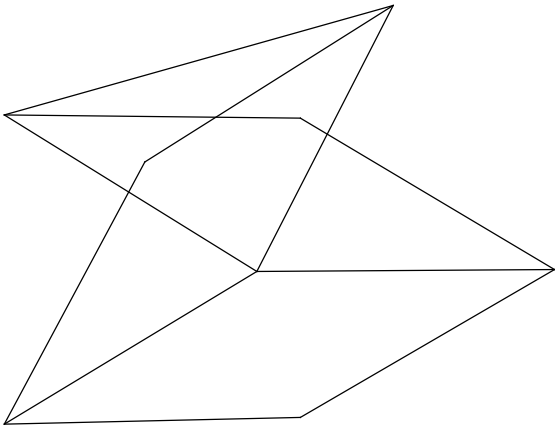
$$\left\{ \frac{8}{3}, 6, 8 \right\}$$



17.

great rhombicuboctahedron

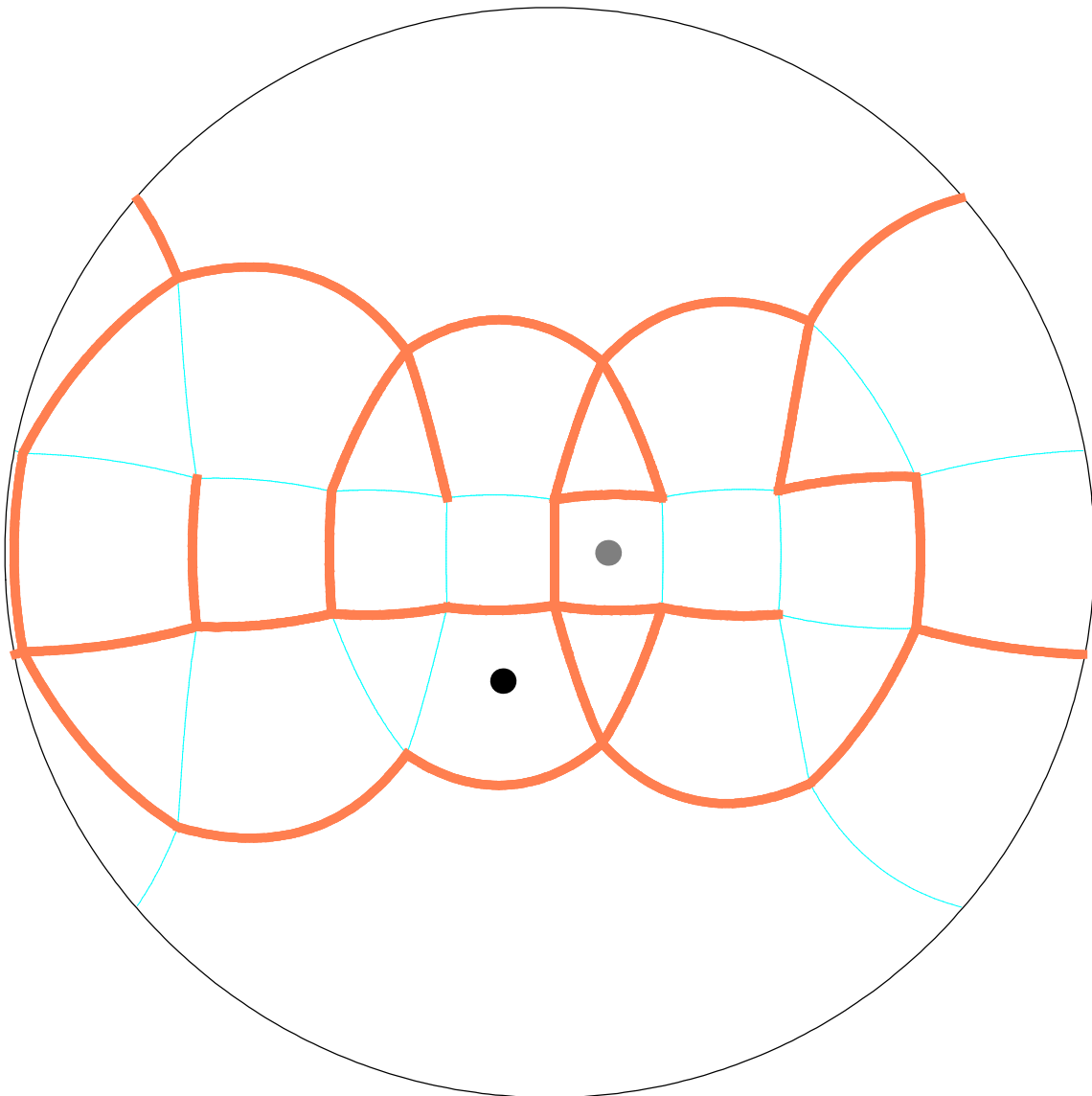
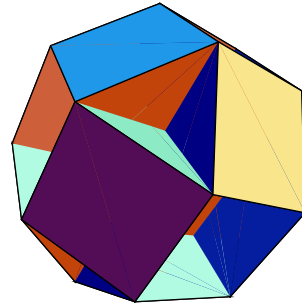
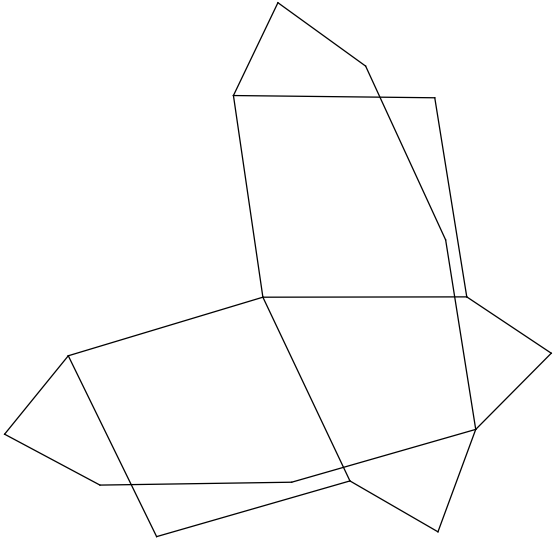
$$\{4, \frac{3}{2}, 4, 4\}$$



18.

small rhombihexahedron

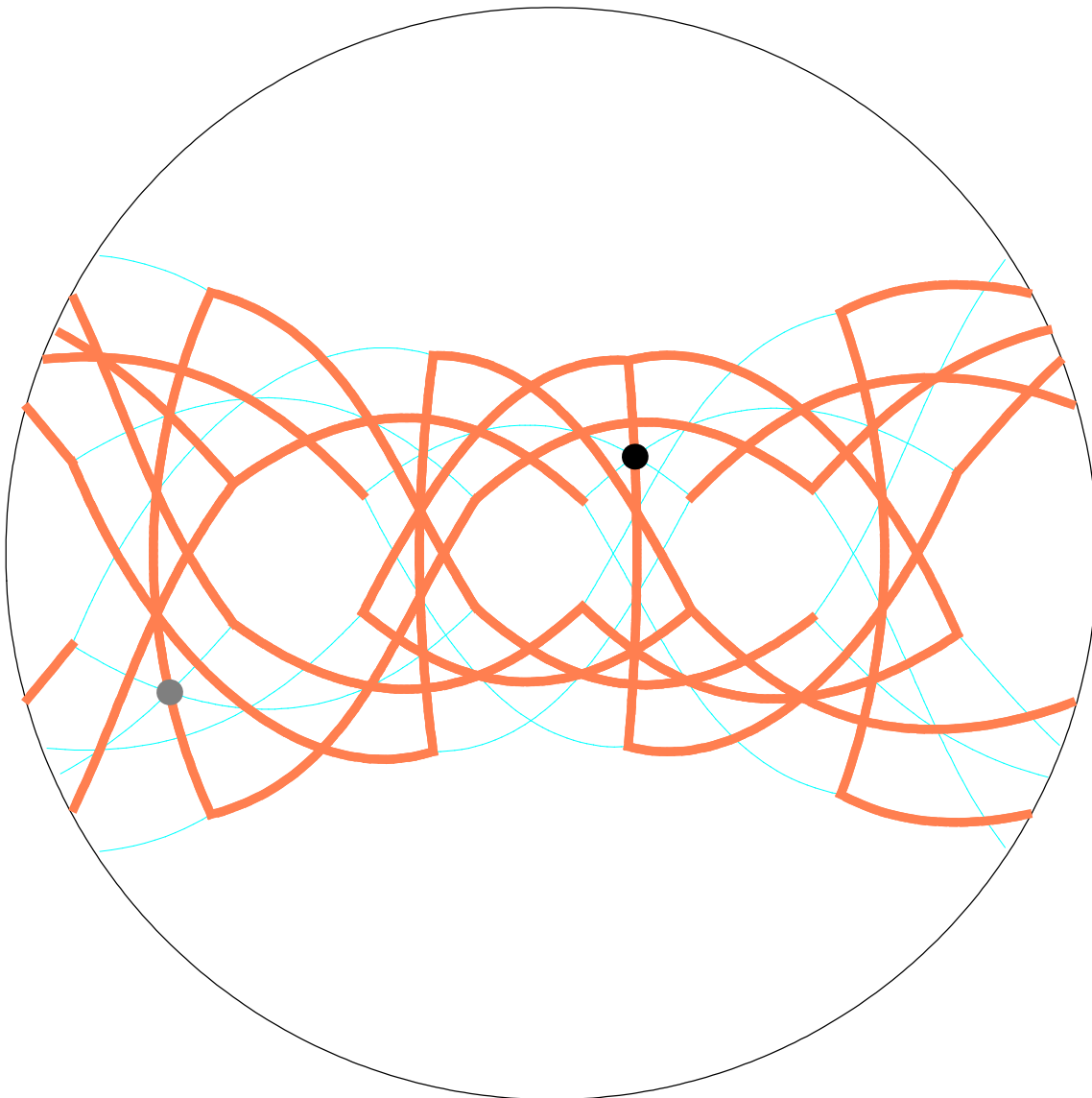
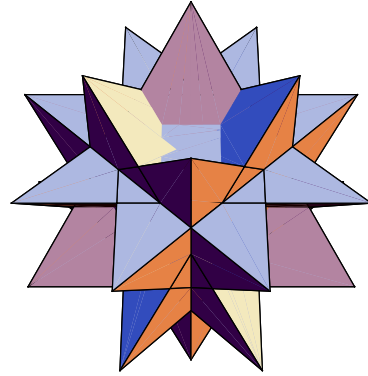
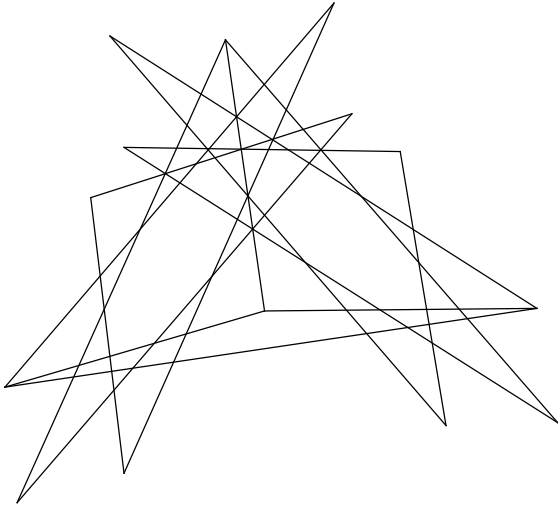
$$\{8, 4, \frac{8}{7}, \frac{4}{3}\}$$



19.

stellated truncated hexahedron

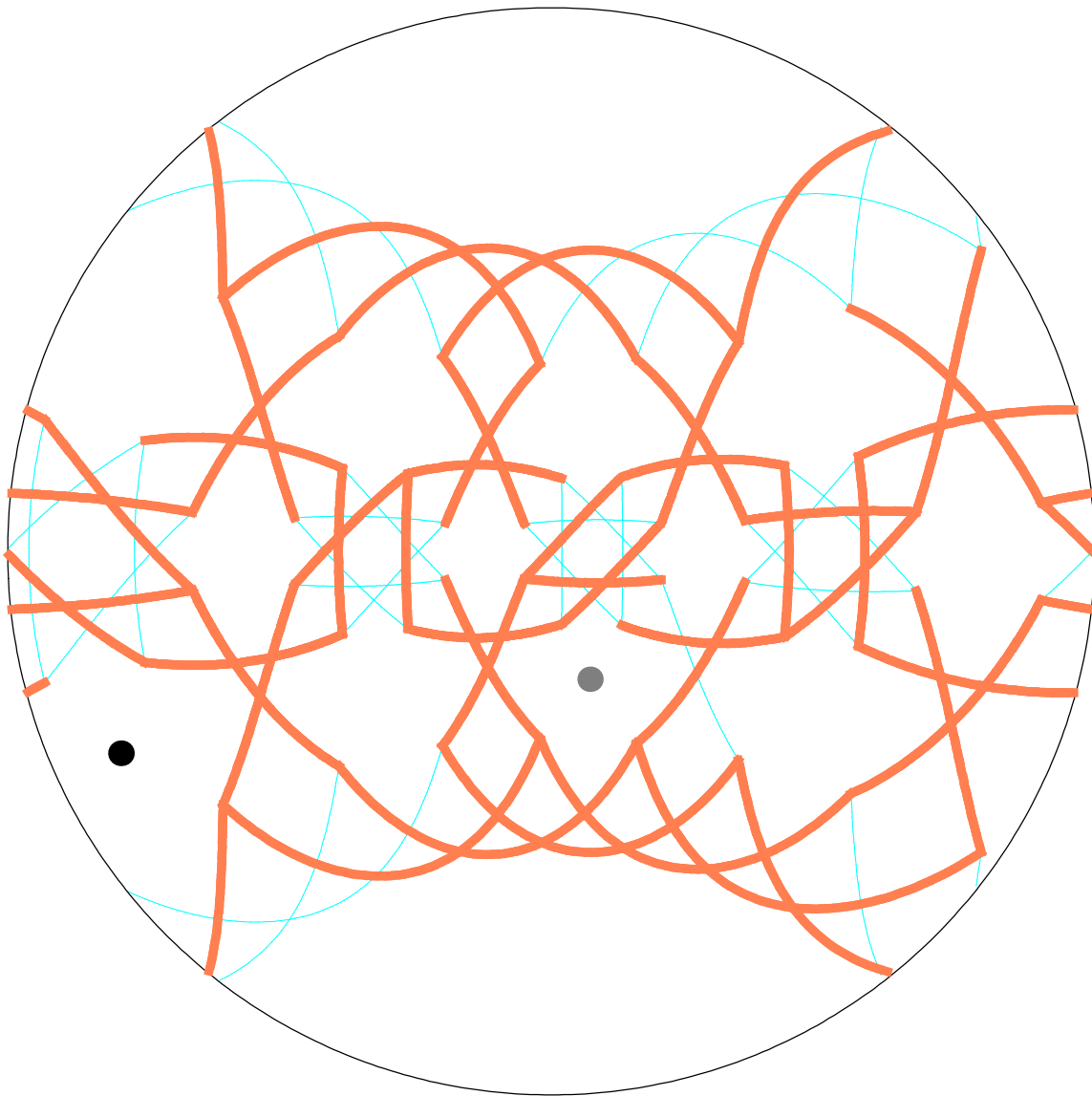
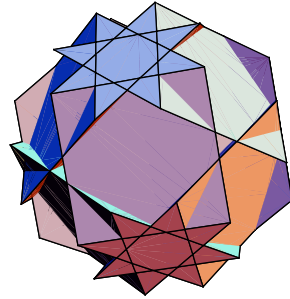
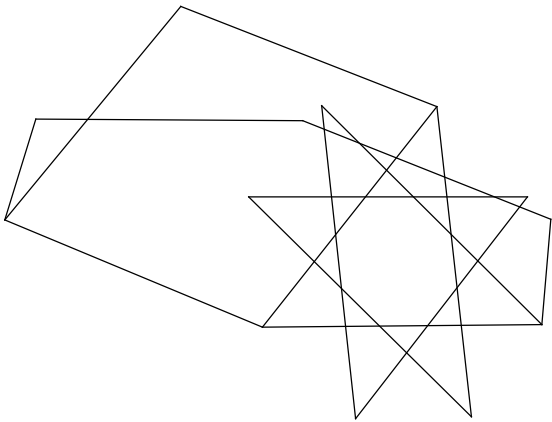
$$\left\{ \frac{8}{3}, \frac{8}{3}, 3 \right\}$$



20.

great truncated cuboctahedron

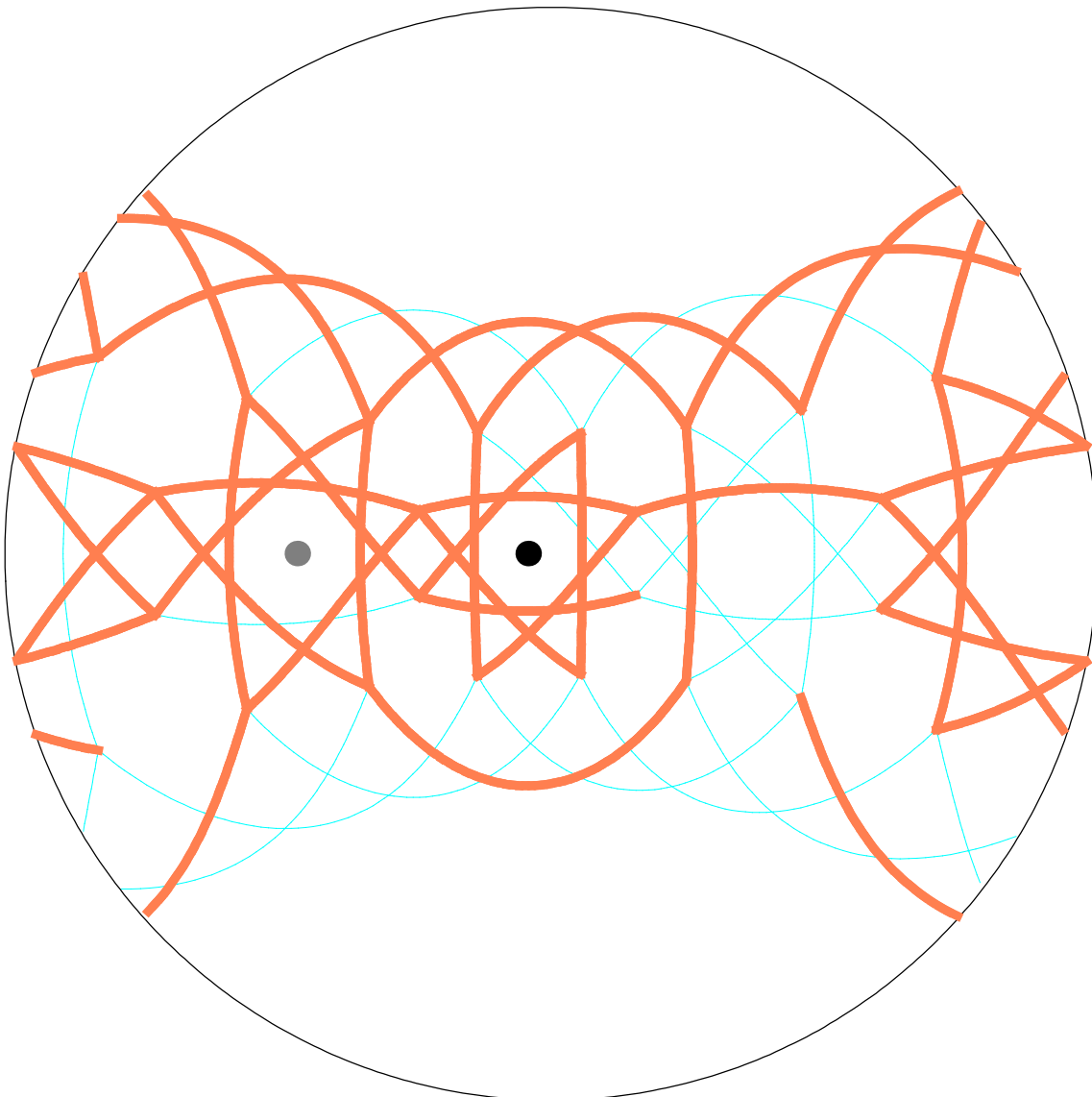
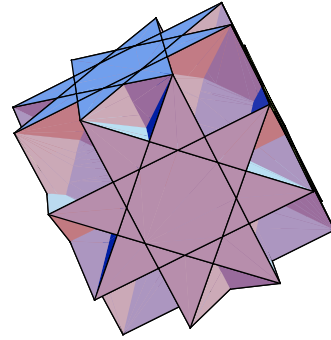
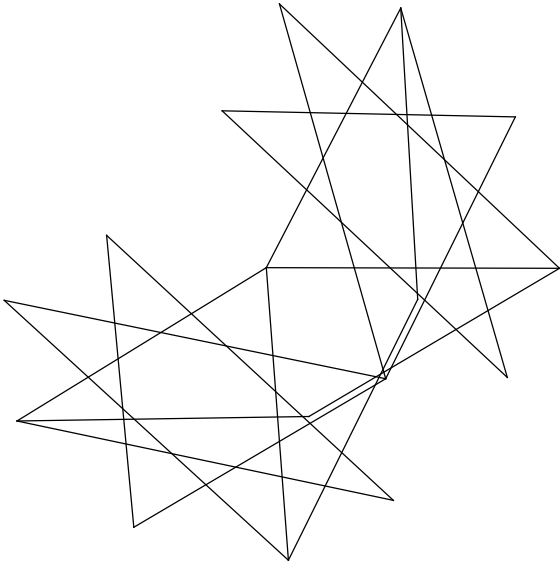
$$\left\{ \frac{8}{3}, 4, 6 \right\}$$



21.

great rhombihexahedron

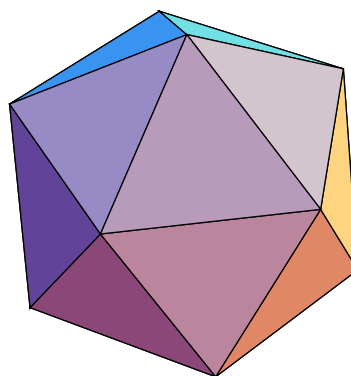
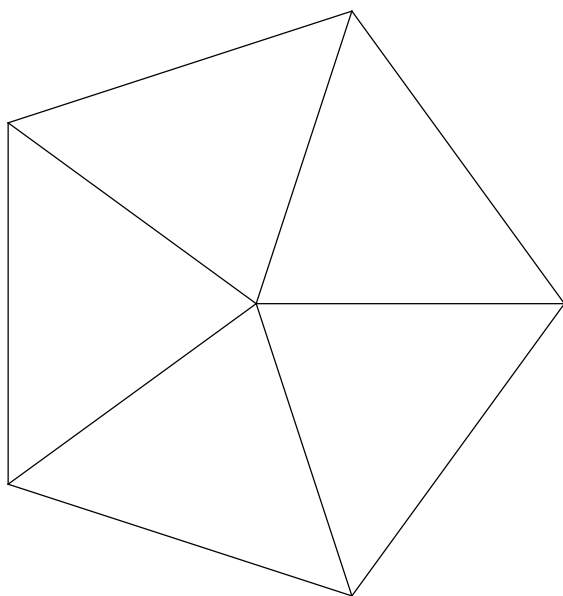
$$\left\{4, \frac{8}{3}, \frac{4}{3}, \frac{8}{5}\right\}$$

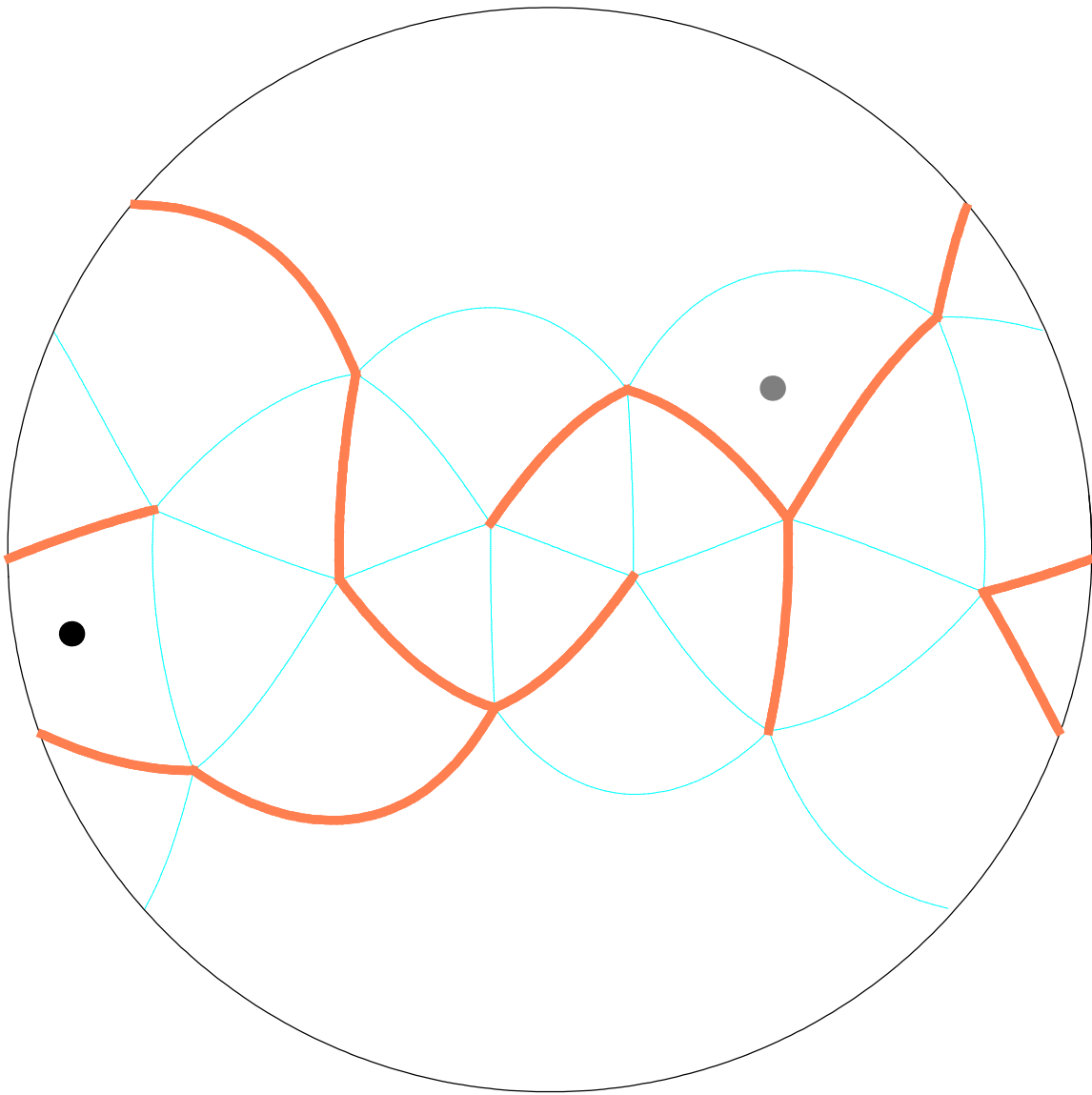


22.

icosahedron

{3, 3, 3, 3, 3}

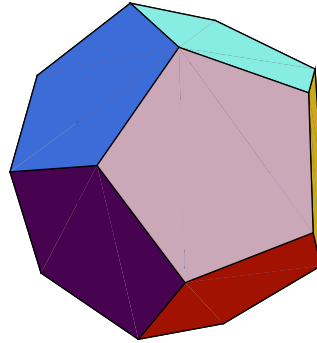
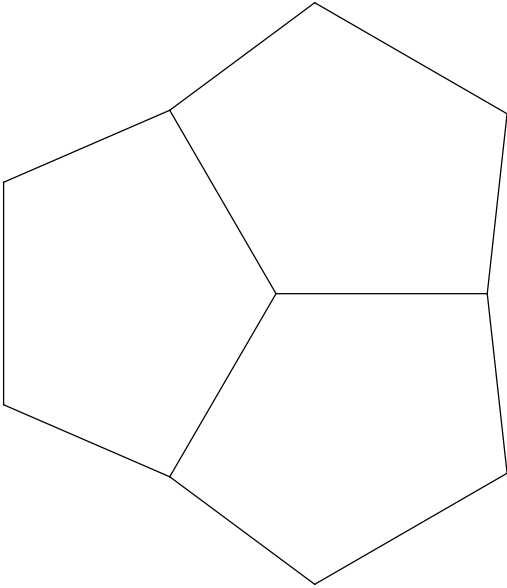




23.

dodecahedron

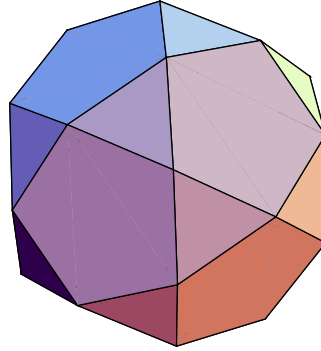
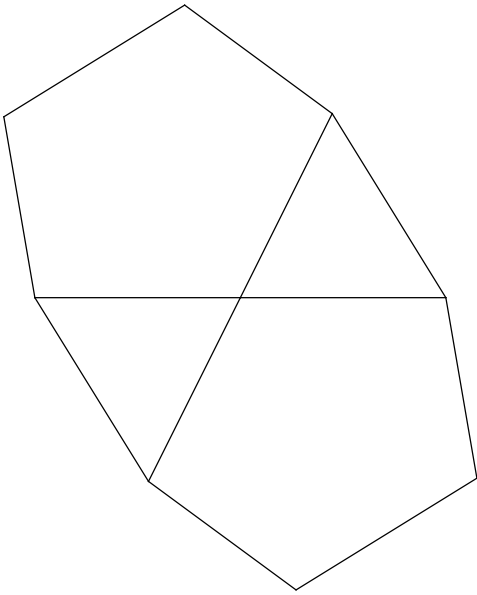
{5, 5, 5}



24.

icosidodecahedron

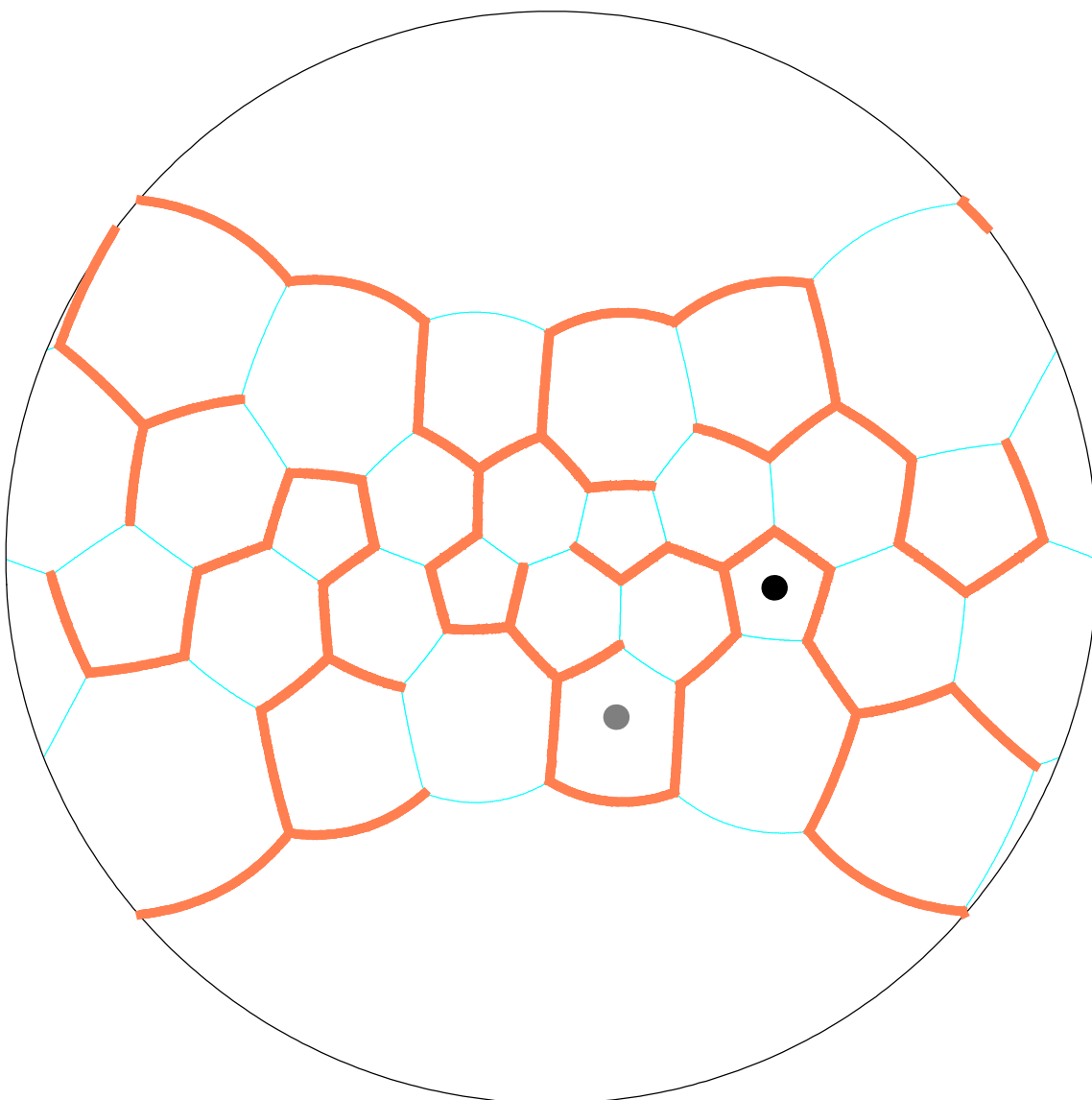
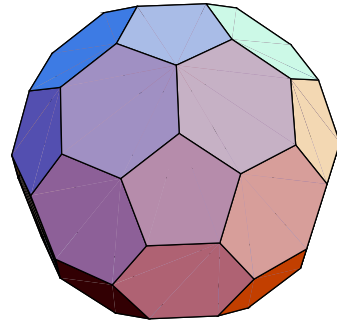
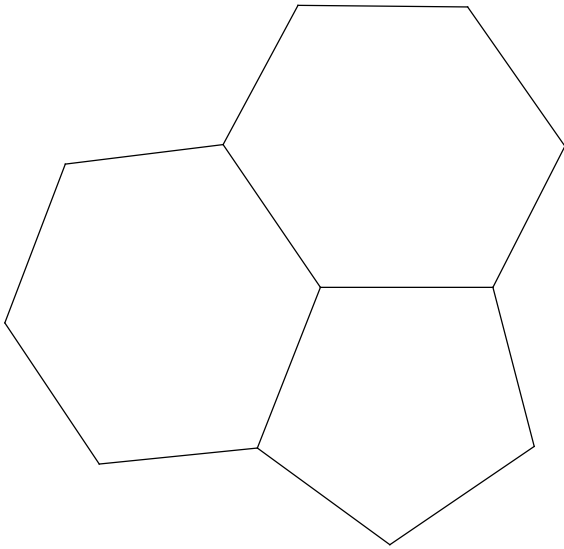
{3, 5, 3, 5}



25.

truncated icosahedron

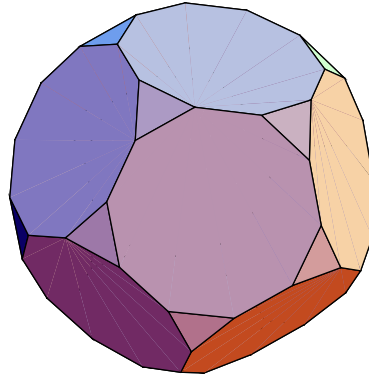
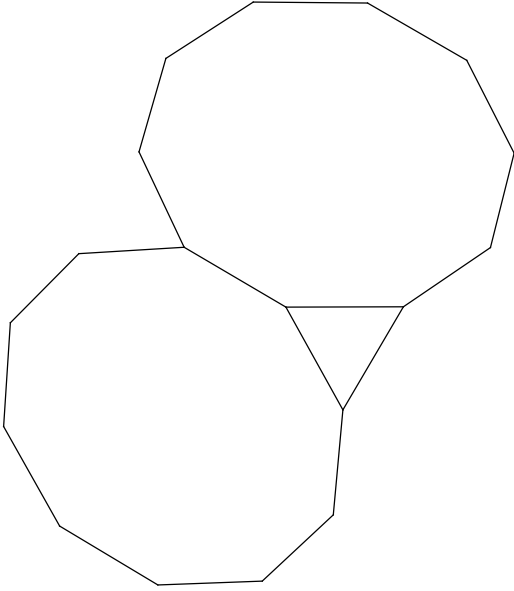
{6, 6, 5}

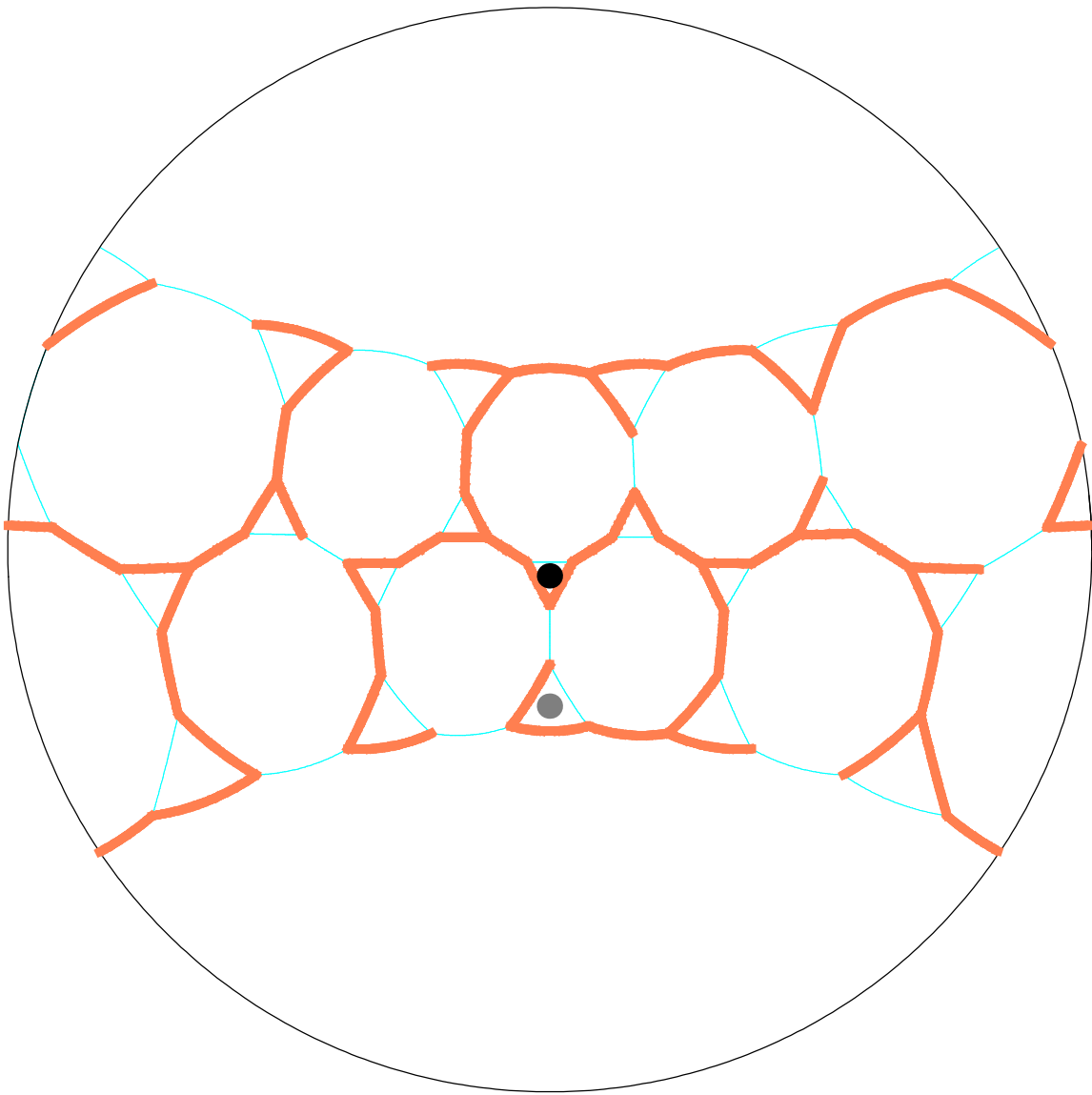


26.

truncated dodecahedron

{10, 10, 3}

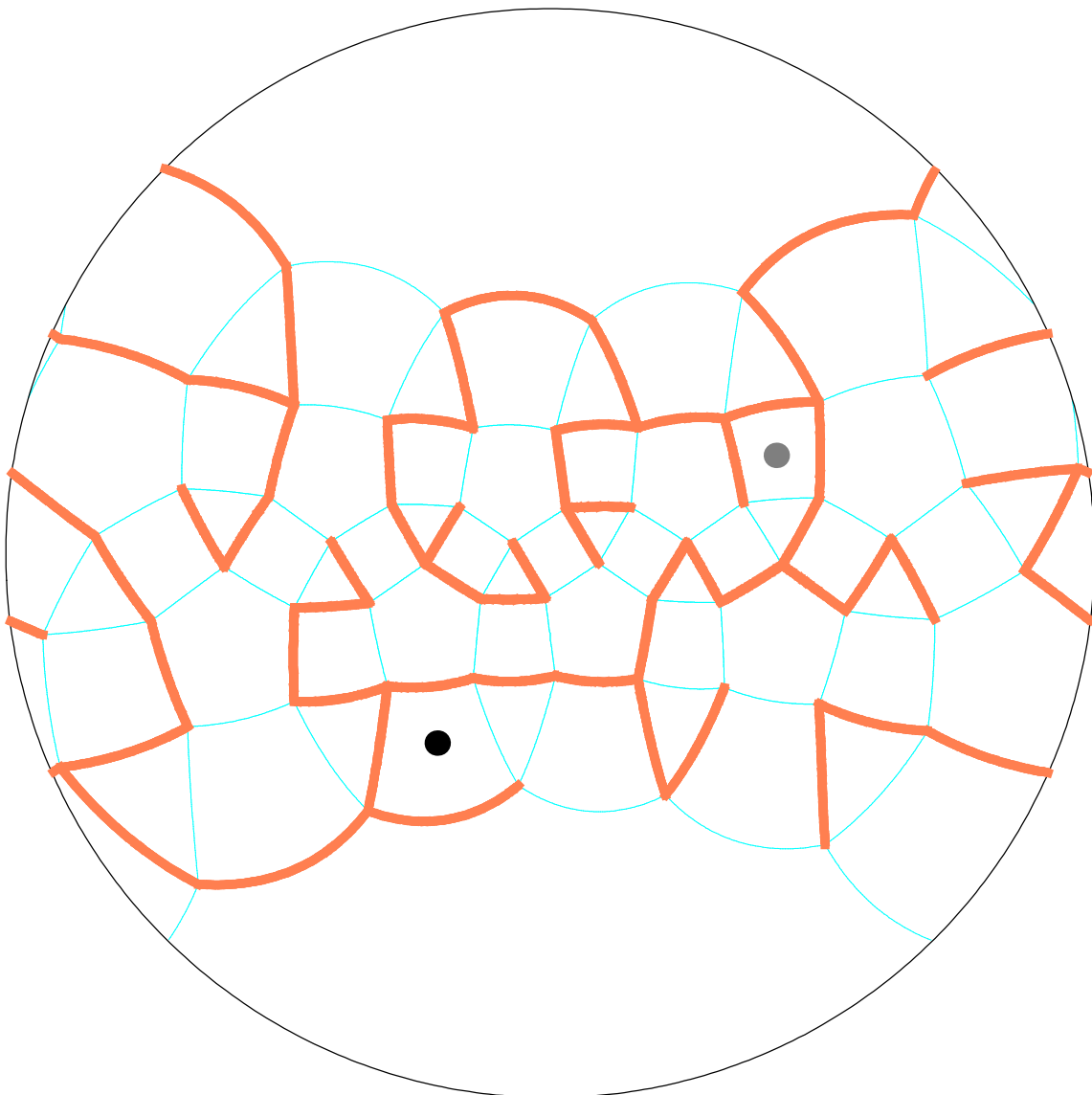
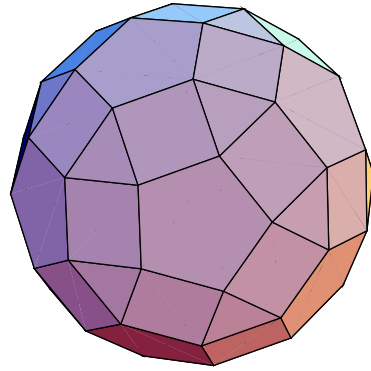
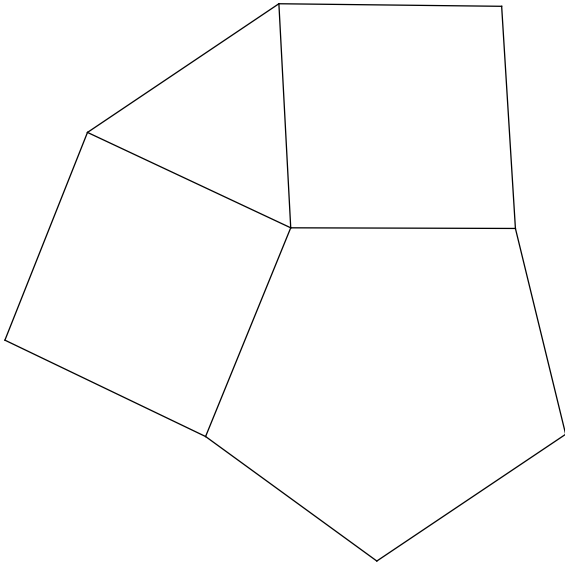




27.

rhombicosidodecahedron

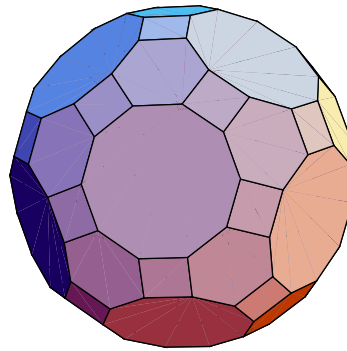
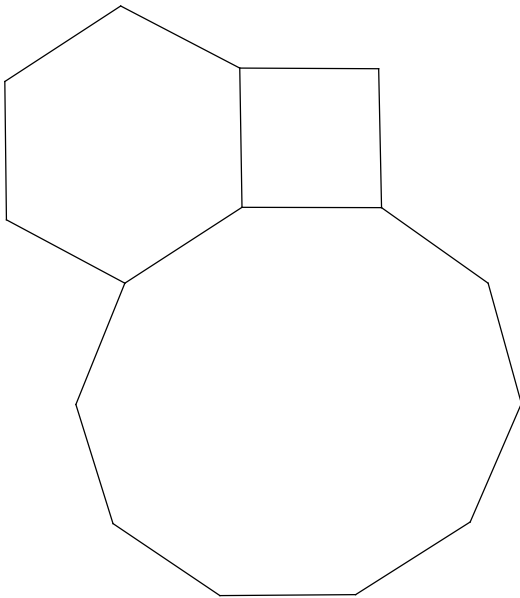
{4, 3, 4, 5}

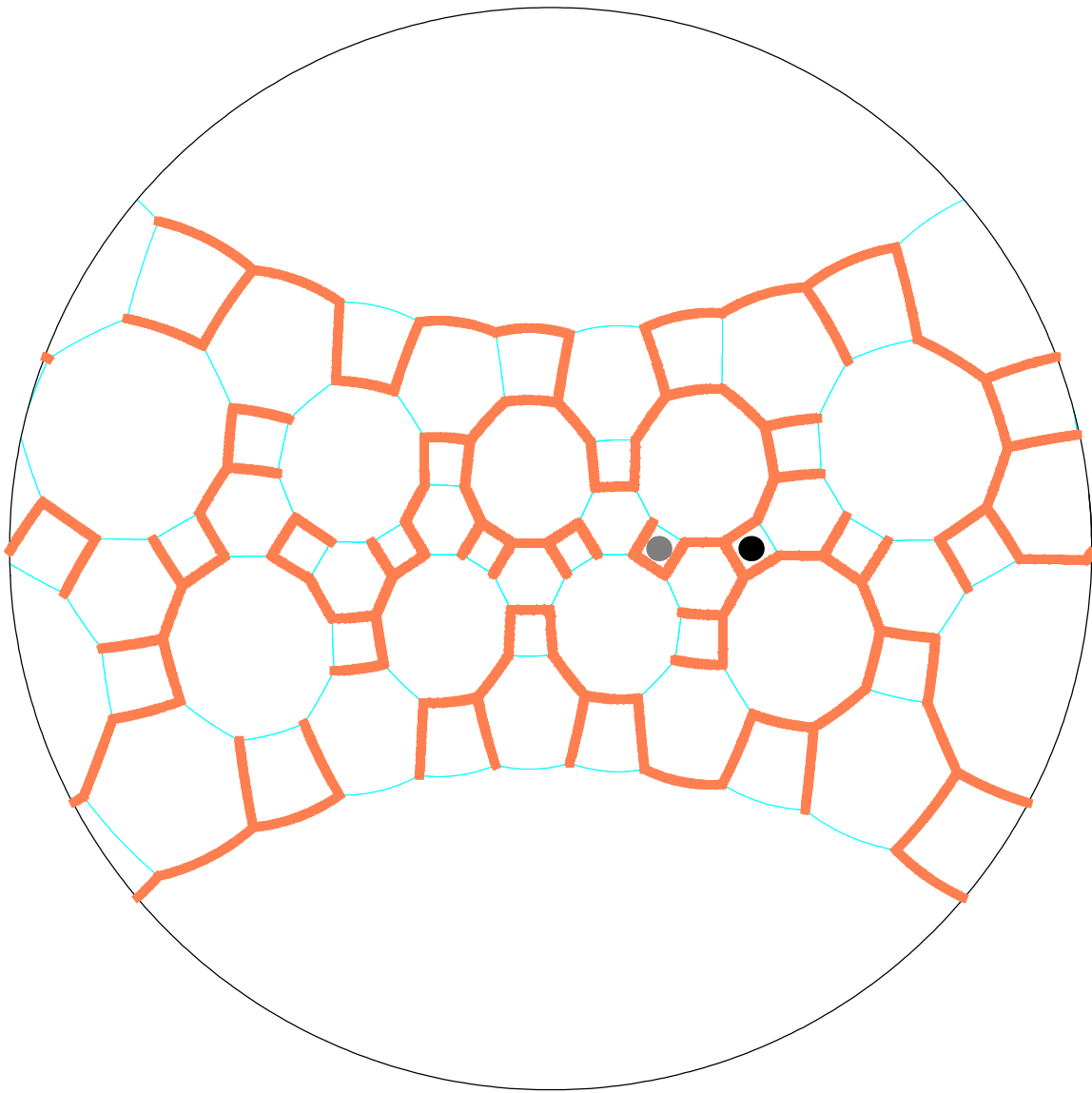


28.

truncated icosidodecahedron

{4, 6, 10}

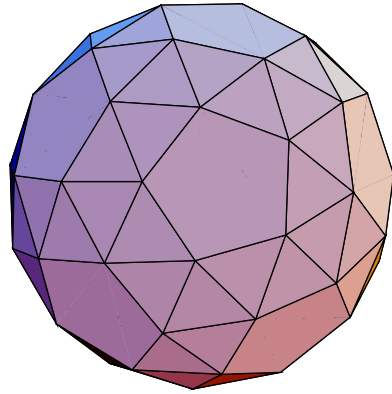
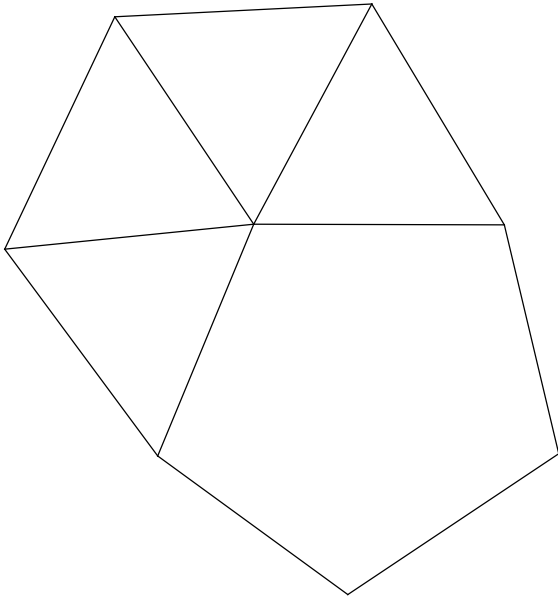


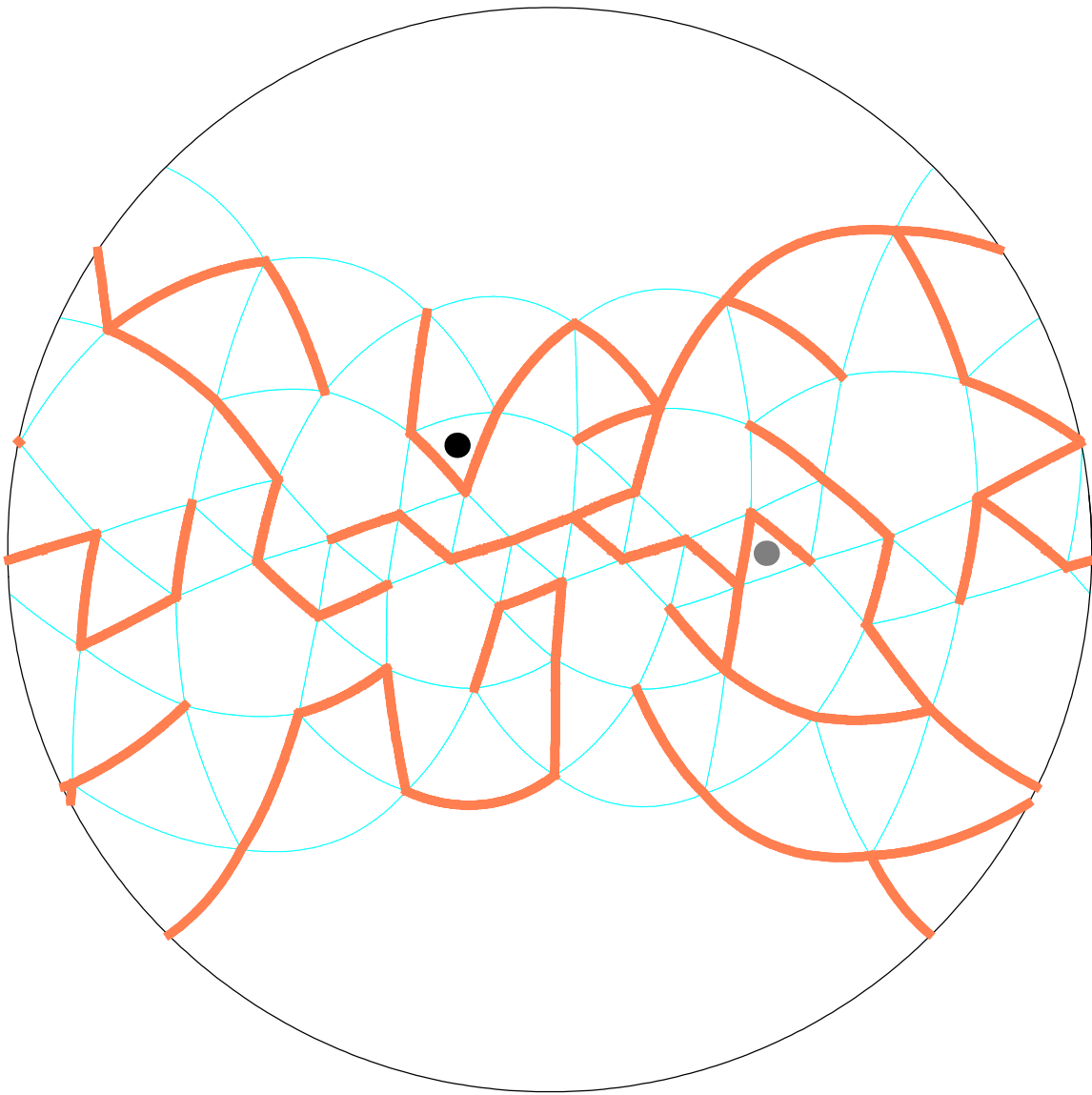


29.

snub dodecahedron

$\{3, 3, 3, 3, 5\}$

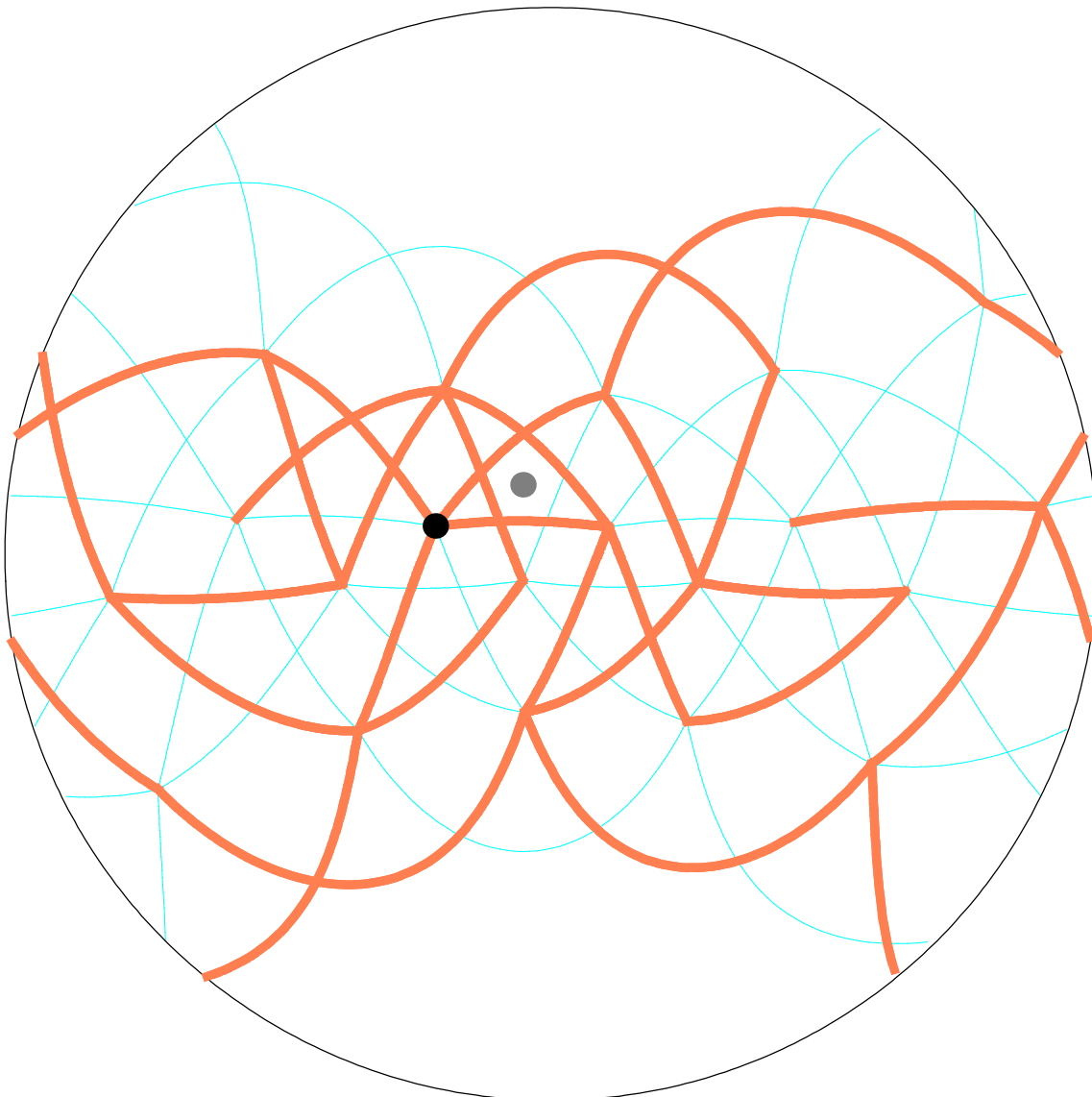
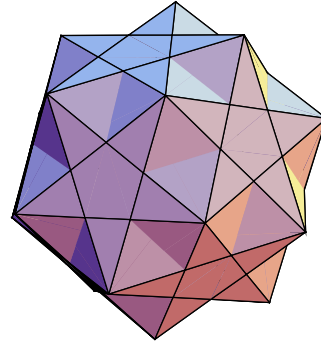
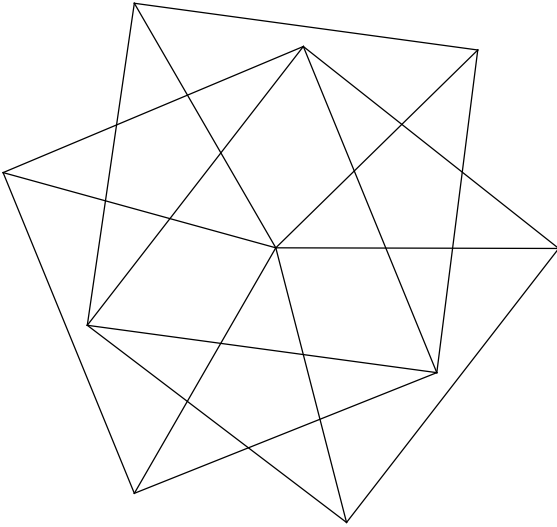




30.

small ditrigonal icosidodecahedron

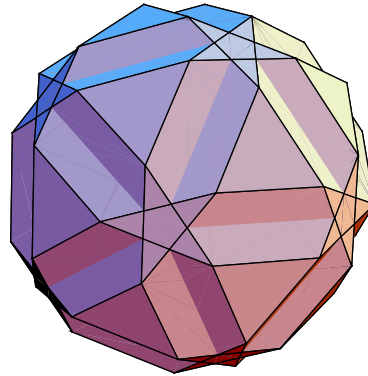
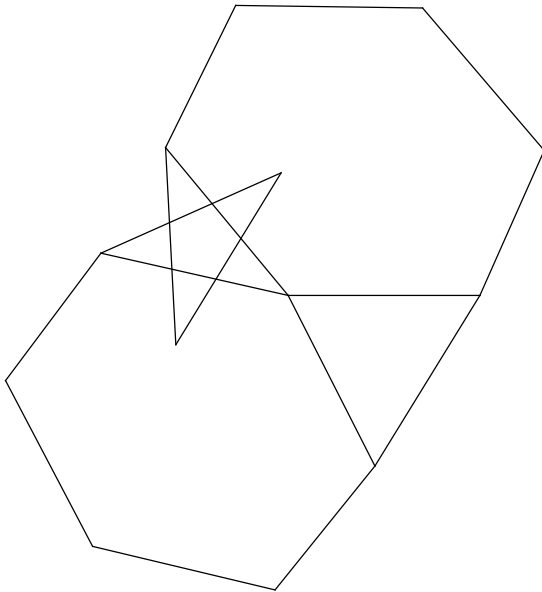
$$\left\{ \frac{5}{2}, 3, \frac{5}{2}, 3, \frac{5}{2}, 3 \right\}$$

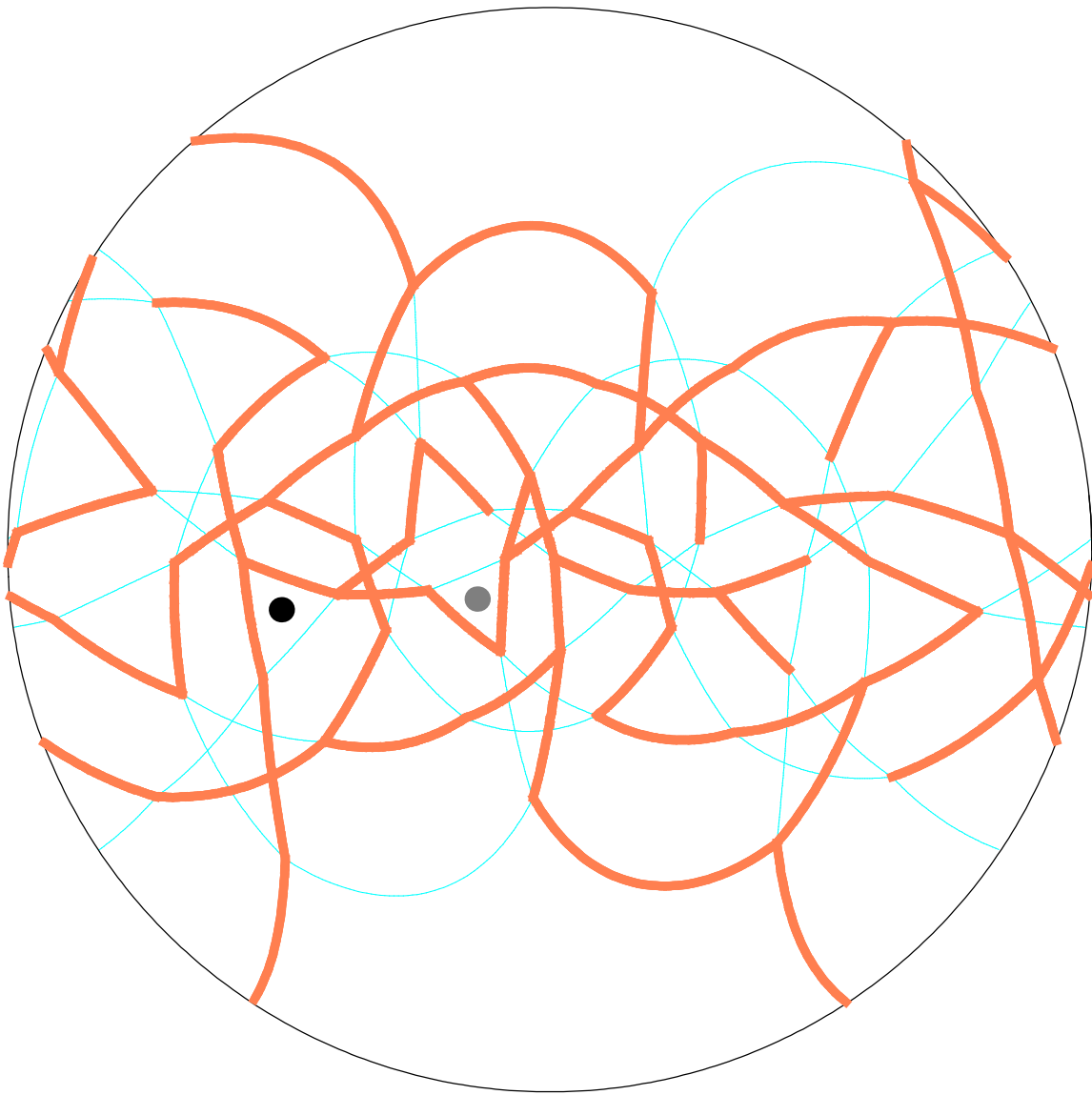


31.

small icosicosidodecahedron

$$\left\{6, \frac{5}{2}, 6, 3\right\}$$

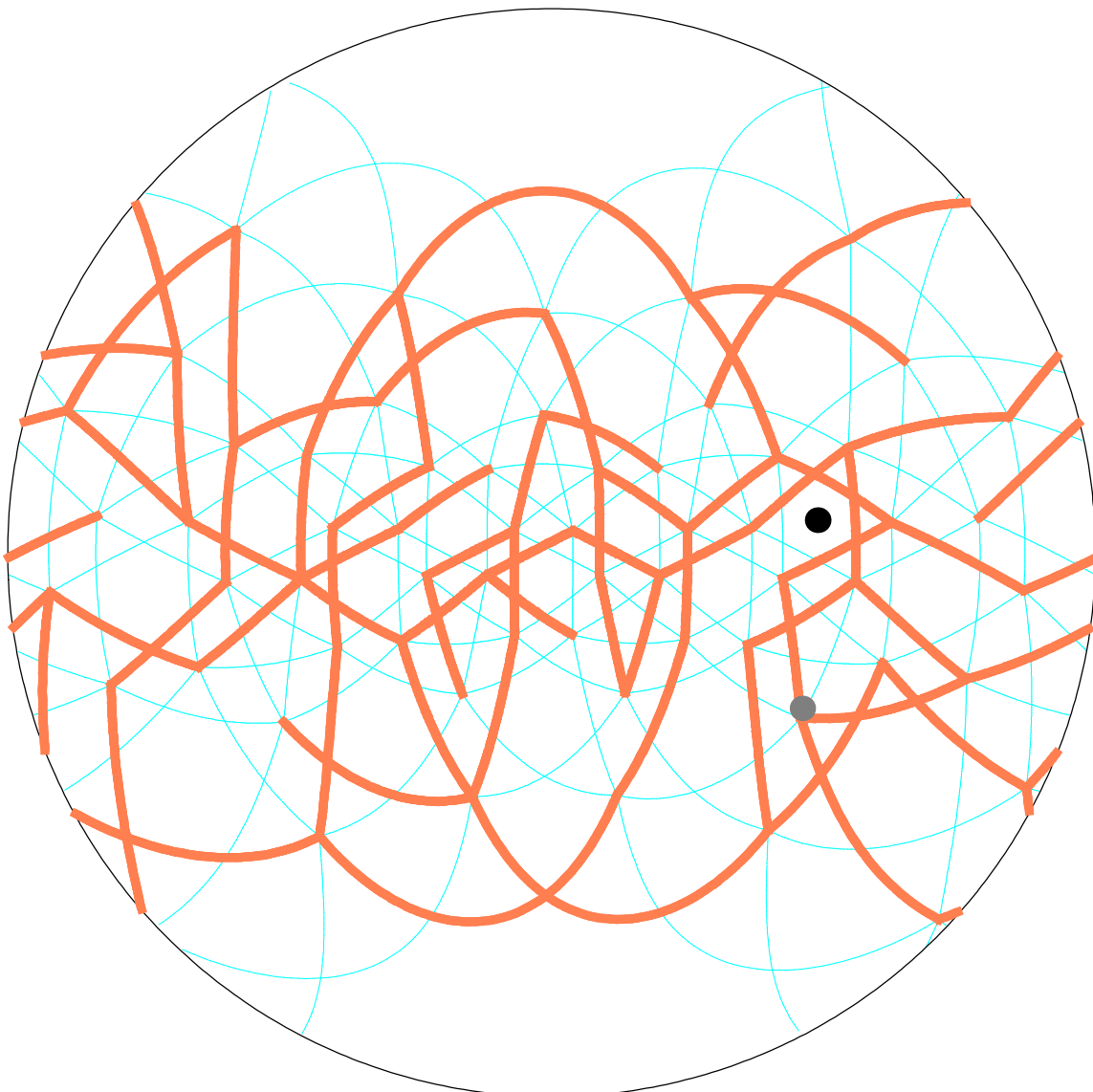
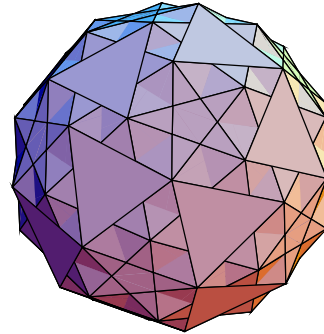
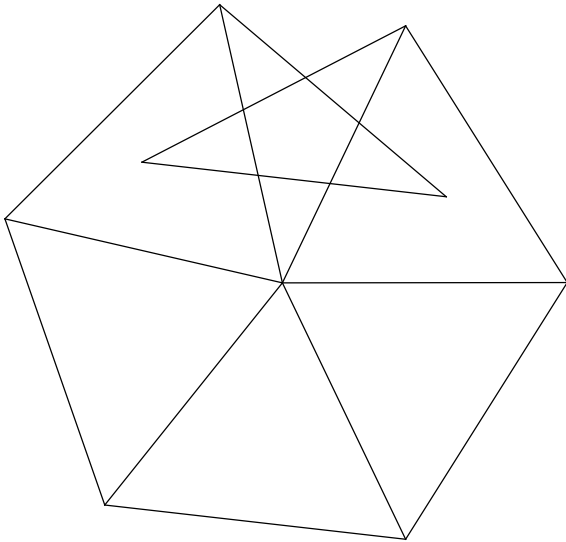




32.

small snub icosicosidodecahedron

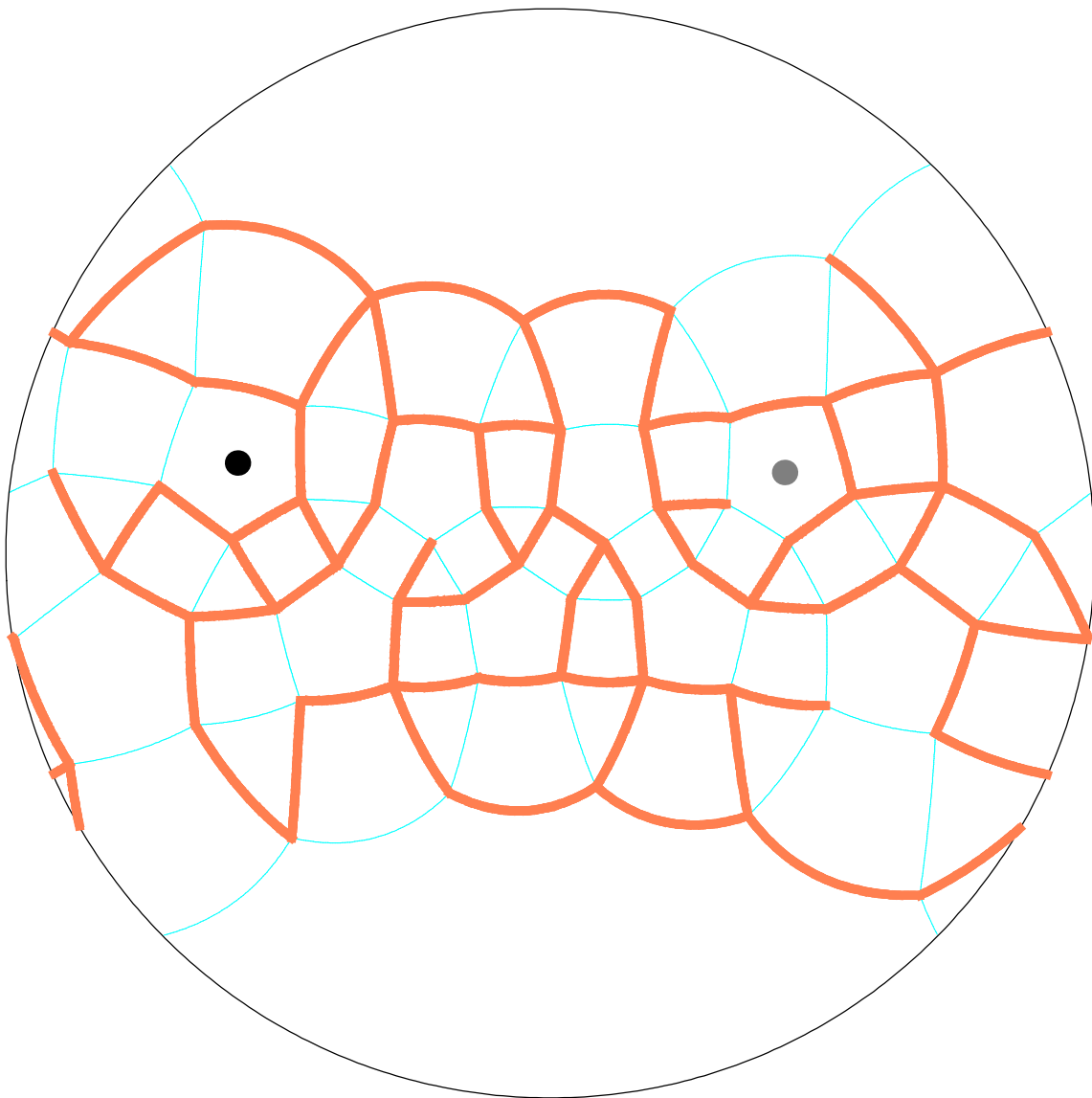
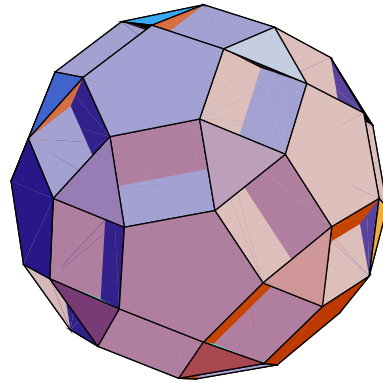
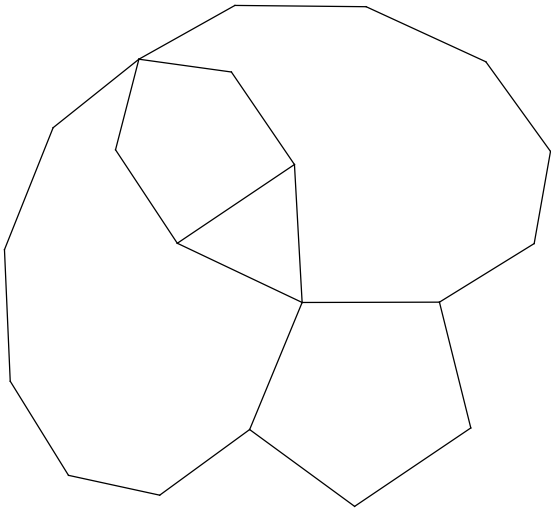
$$\left\{3, \frac{5}{2}, 3, 3, 3, 3\right\}$$



33.

small dodecicosidodecahedron

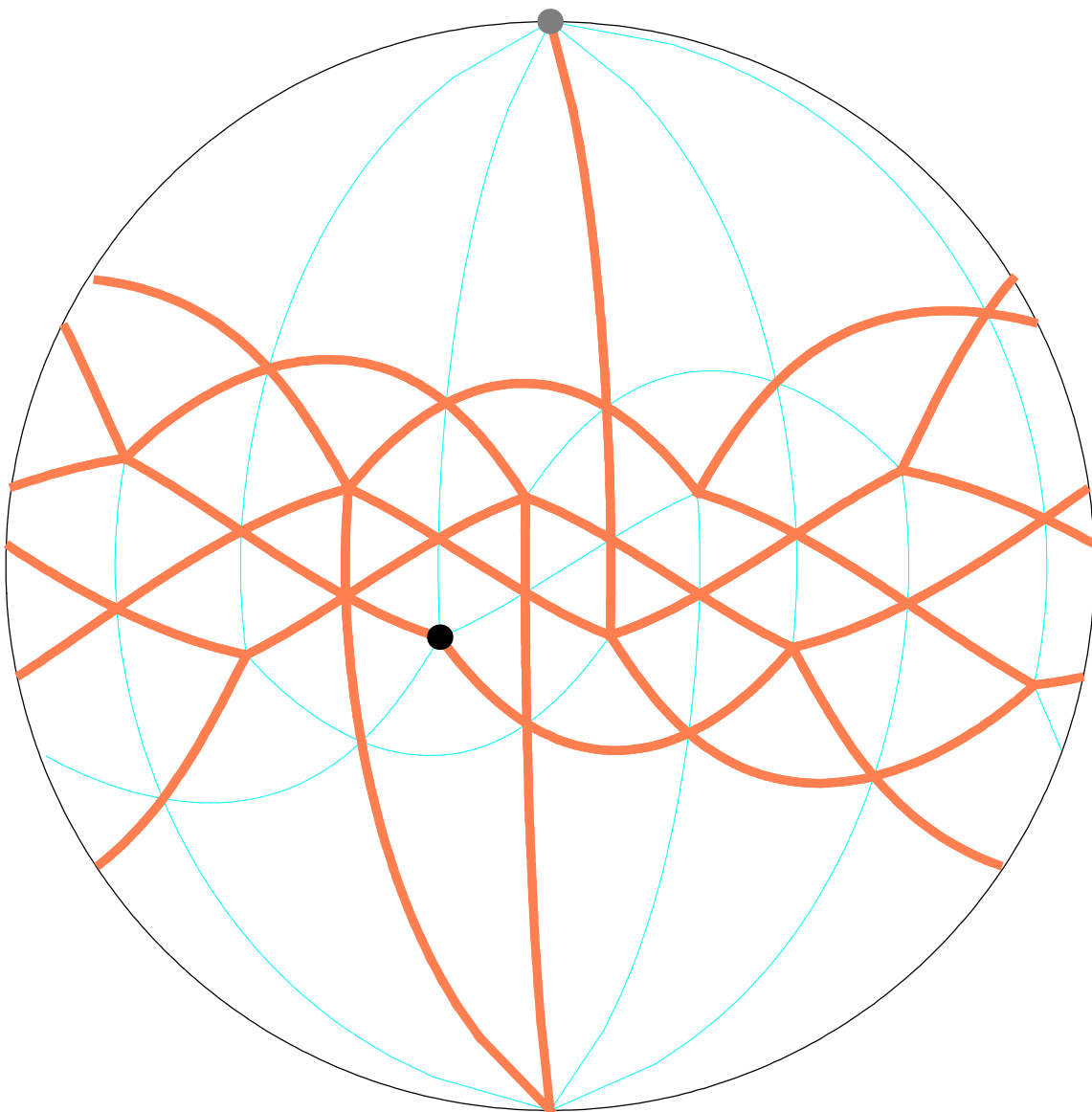
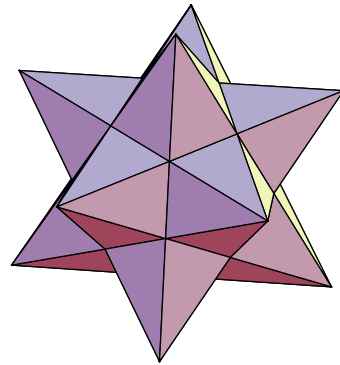
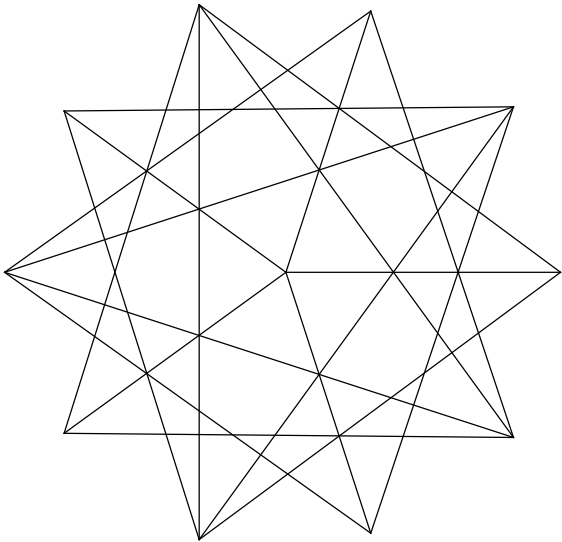
$$\left\{10, \frac{3}{2}, 10, 5\right\}$$



34.

small stellated dodecahedron

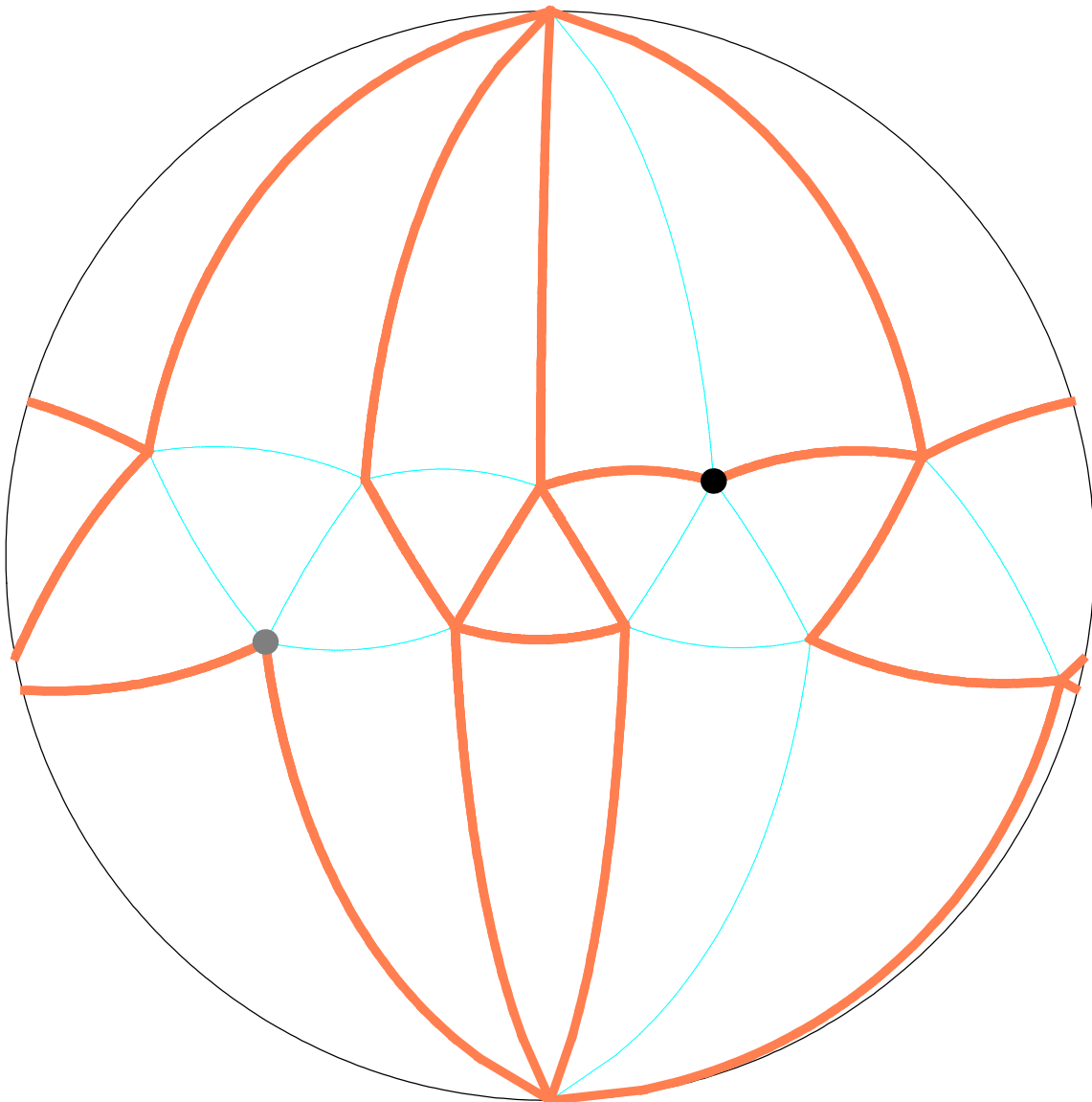
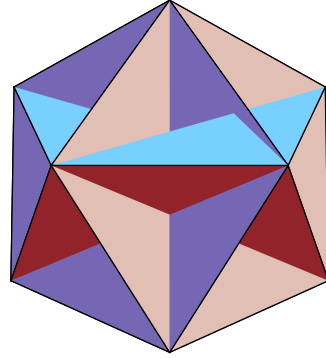
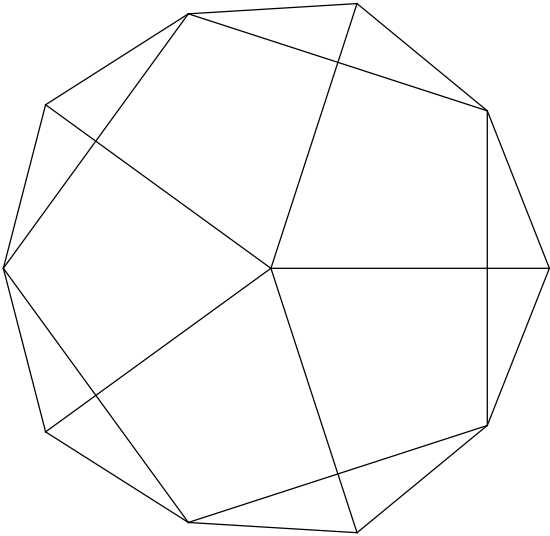
$$\left\{ \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2} \right\}$$



35.

great dodecahedron

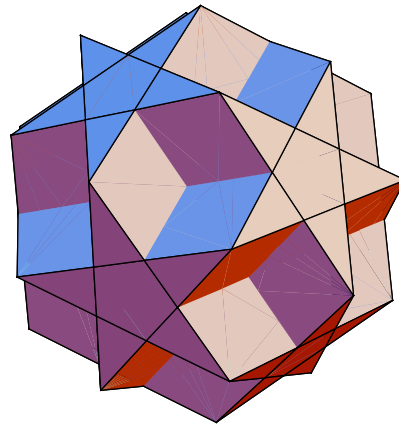
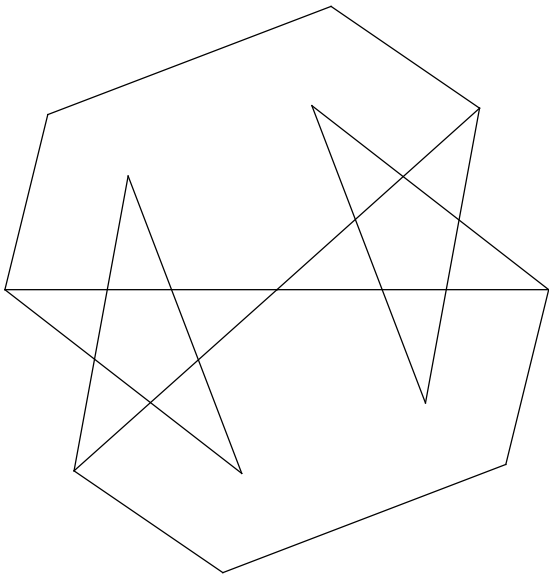
$$\frac{1}{2} \{5, 5, 5, 5\}$$

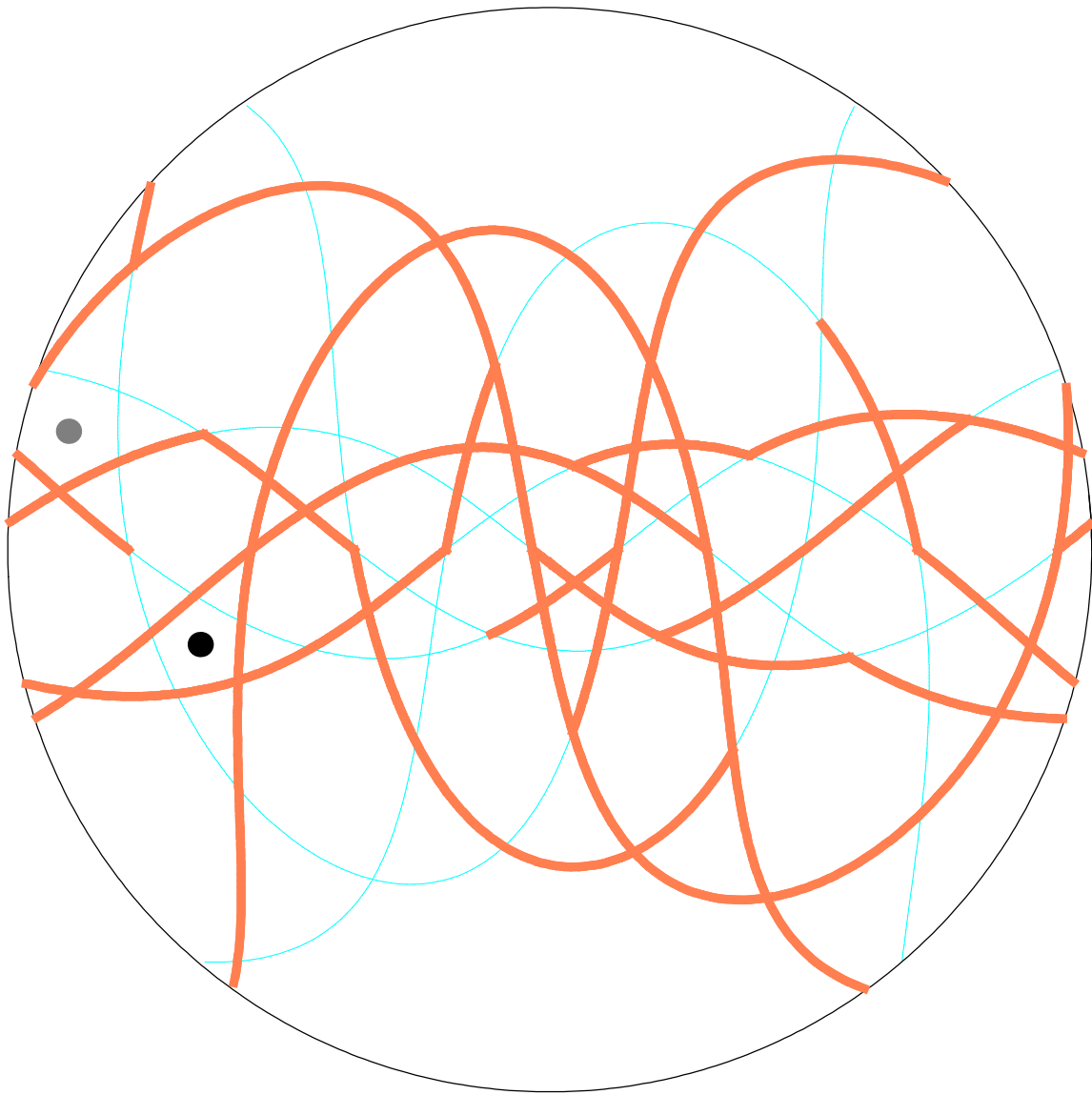


36.

dodecadodecahedron

$$\left\{ \frac{5}{2}, 5, \frac{5}{2}, 5 \right\}$$

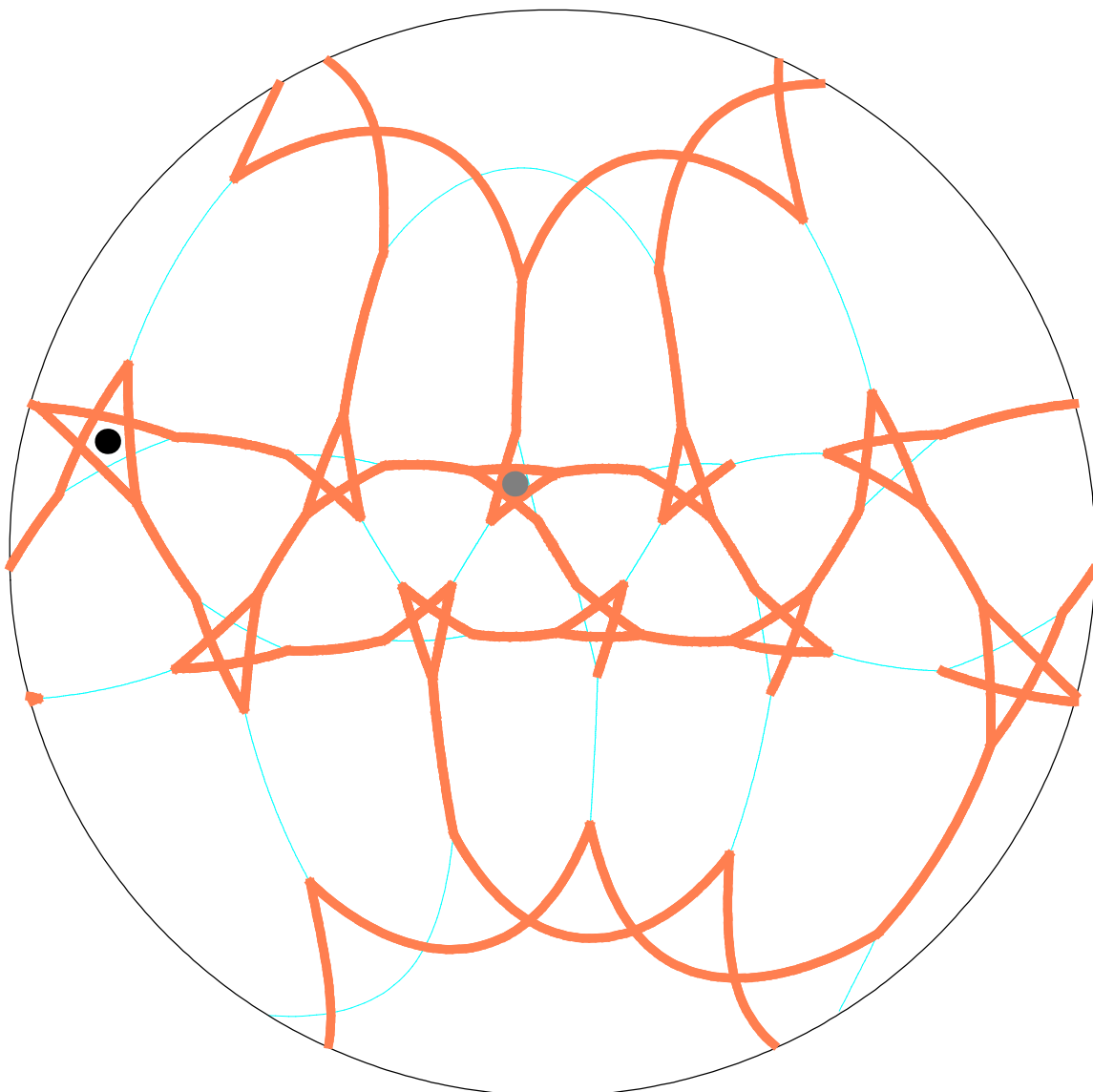
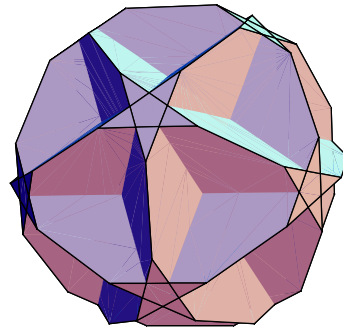
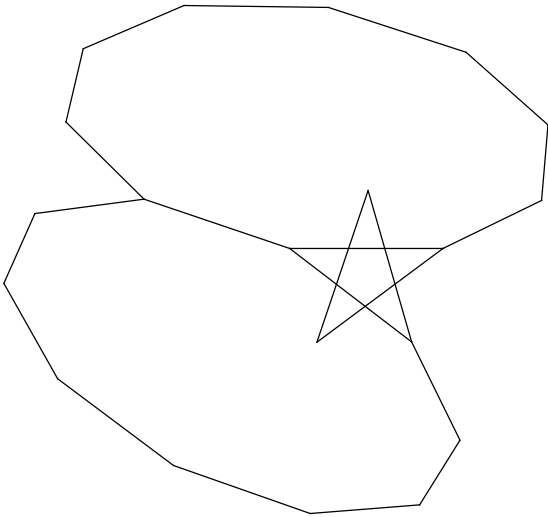




37.

truncated great dodecahedron

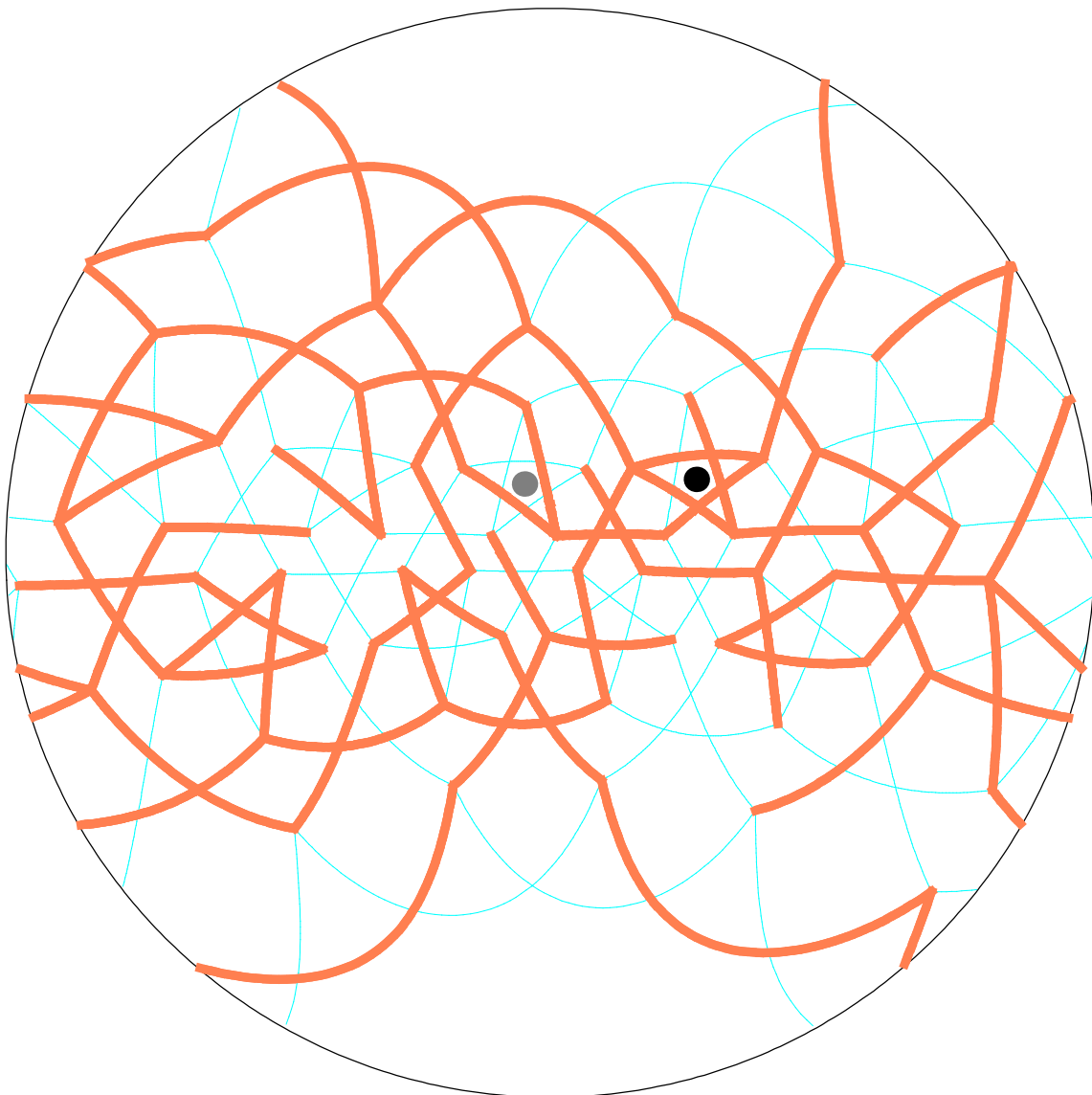
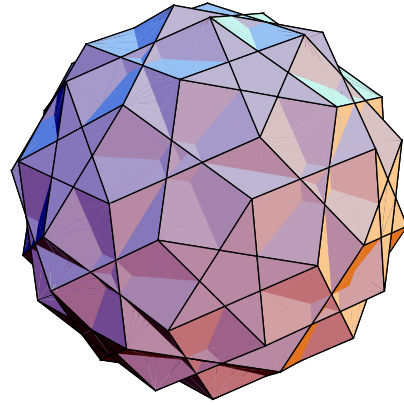
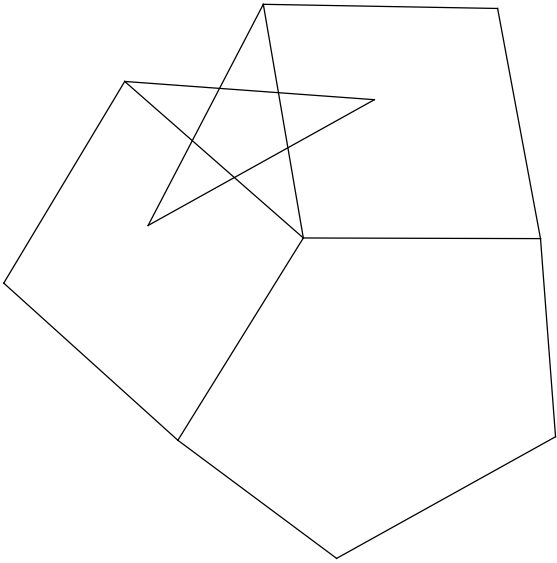
$$\{10, 10, \frac{5}{2}\}$$



38.

rhombidodecadodecahedron

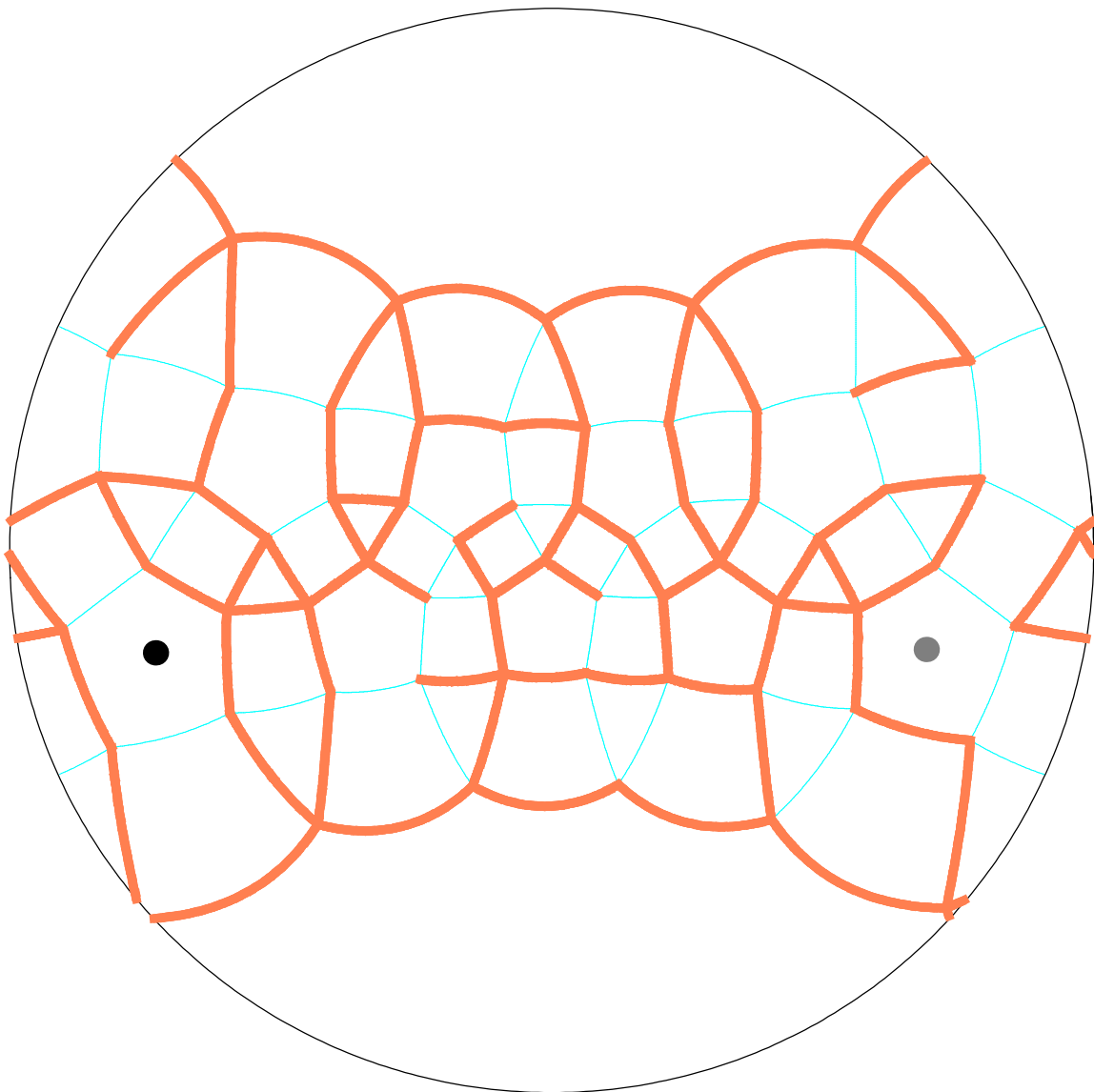
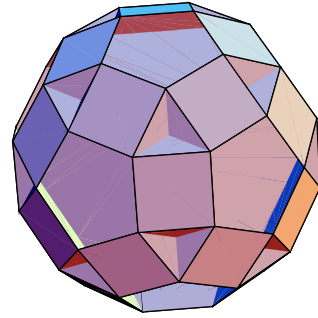
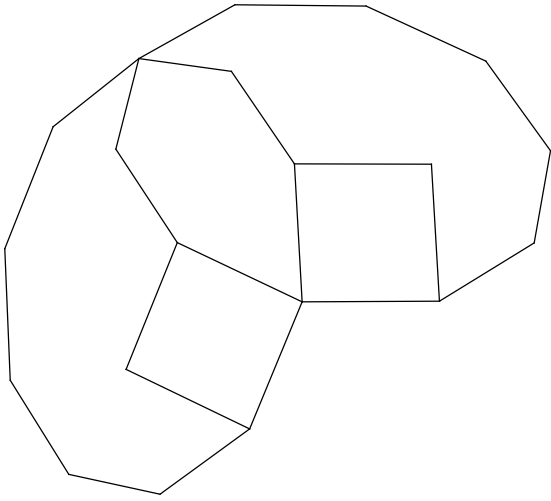
$$\{4, \frac{5}{2}, 4, 5\}$$



39.

small rhombidodecahedron

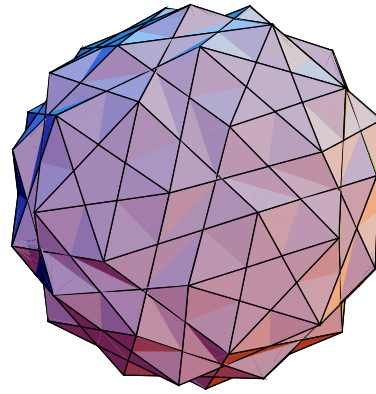
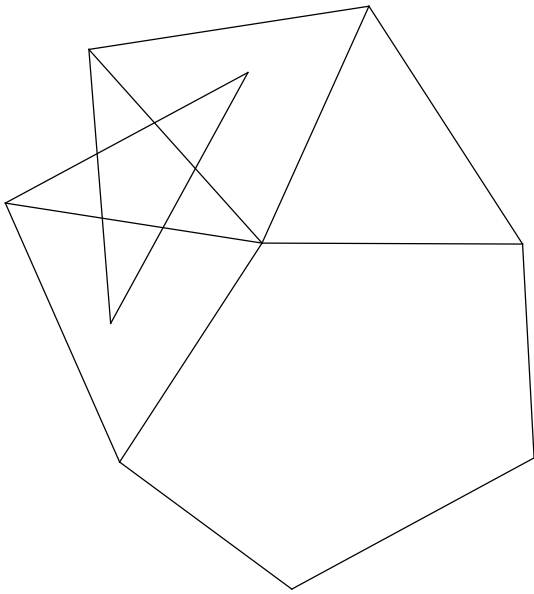
$$\left\{10, 4, \frac{10}{9}, \frac{4}{3}\right\}$$

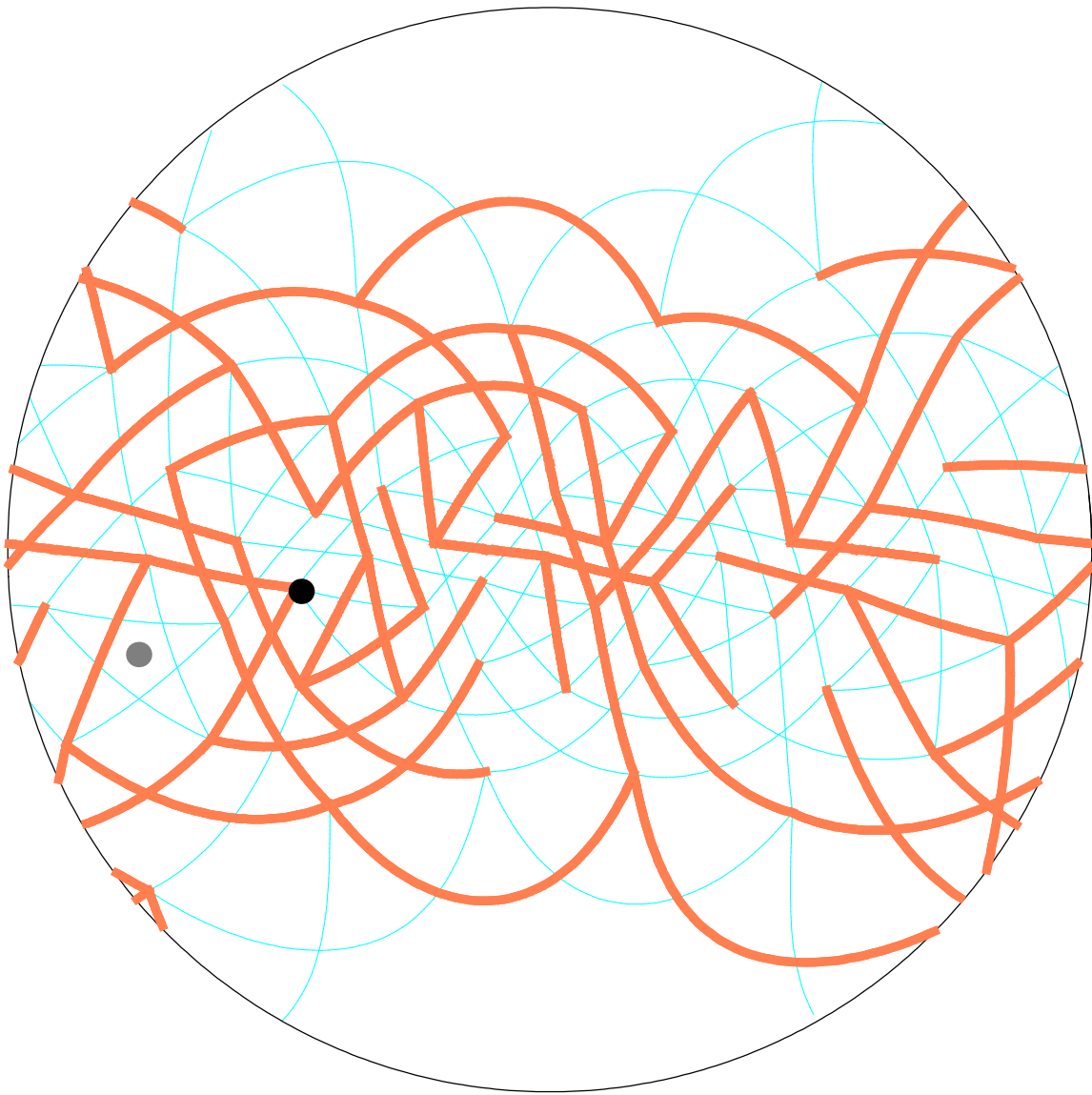


40.

snub dodecadodecahedron

$$\left\{3, 3, \frac{5}{2}, 3, 5\right\}$$



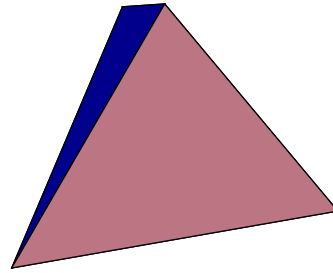
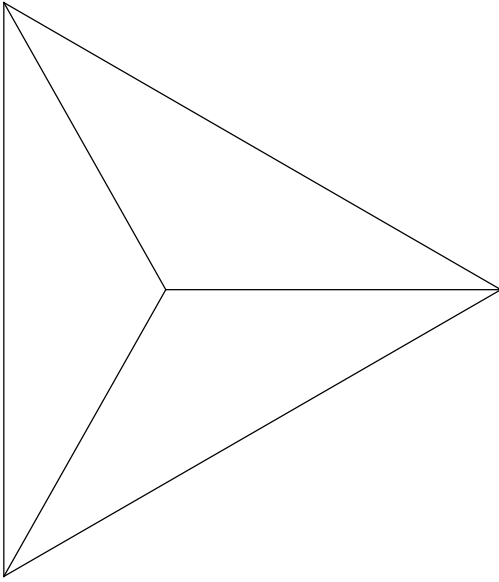


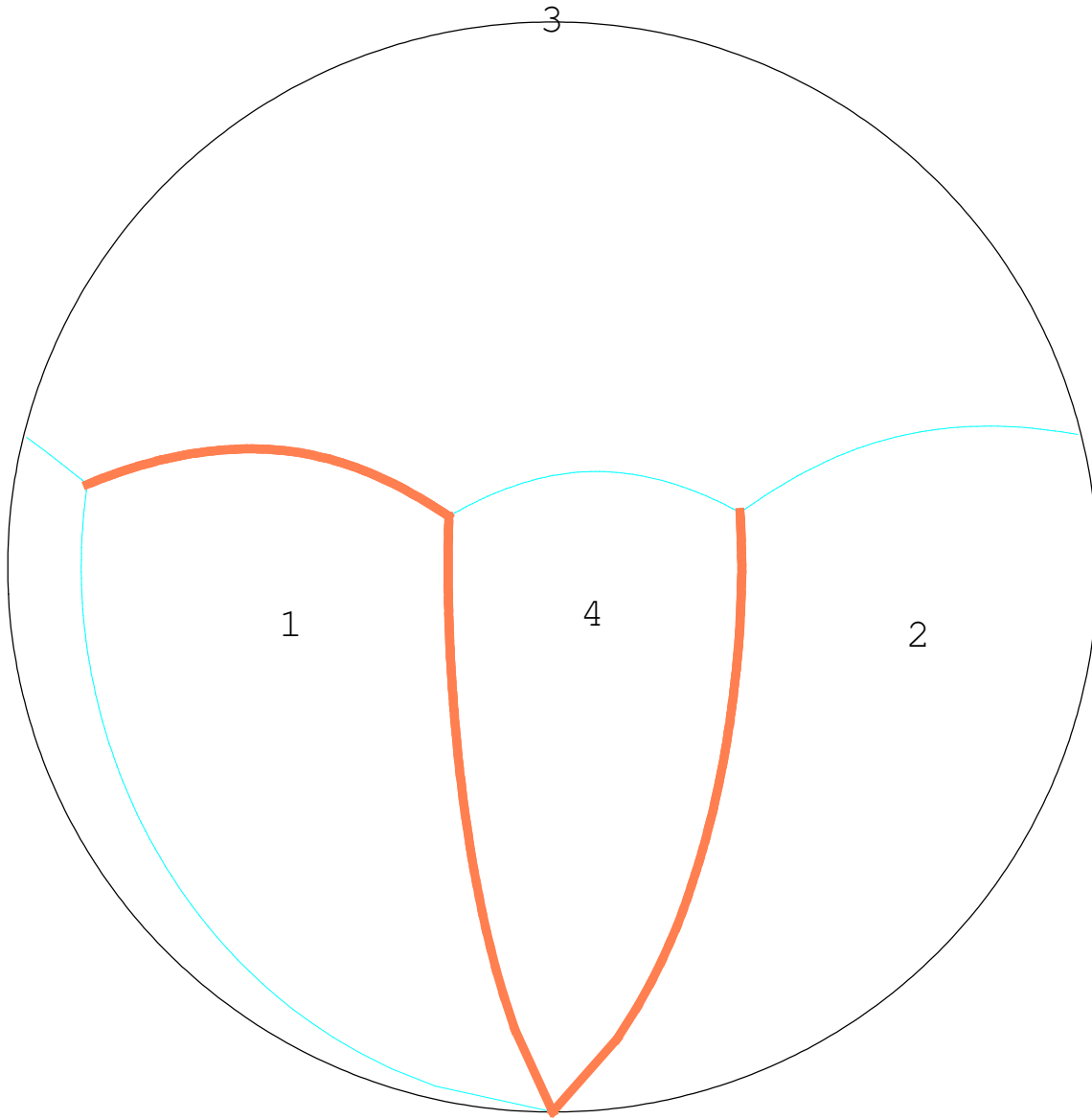
Solutions

1.

tetrahedron

{3, 3, 3}

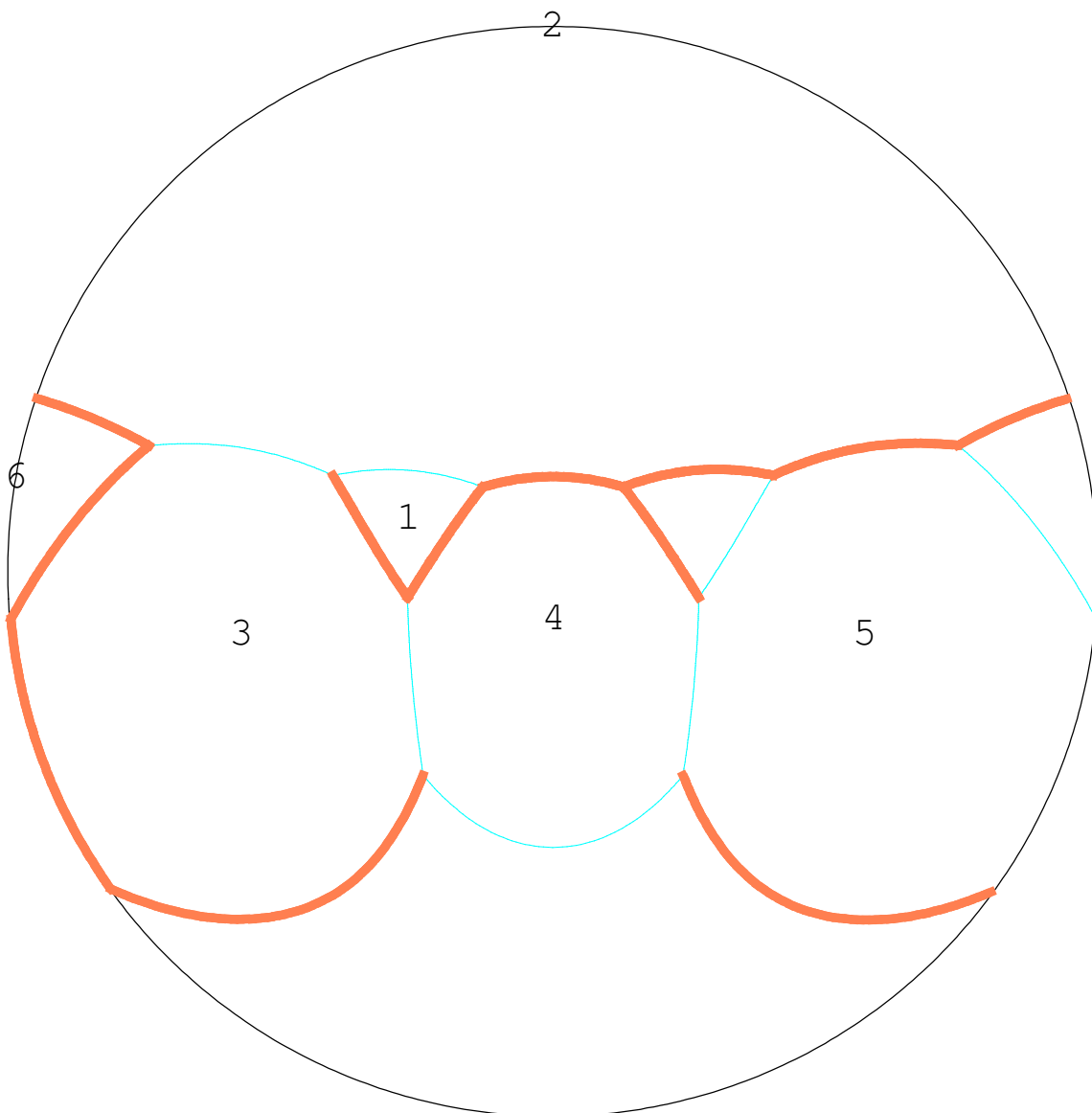
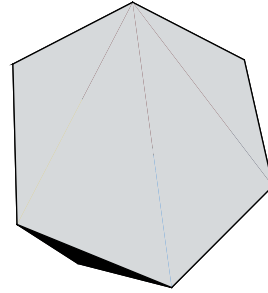
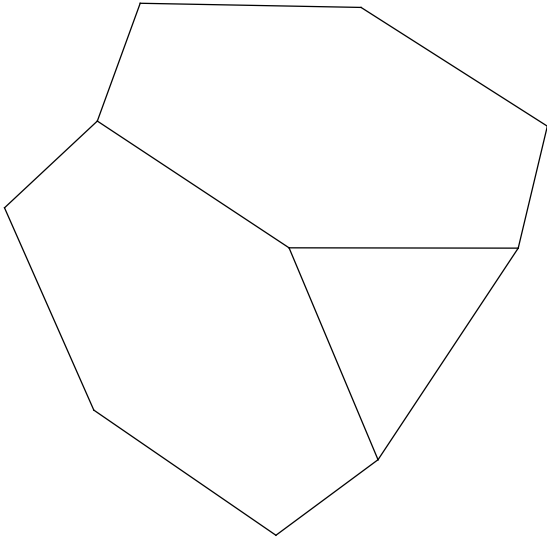




2.

truncated tetrahedron

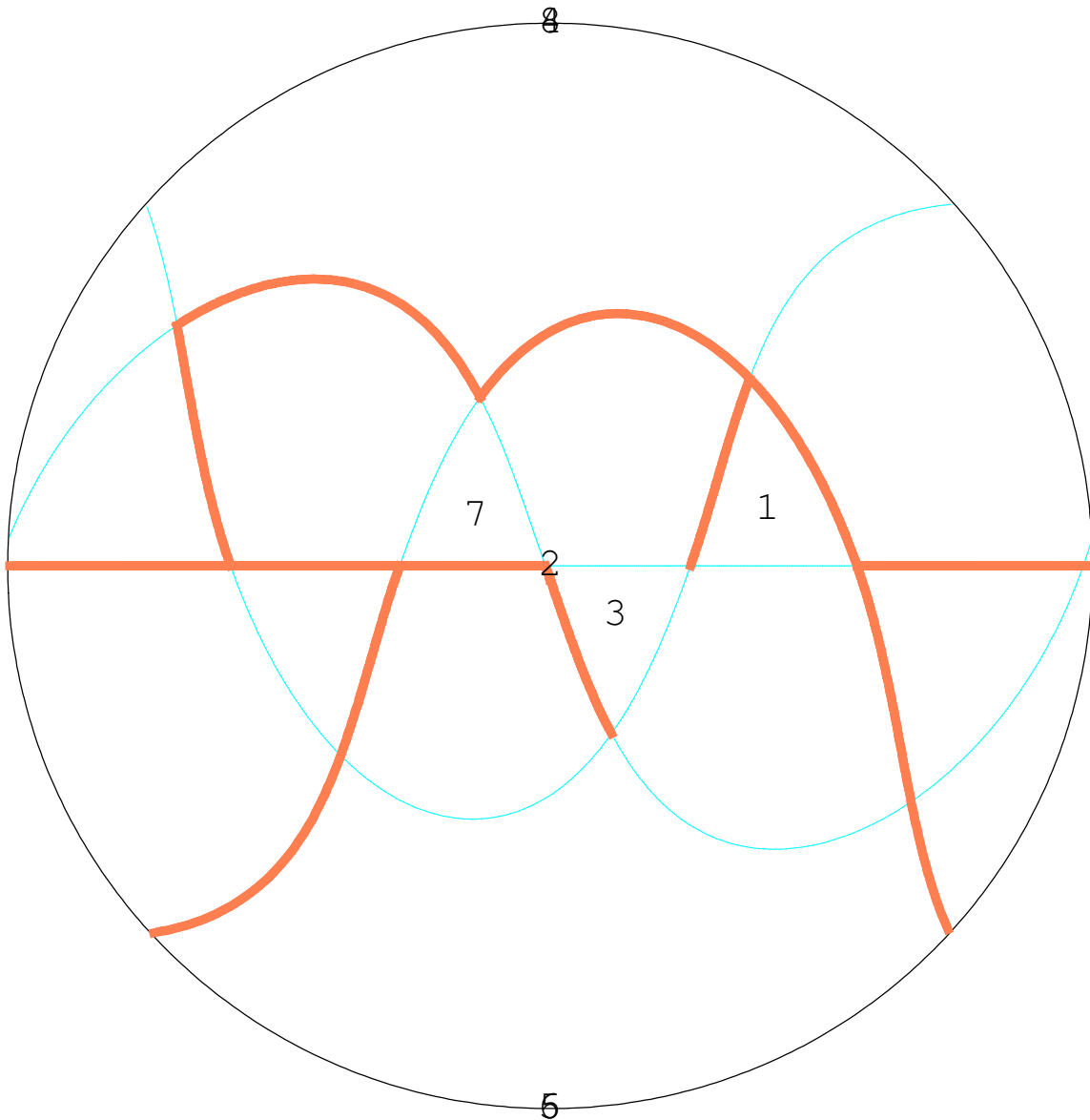
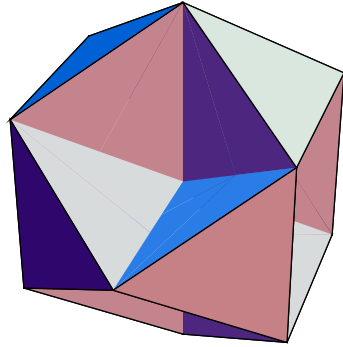
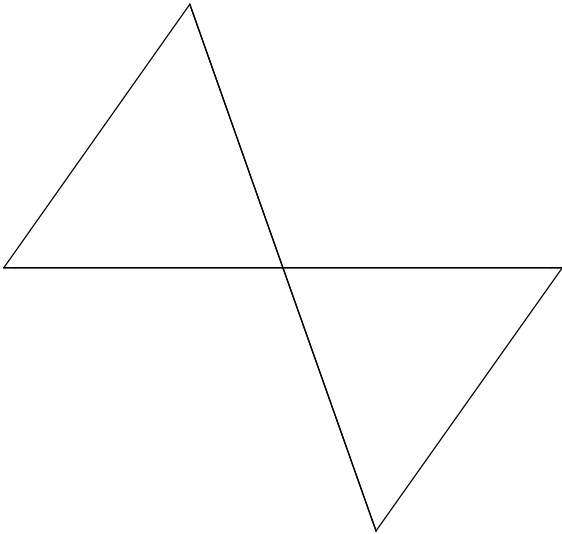
{6, 6, 3}



3.

octahemioctahedron

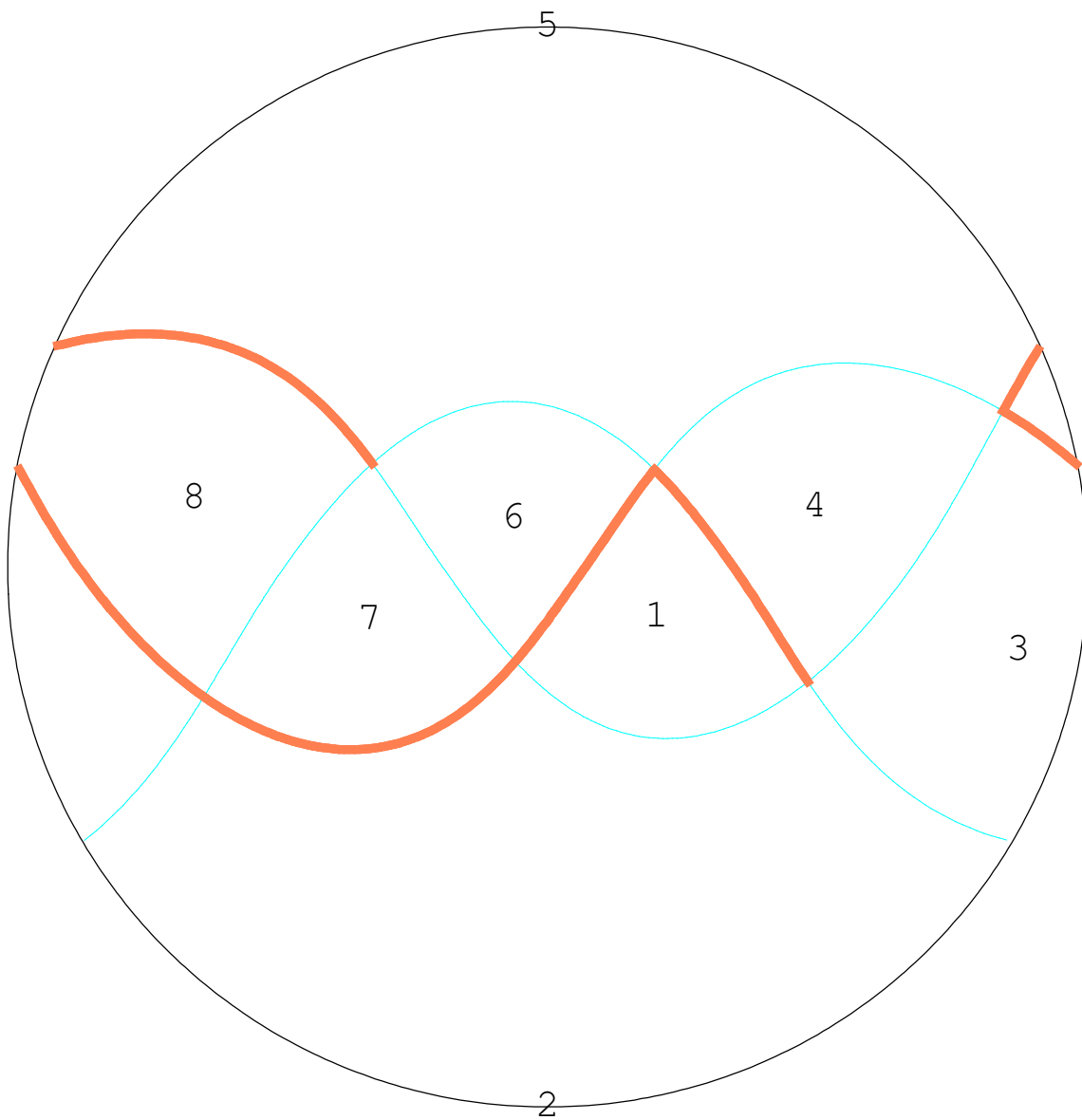
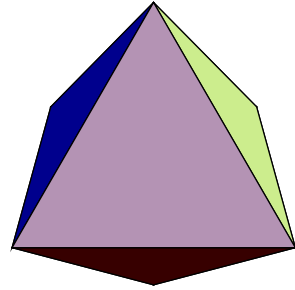
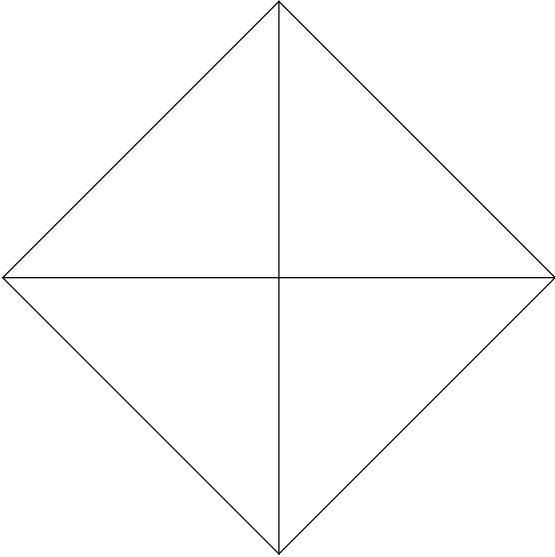
$$\{6, \frac{3}{2}, 6, 3\}$$



5.

octahedron

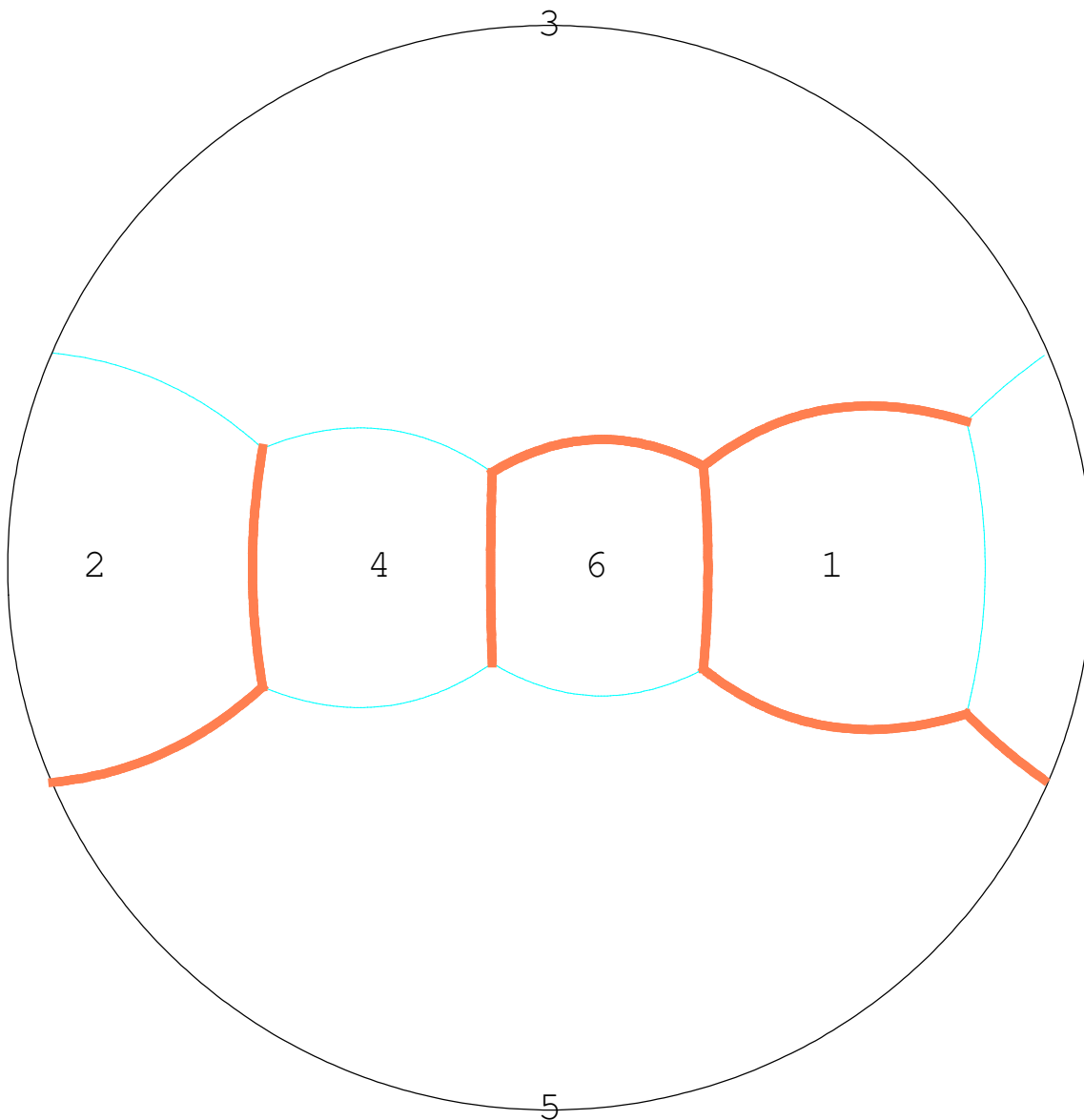
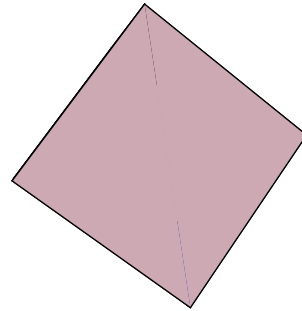
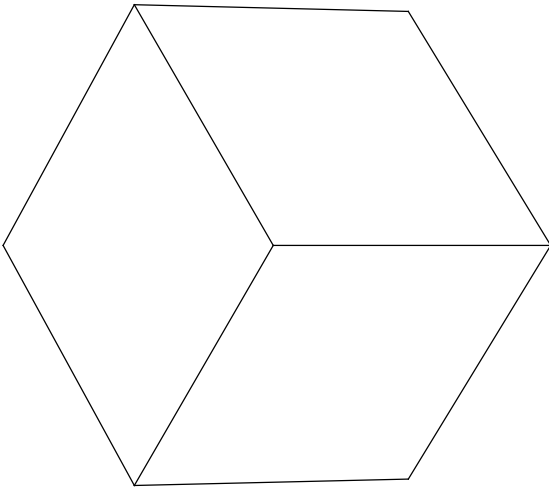
{3, 3, 3, 3}



6.

cube

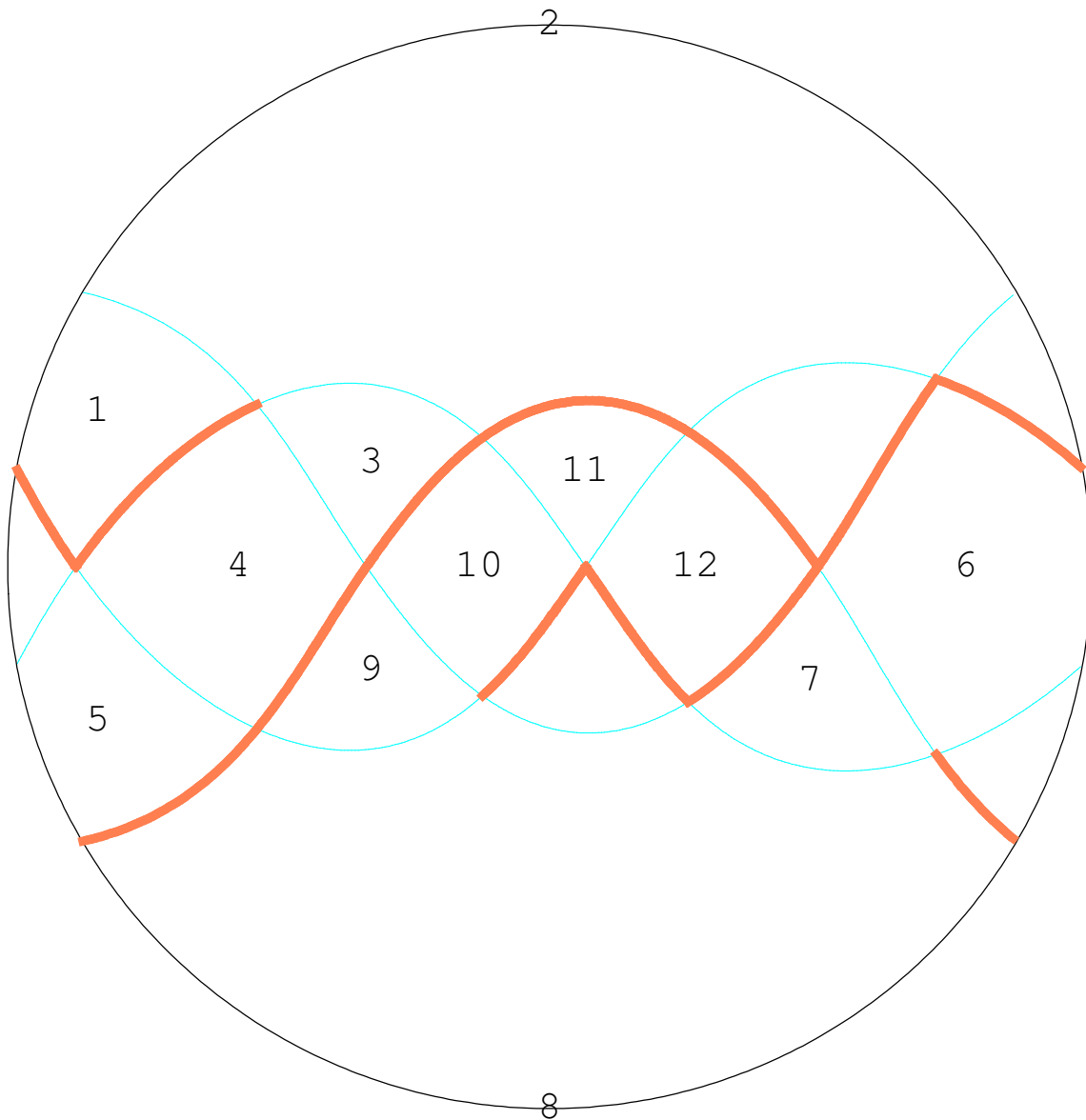
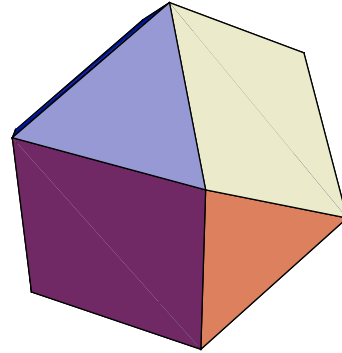
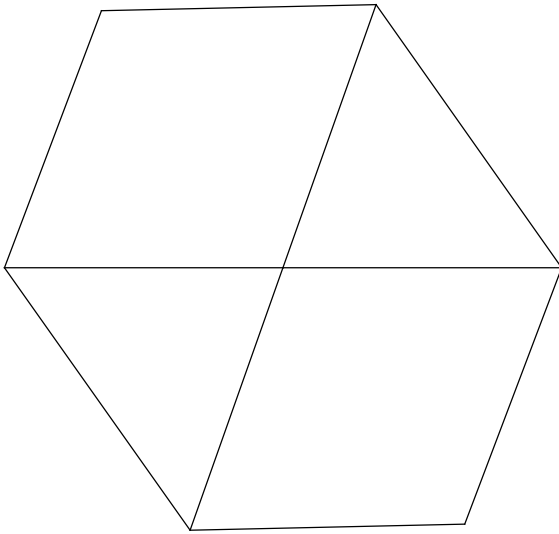
{4, 4, 4}



7.

cuboctahedron

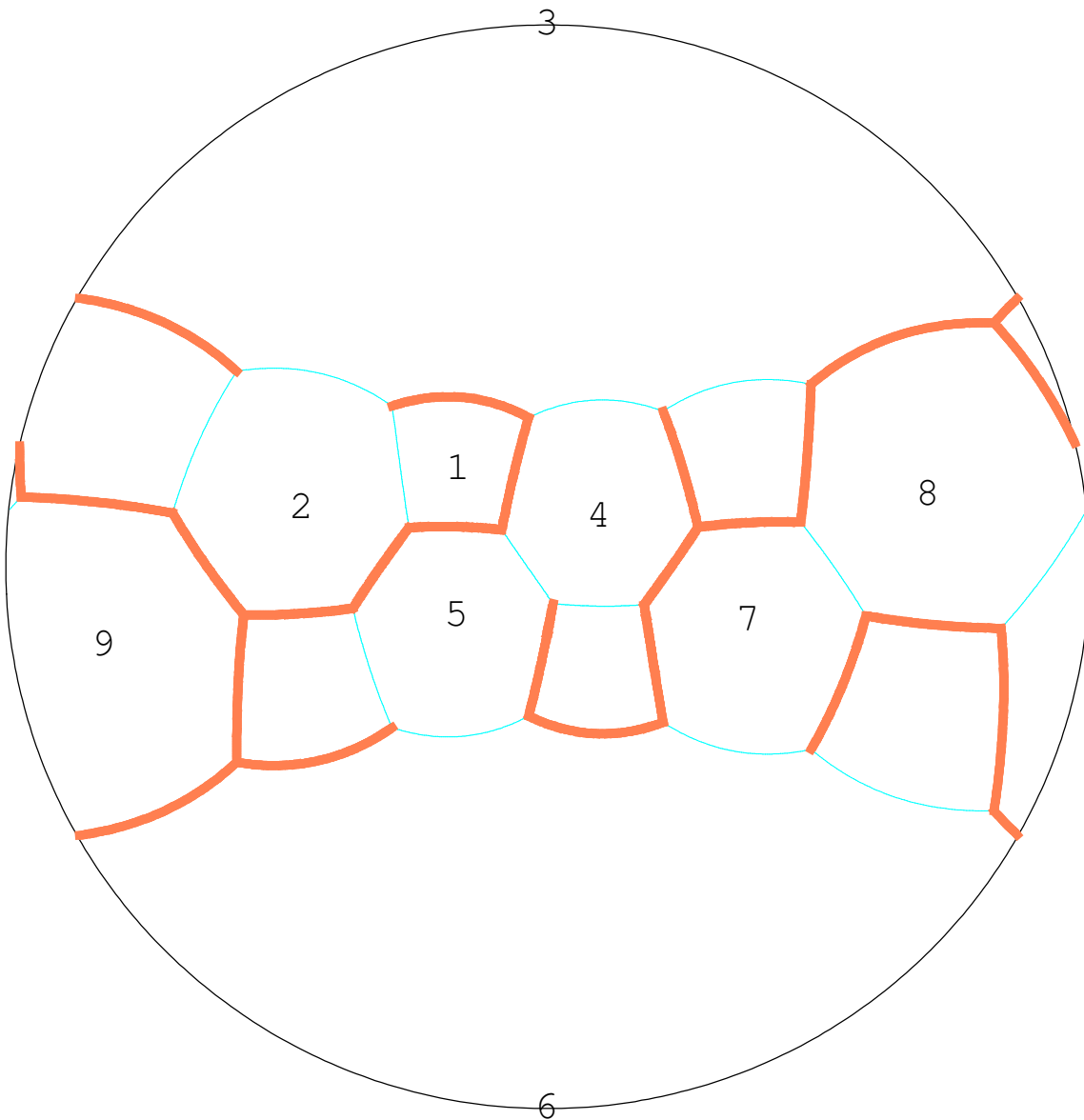
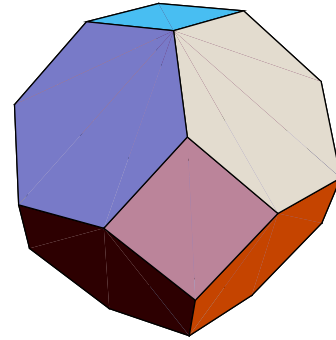
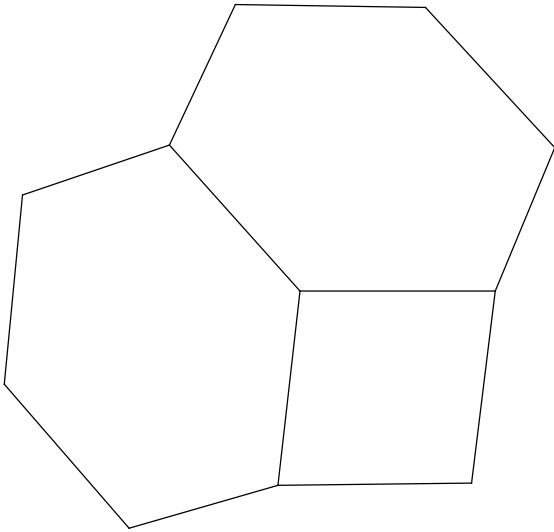
{3, 4, 3, 4}



8.

truncated octahedron

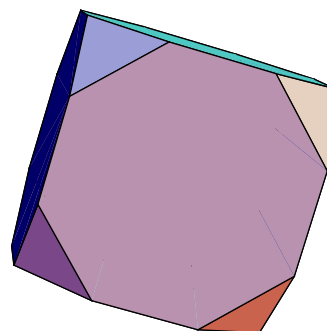
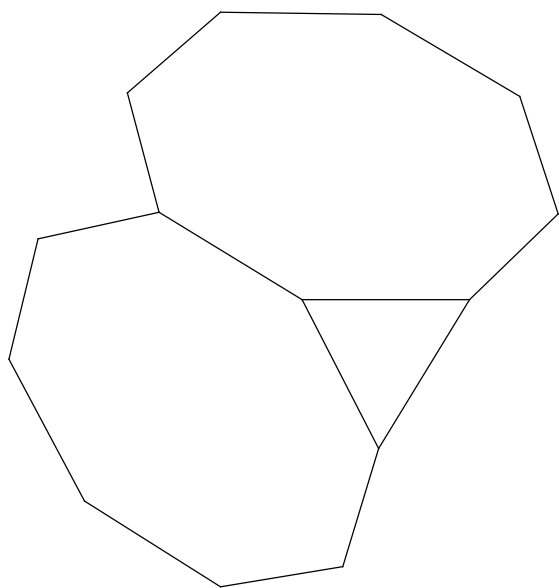
{6, 6, 4}

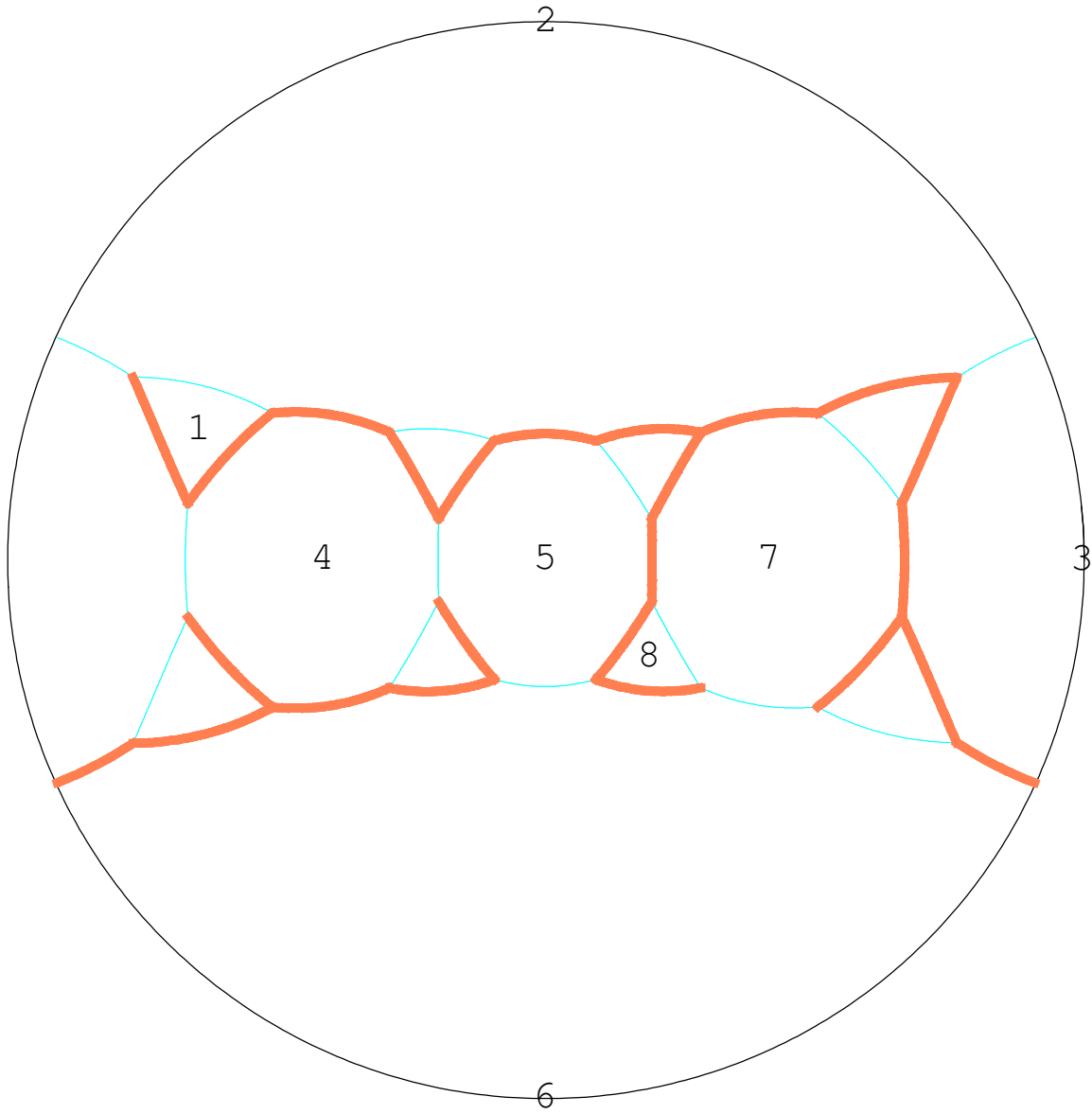


9.

truncated cube

{8, 8, 3}

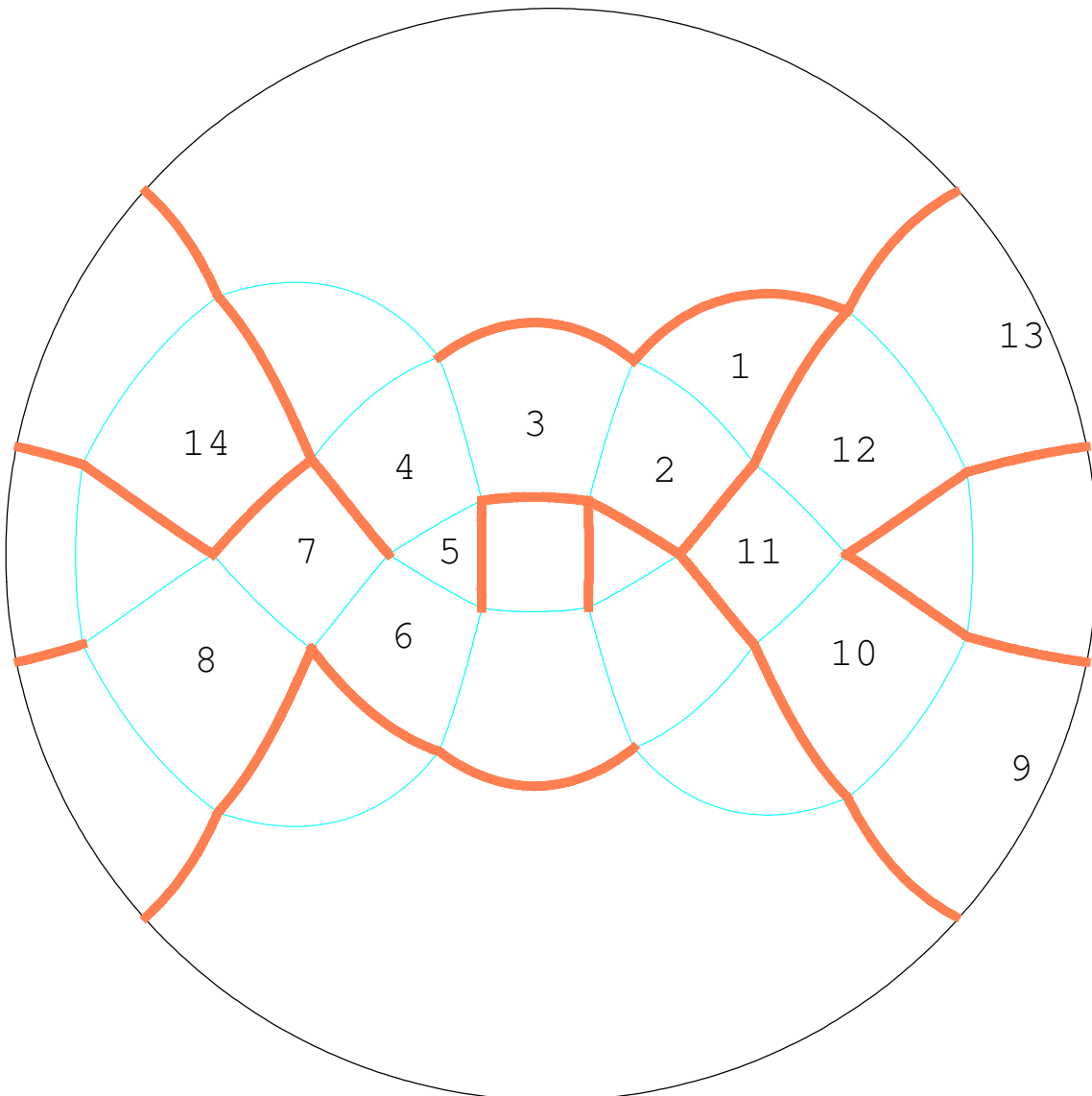
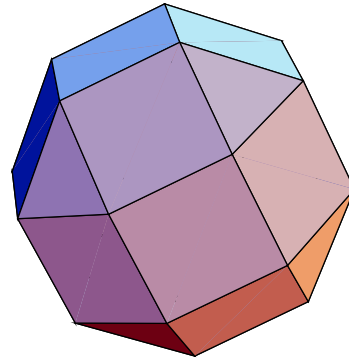
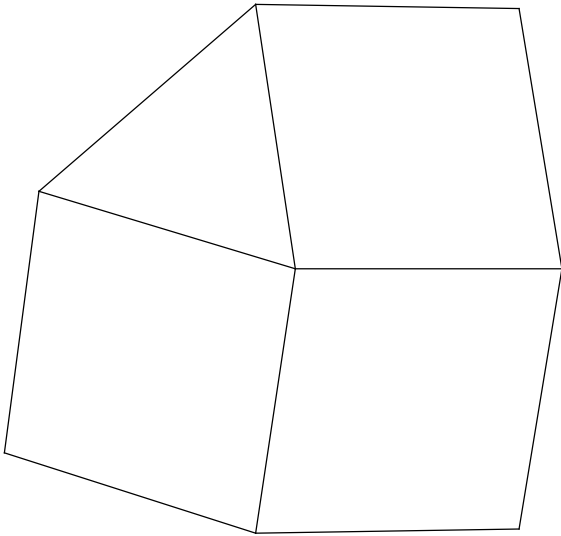




10.

rhombicuboctahedron

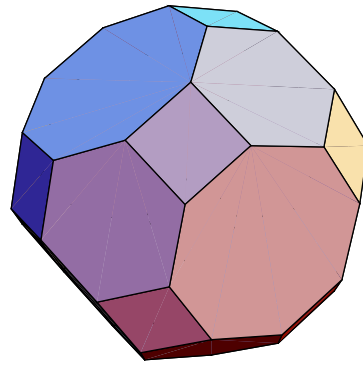
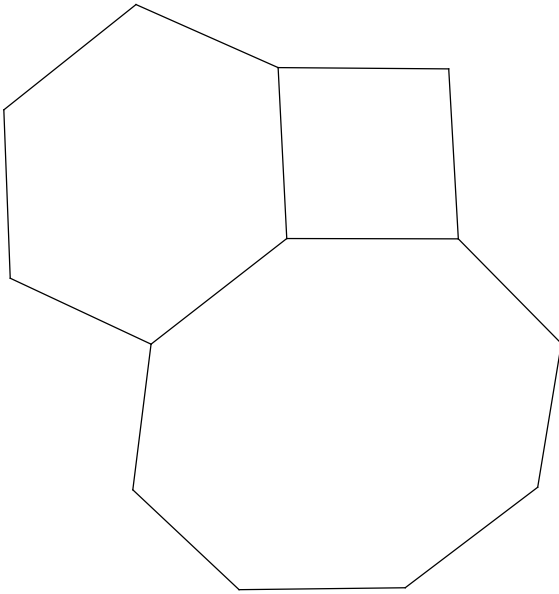
{4, 3, 4, 4}

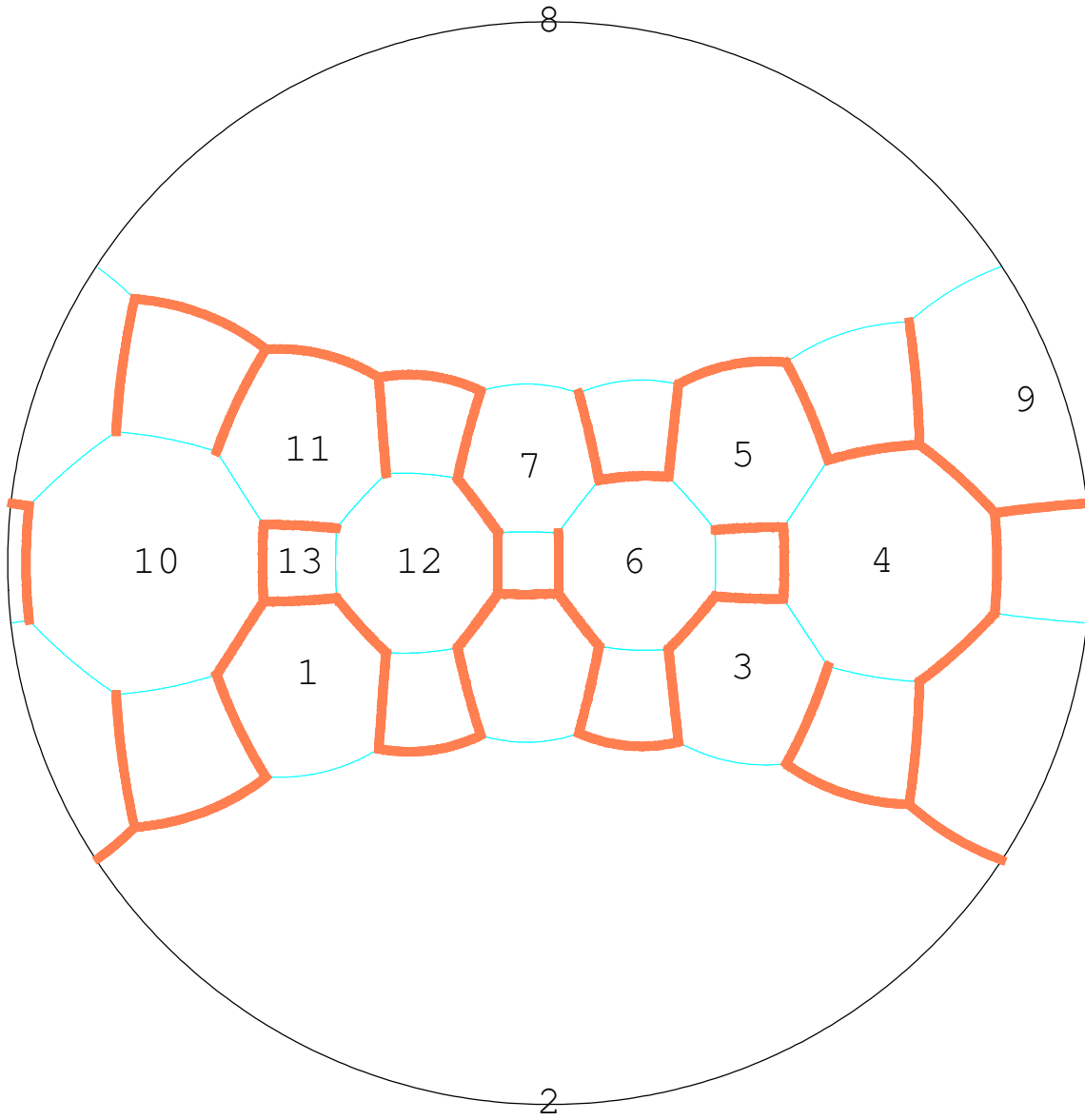


11.

truncated cuboctahedron

{4, 6, 8}

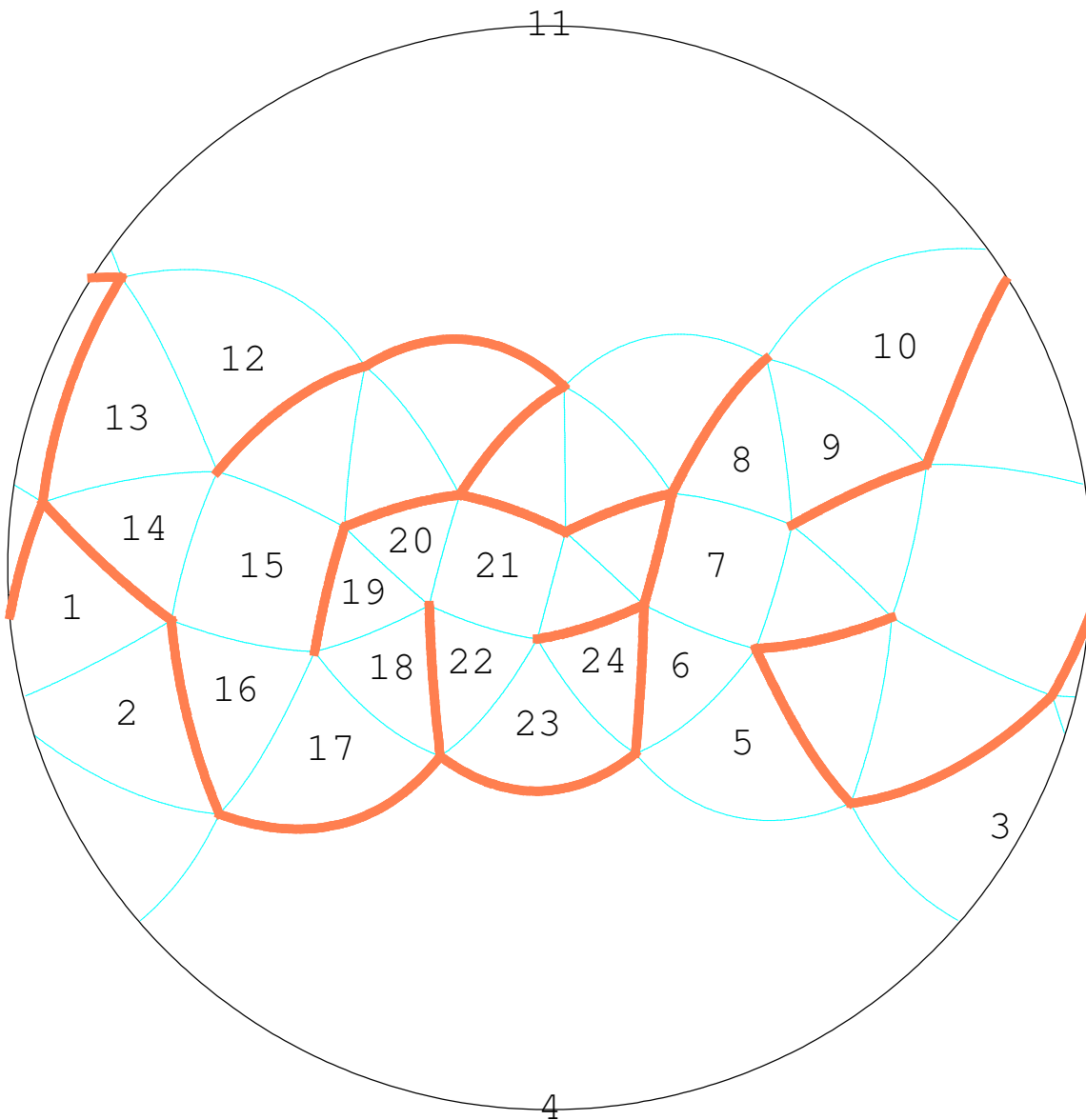
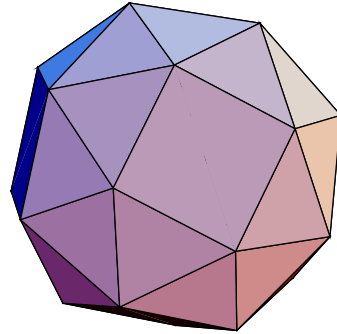
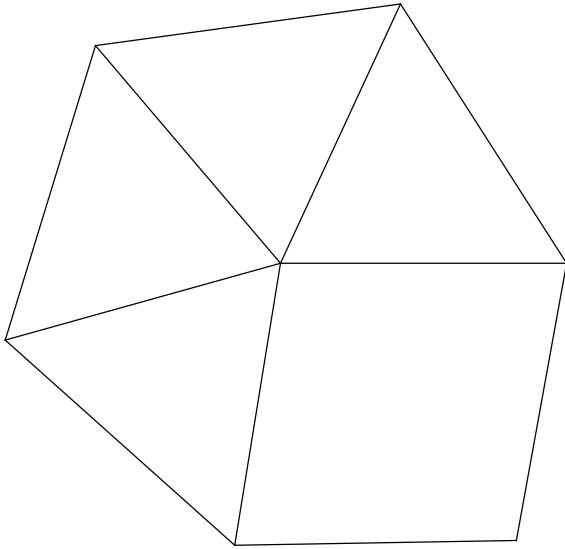




12.

snub cube

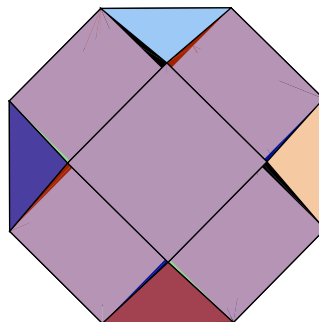
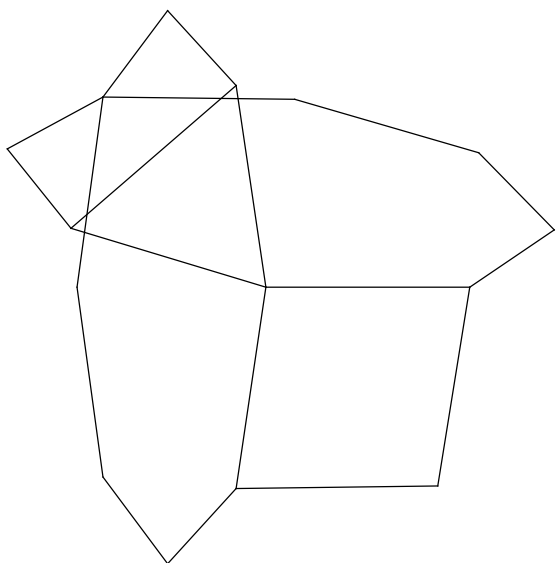
{3, 3, 3, 3, 4}



13.

small cubicuboctahedron

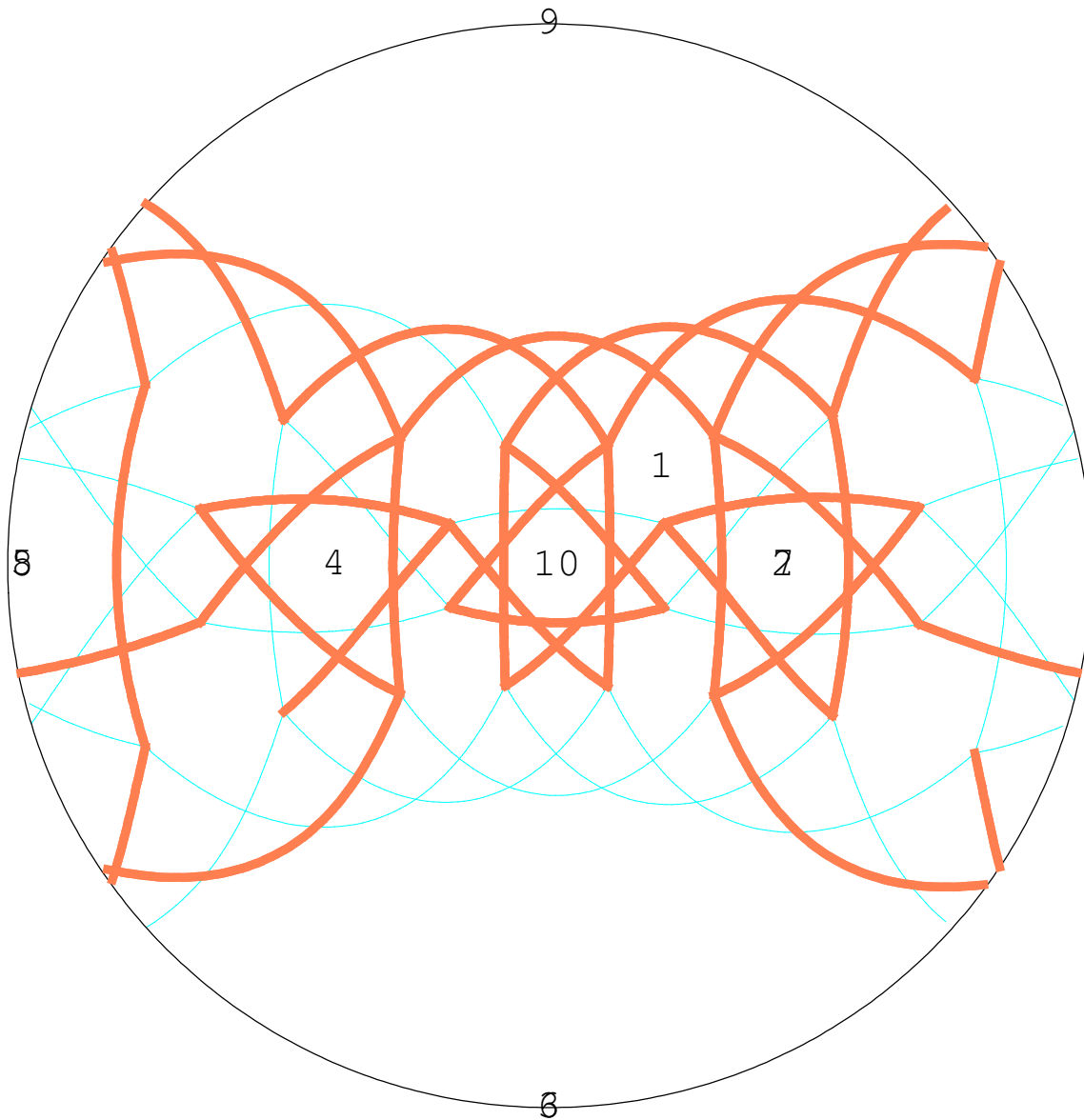
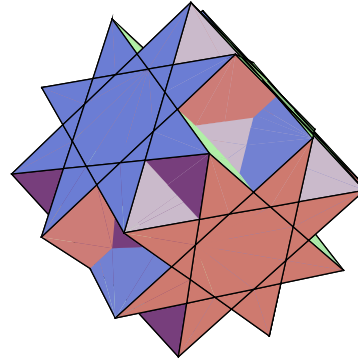
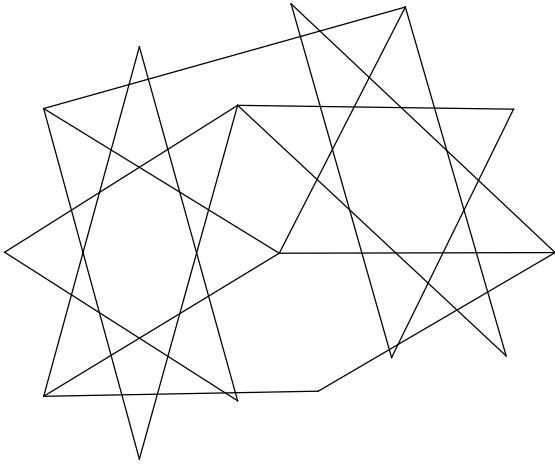
$$\left\{8, \frac{3}{2}, 8, 4\right\}$$



14.

great cubicuboctahedron

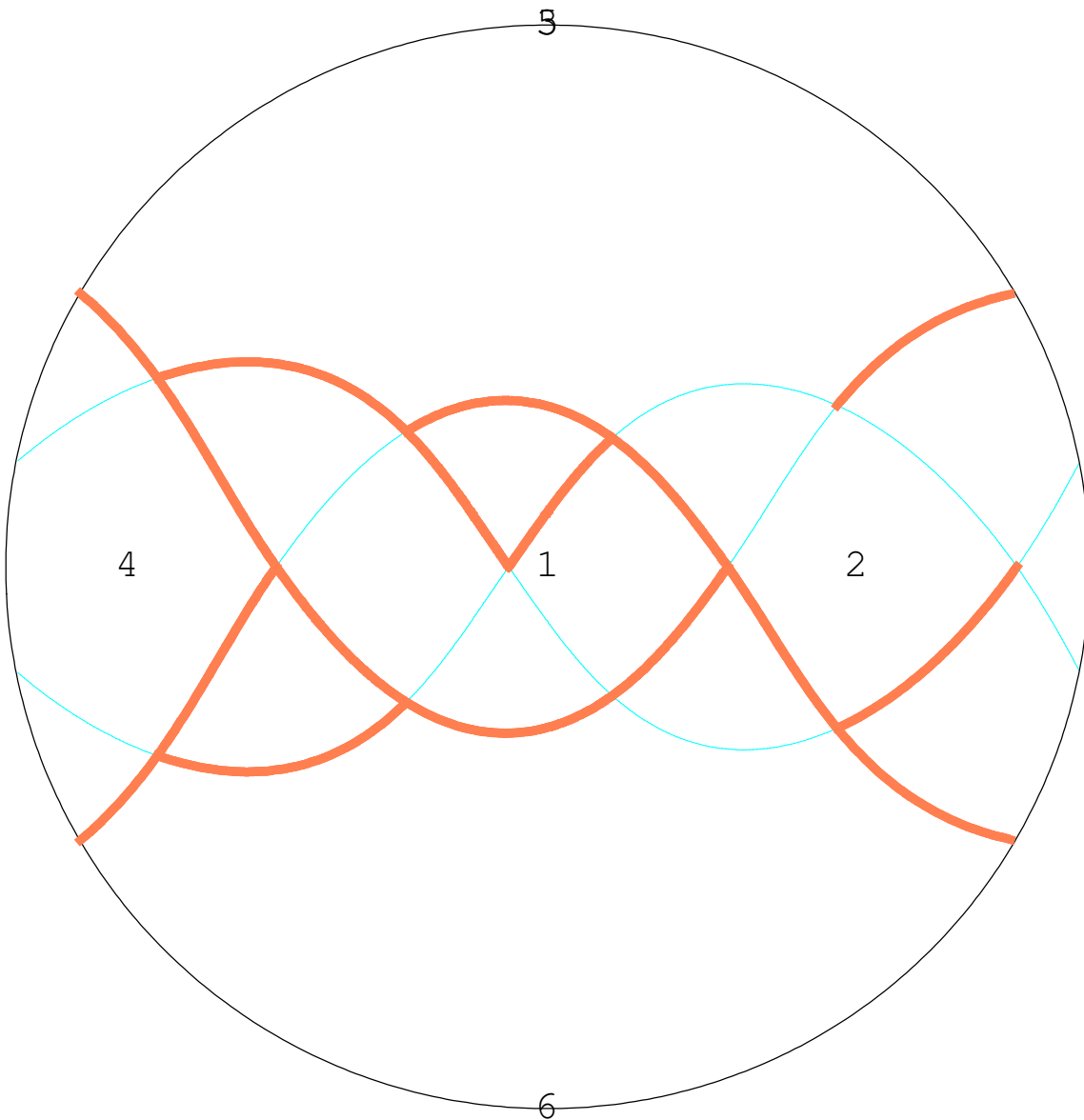
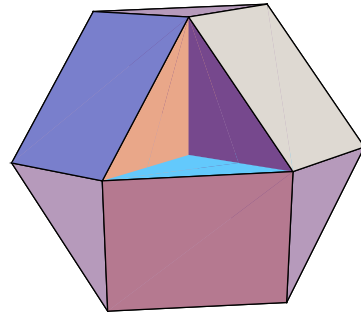
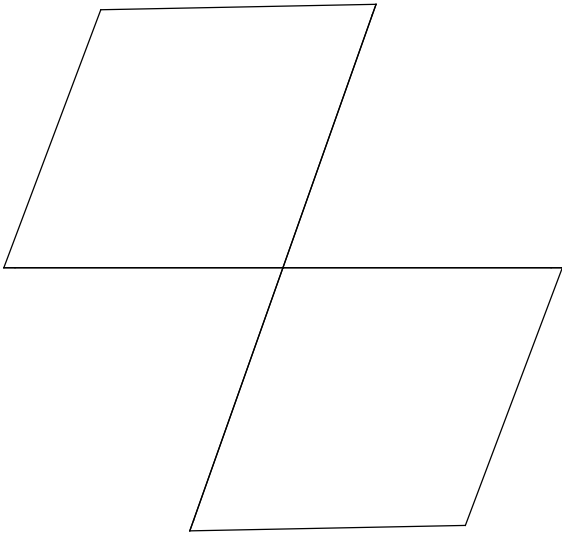
$$\left\{ \frac{8}{3}, 3, \frac{8}{3}, 4 \right\}$$



15.

cubohemioctahedron

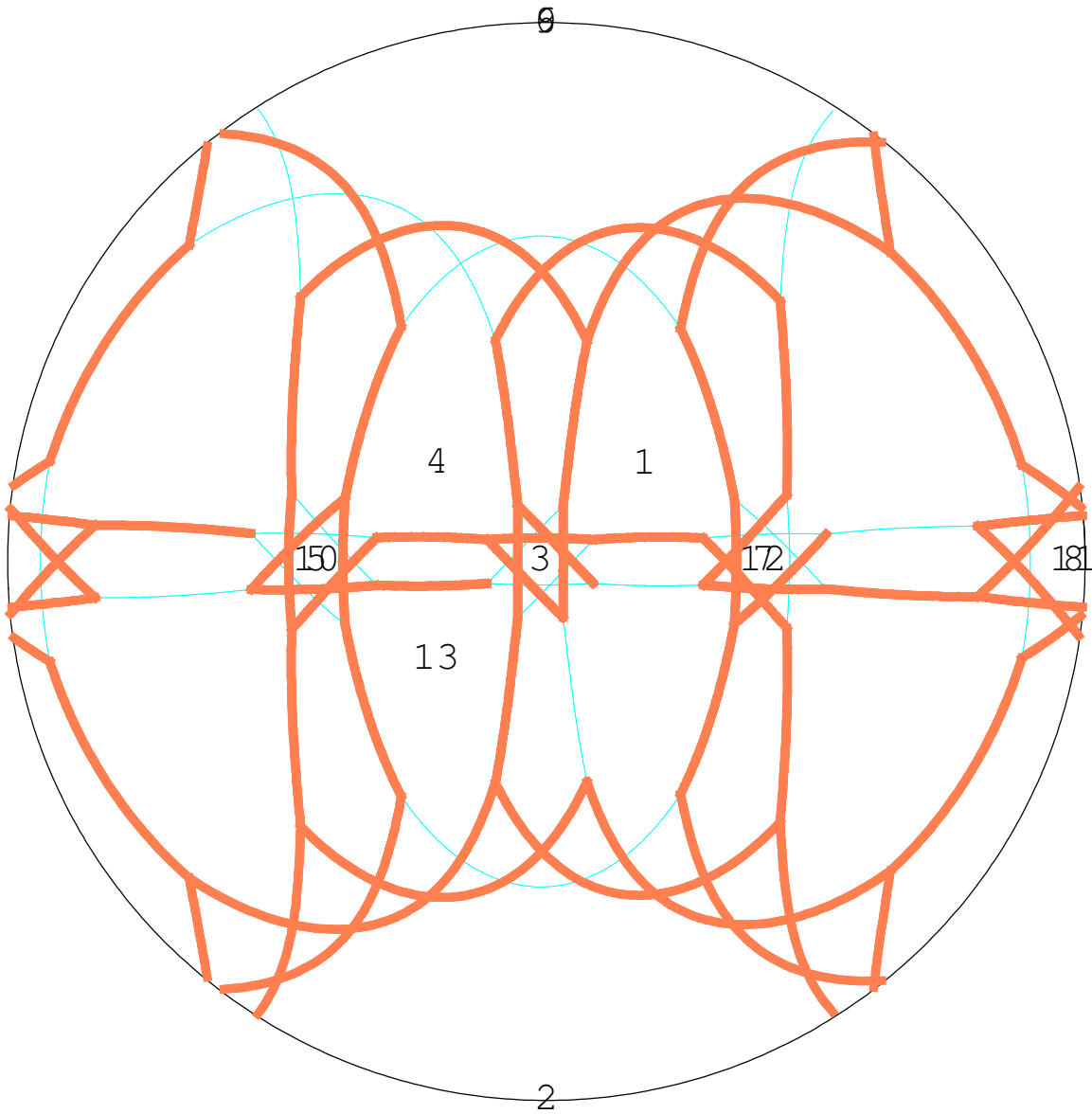
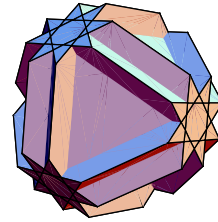
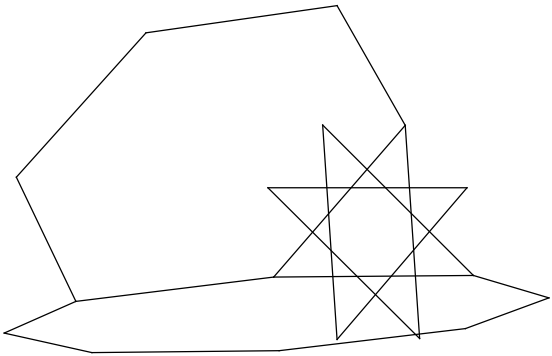
$$\left\{6, \frac{4}{3}, 6, 4\right\}$$



16.

cubitruncated cuboctahedron

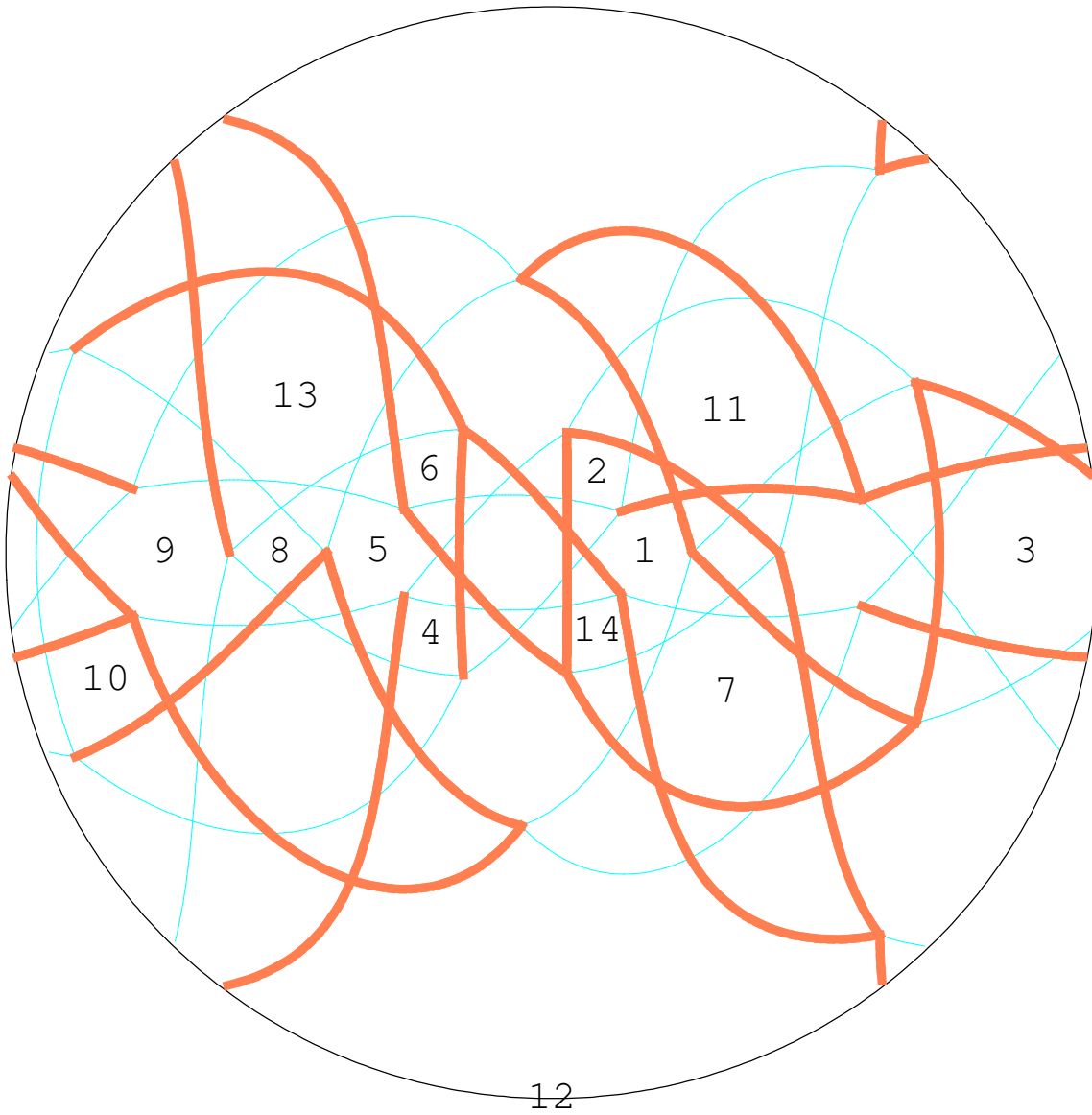
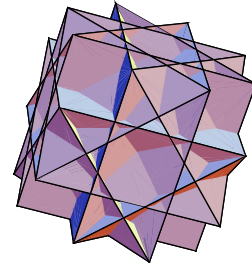
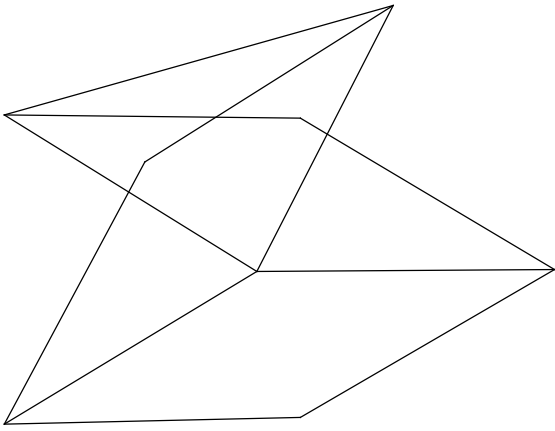
$$\left\{ \frac{8}{3}, 6, 8 \right\}$$



17.

great rhombicuboctahedron

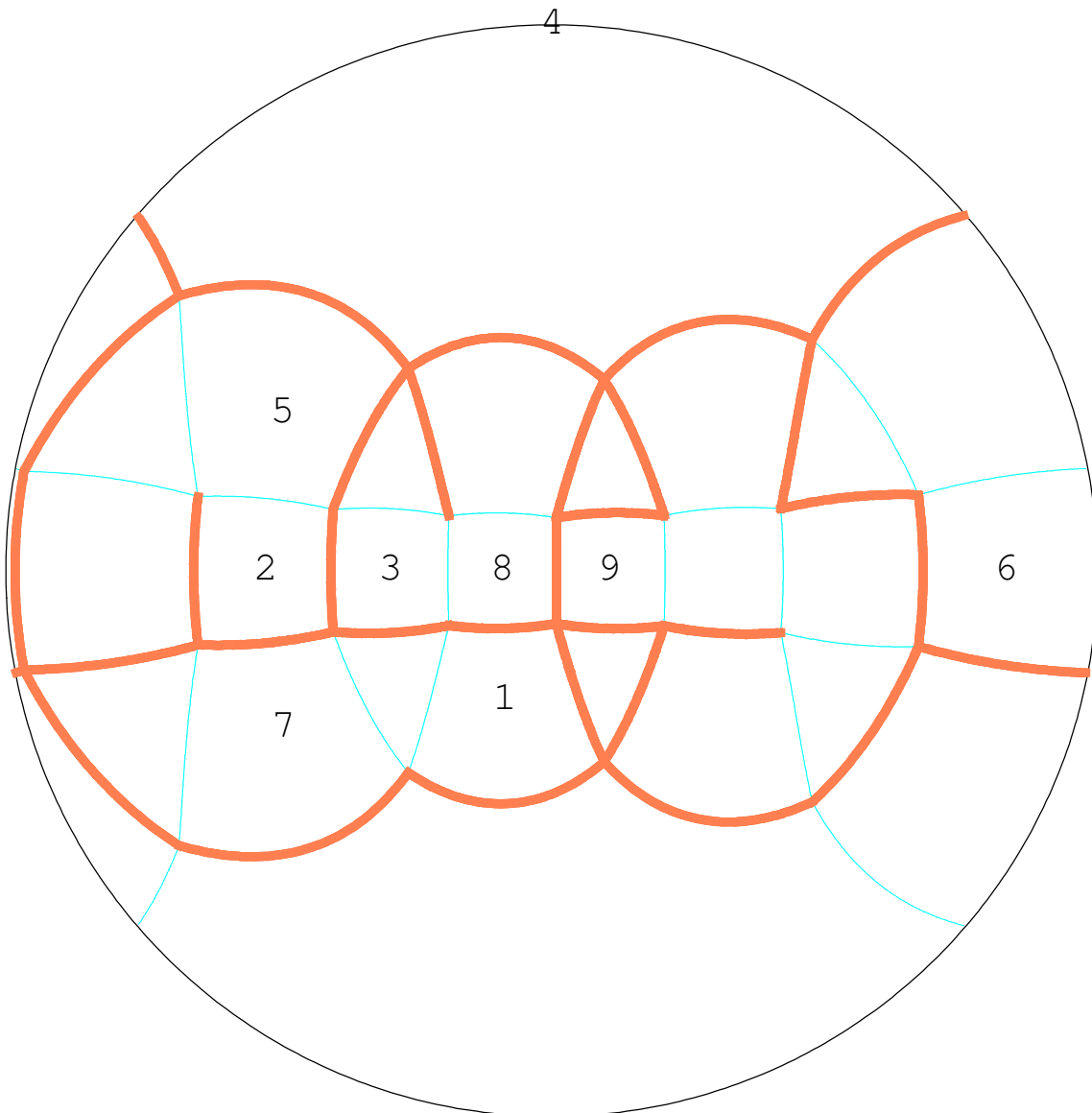
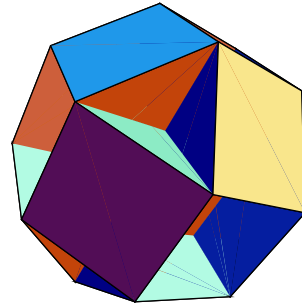
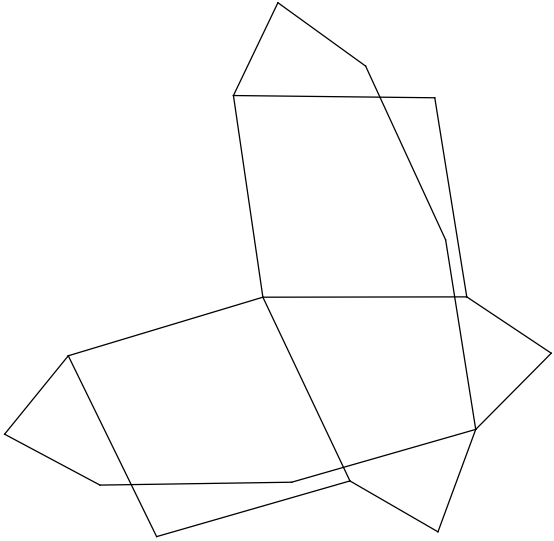
$$\{4, \frac{3}{2}, 4, 4\}$$



18.

small rhombihexahedron

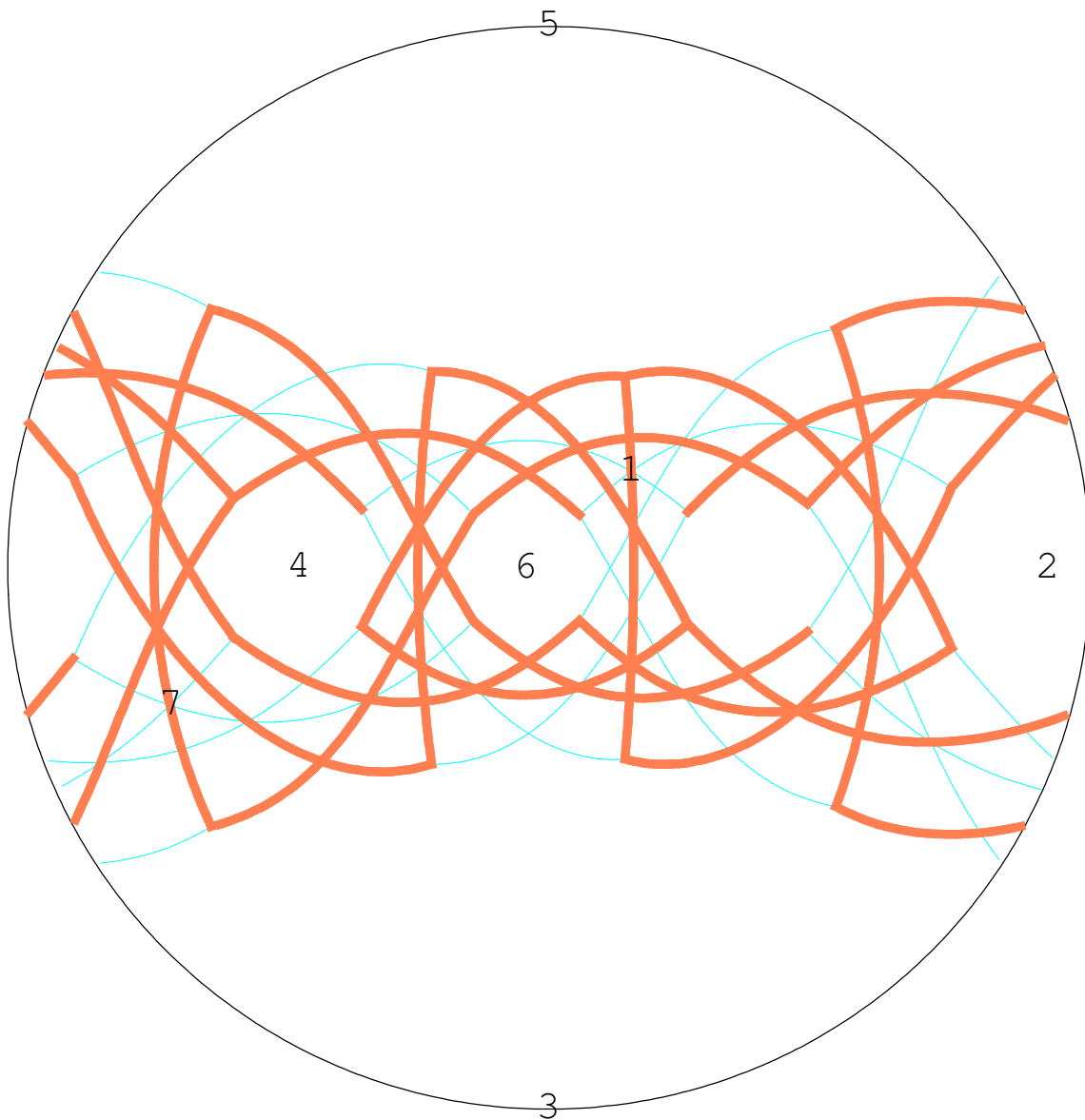
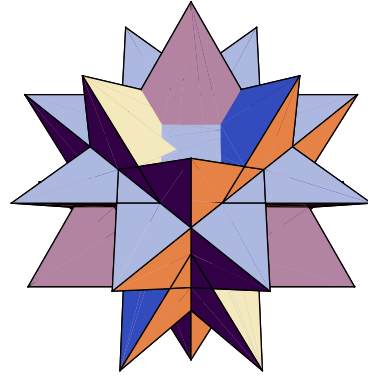
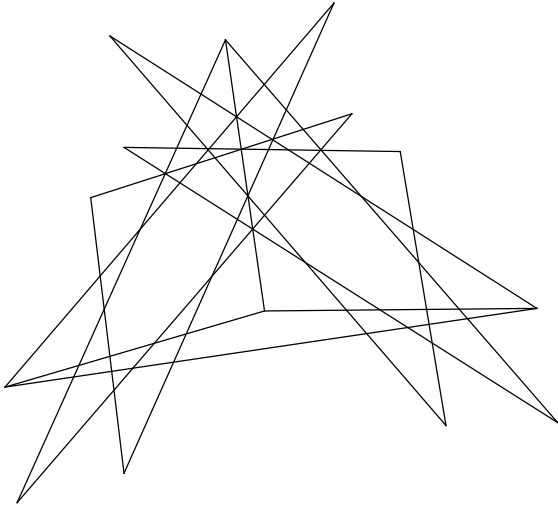
$$\{8, 4, \frac{8}{7}, \frac{4}{3}\}$$



19.

stellated truncated hexahedron

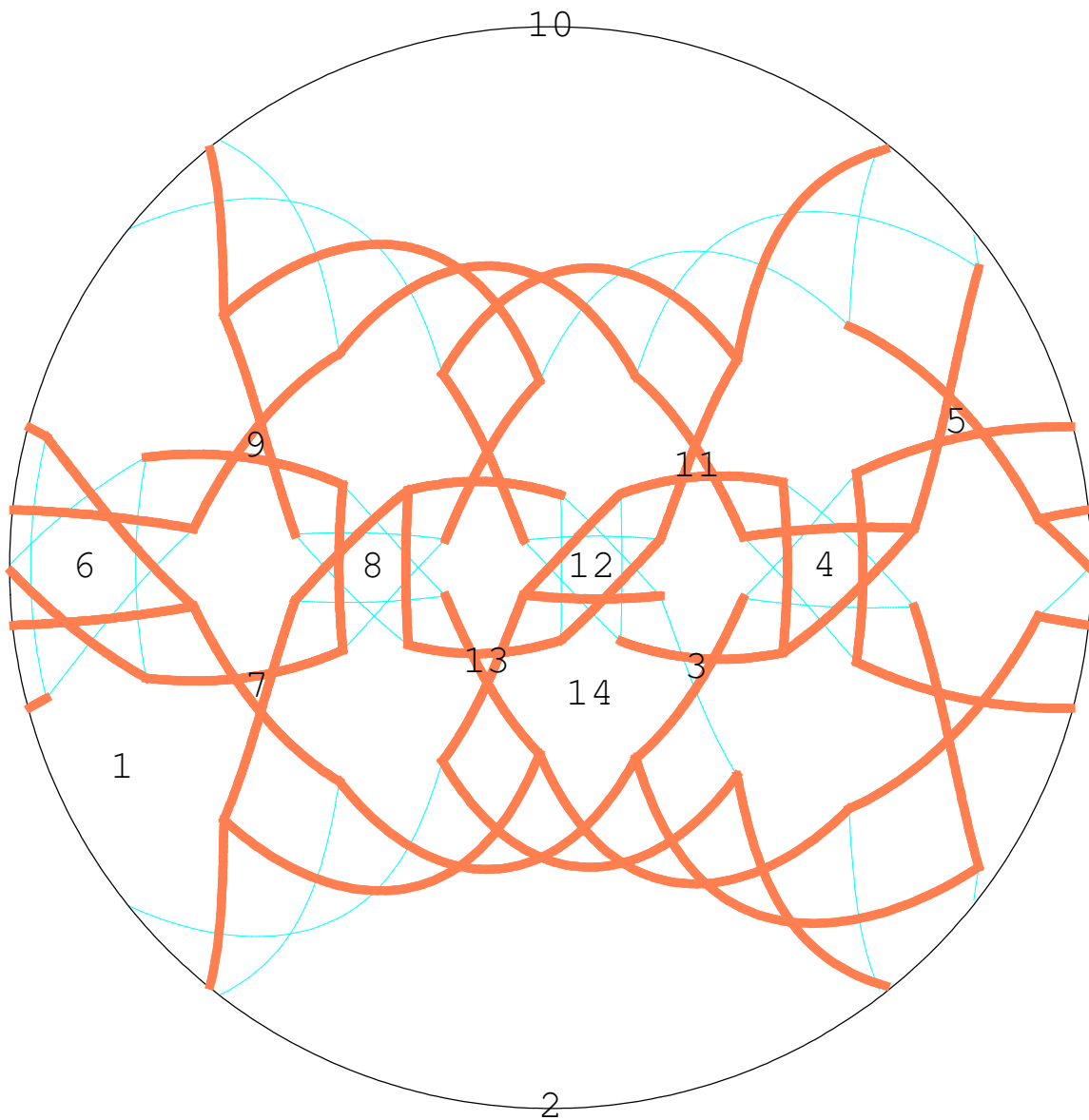
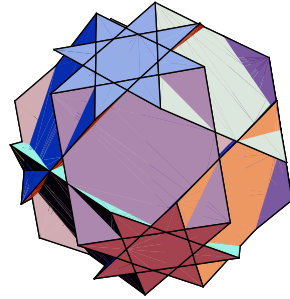
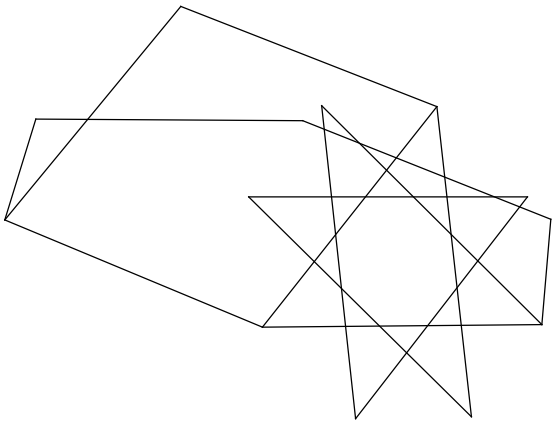
$$\left\{ \frac{8}{3}, \frac{8}{3}, 3 \right\}$$



20.

great truncated cuboctahedron

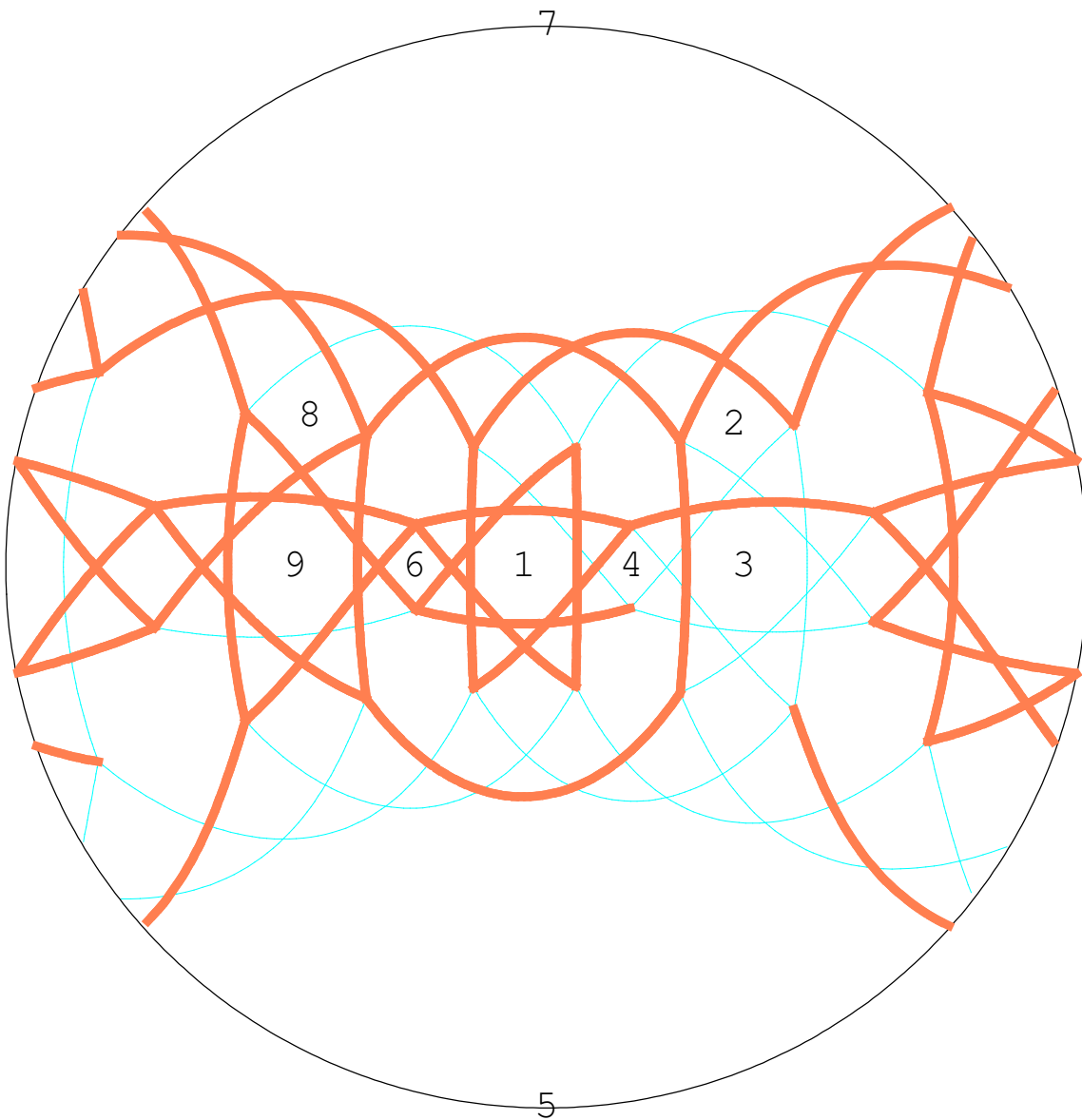
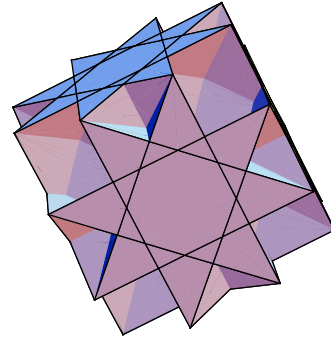
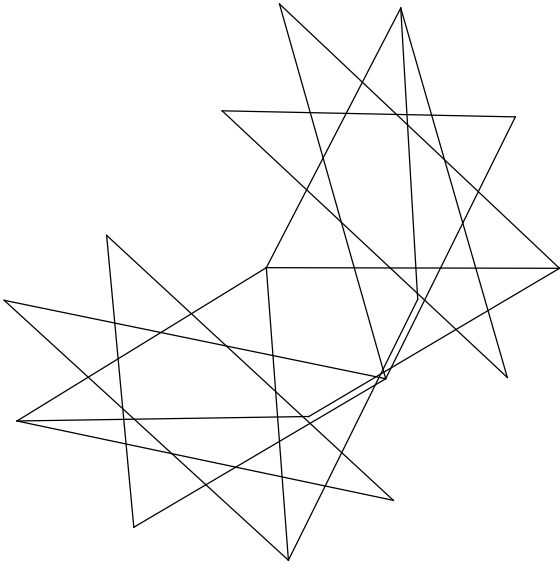
$$\left\{ \frac{8}{3}, 4, 6 \right\}$$



21.

great rhombihexahedron

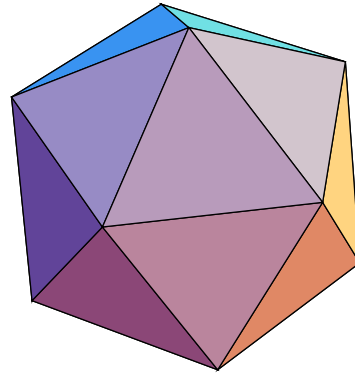
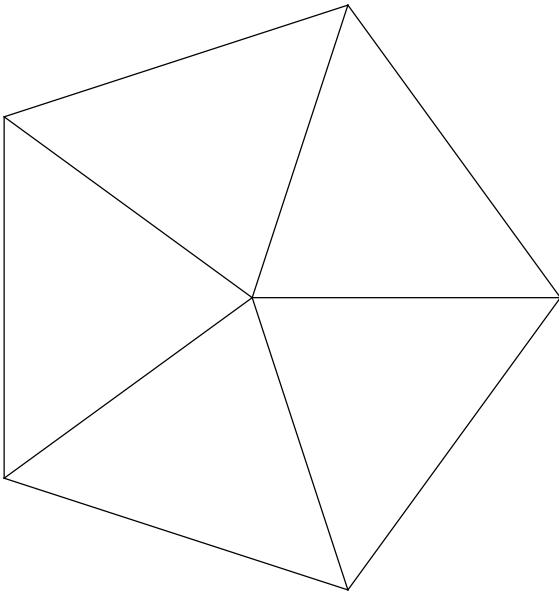
$$\left\{4, \frac{8}{3}, \frac{4}{3}, \frac{8}{5}\right\}$$

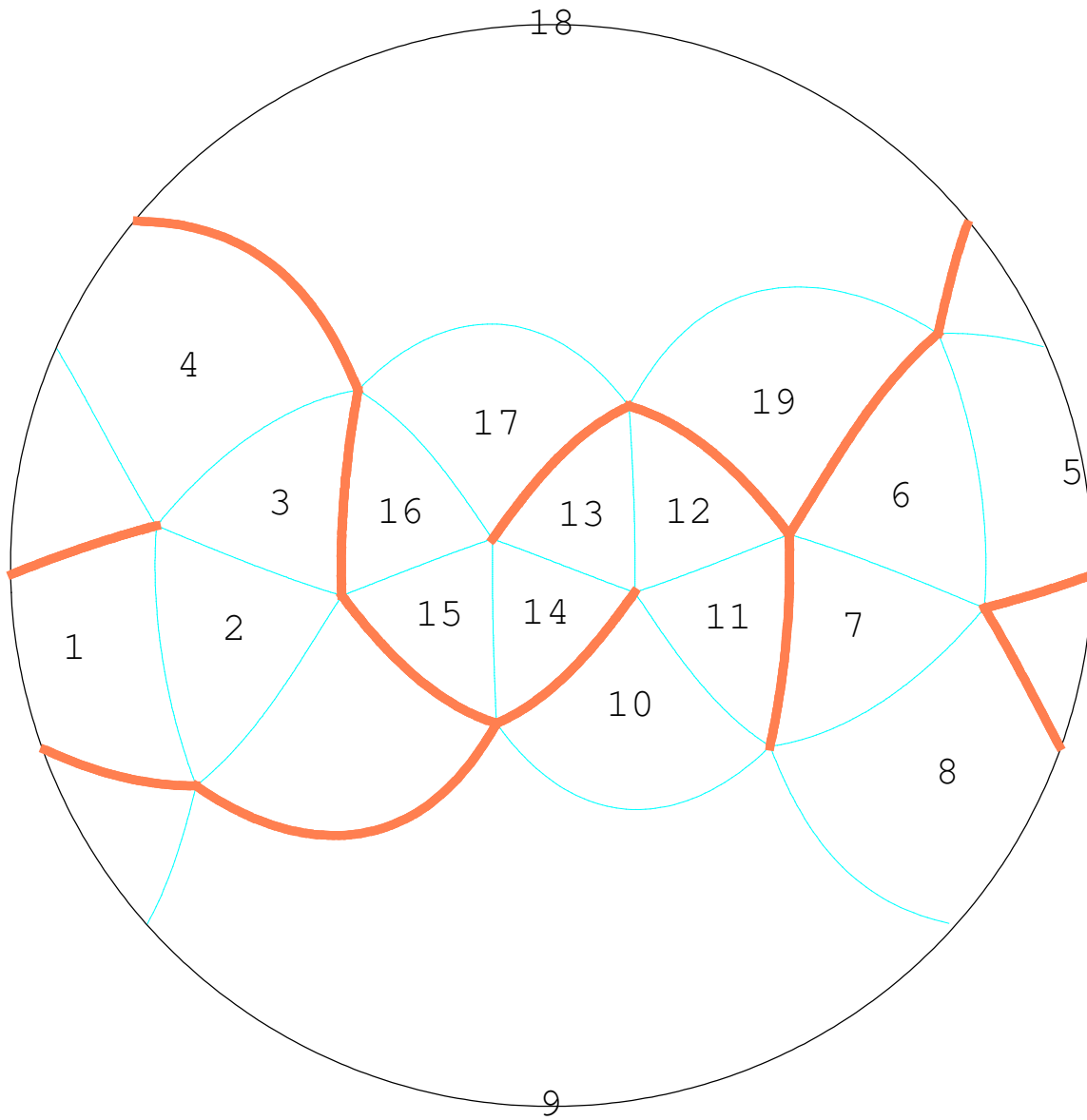


22.

icosahedron

{3, 3, 3, 3}

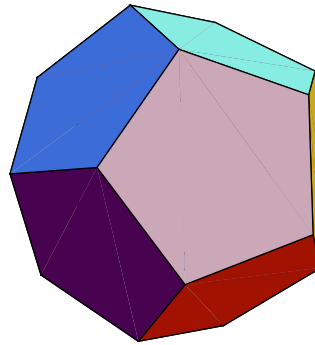
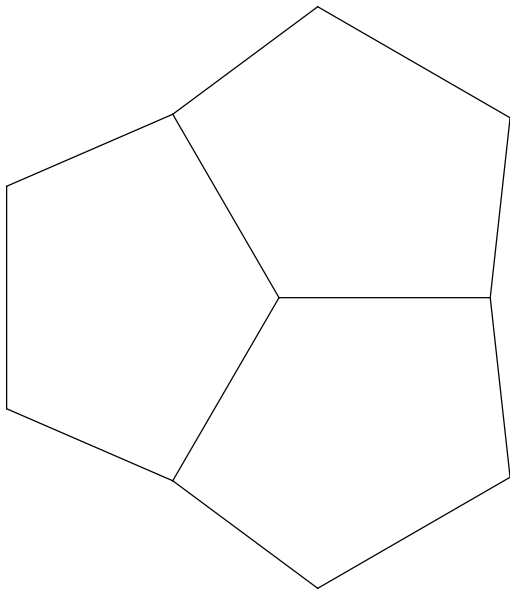


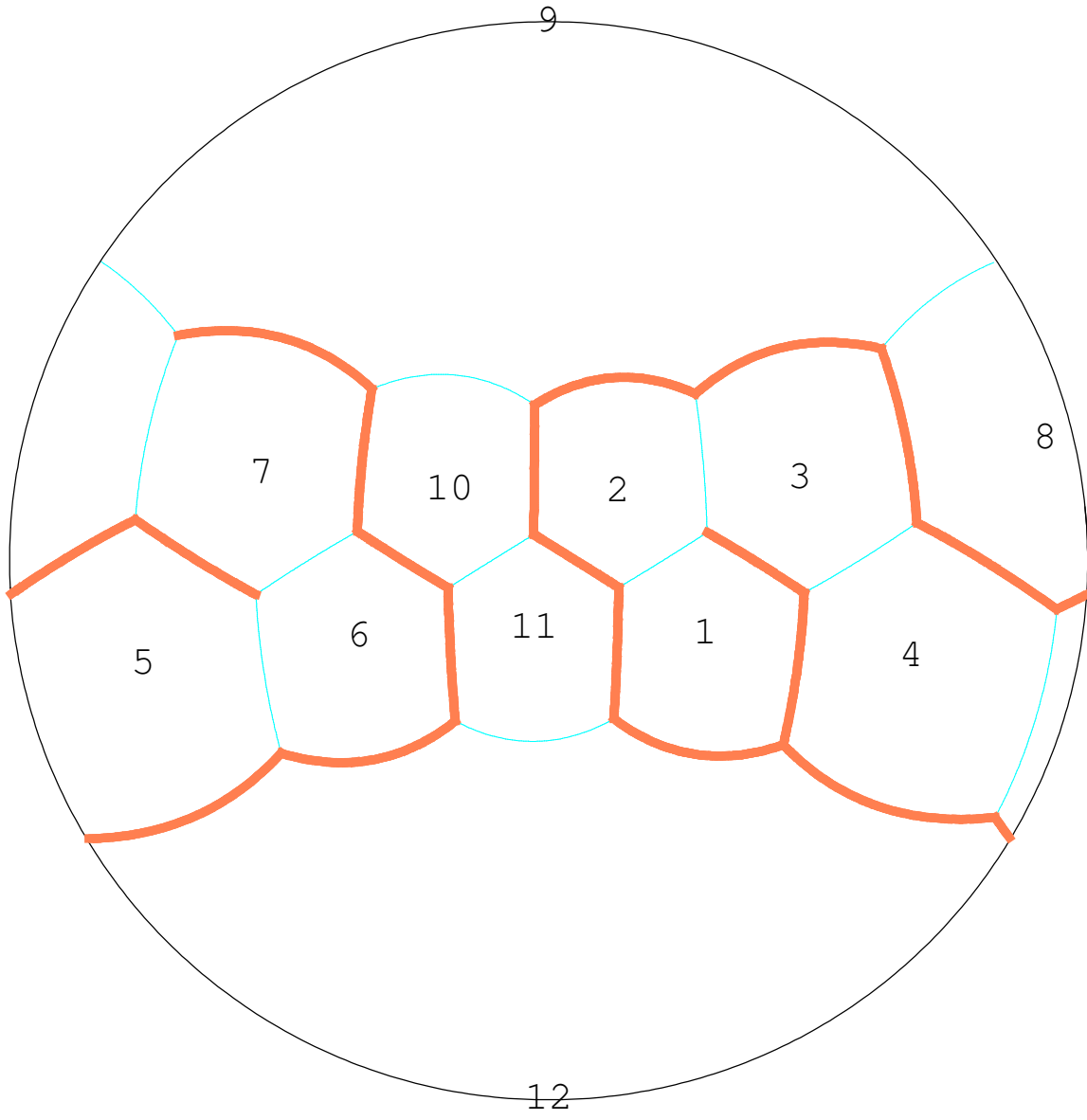


23.

dodecahedron

{5, 5, 5}

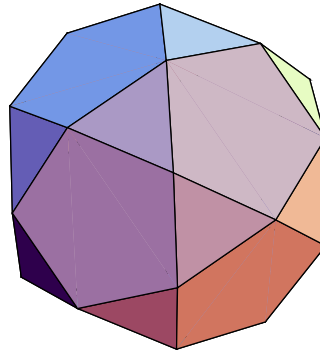
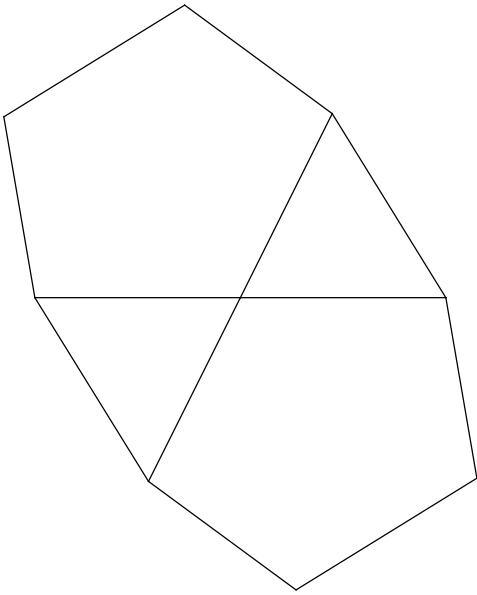


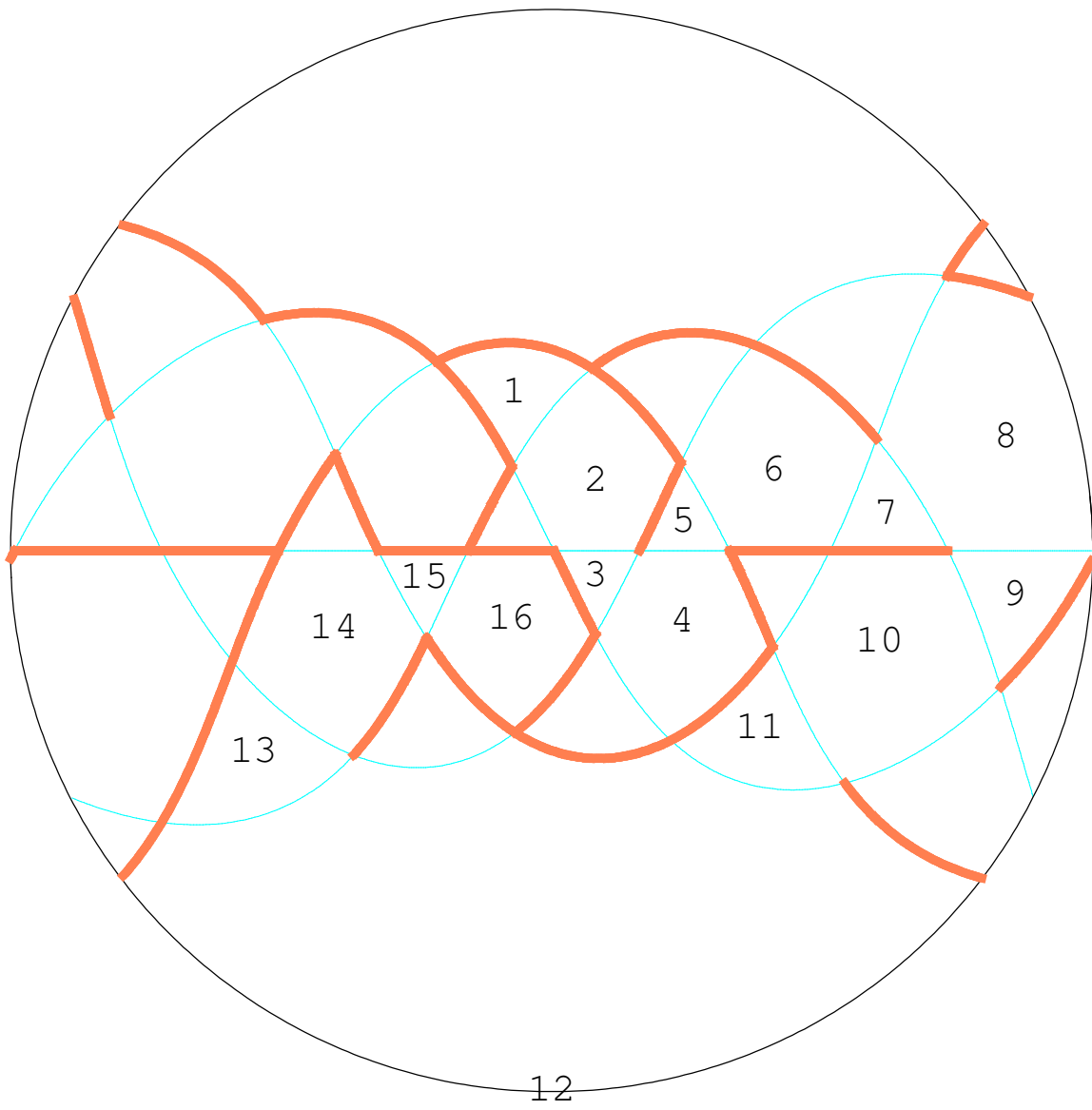


24.

icosidodecahedron

{3, 5, 3, 5}

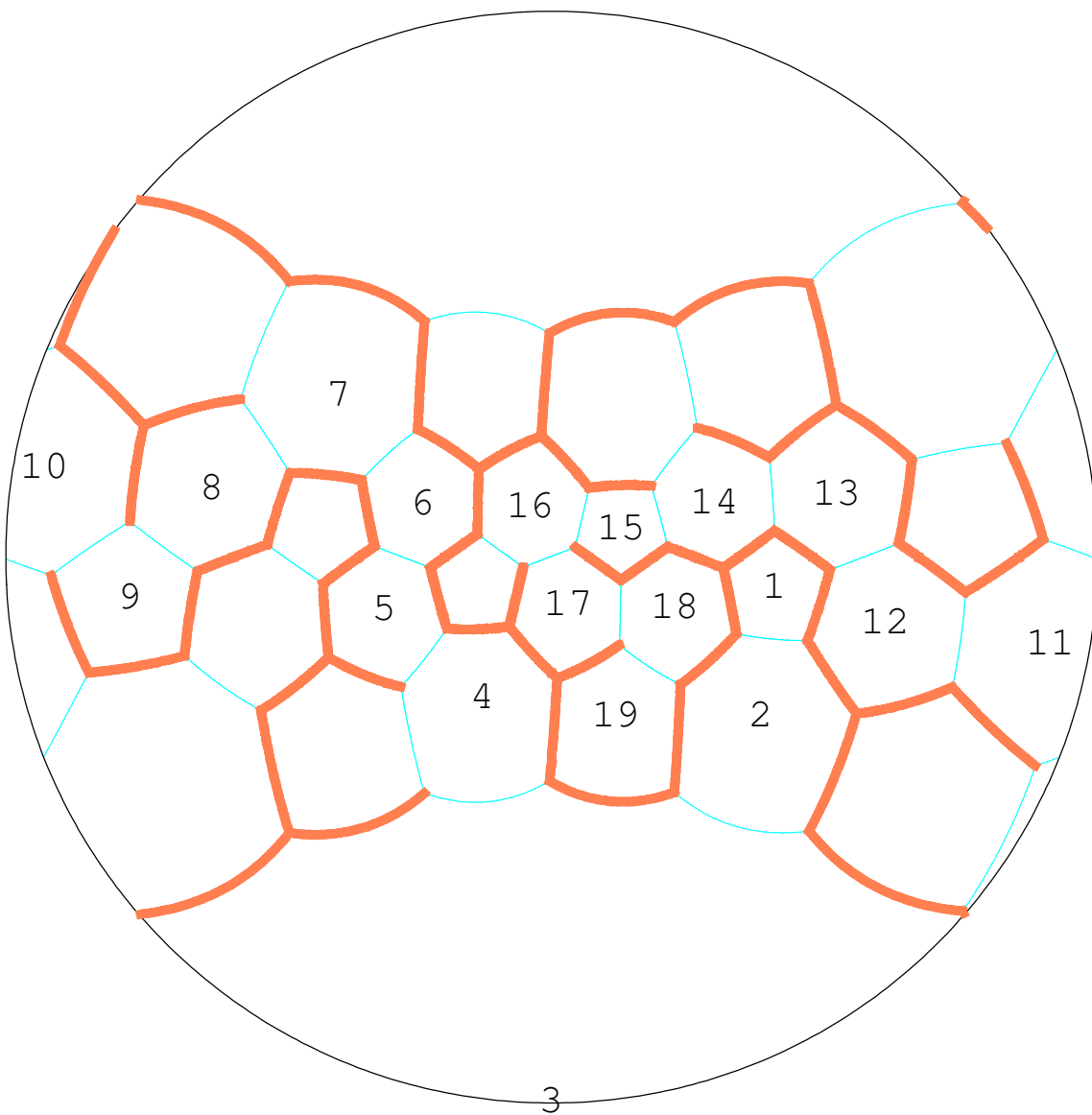
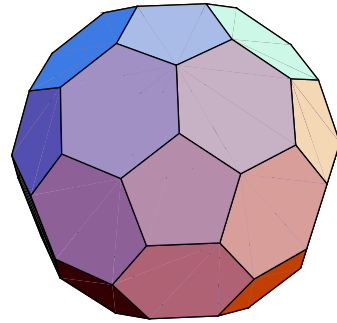
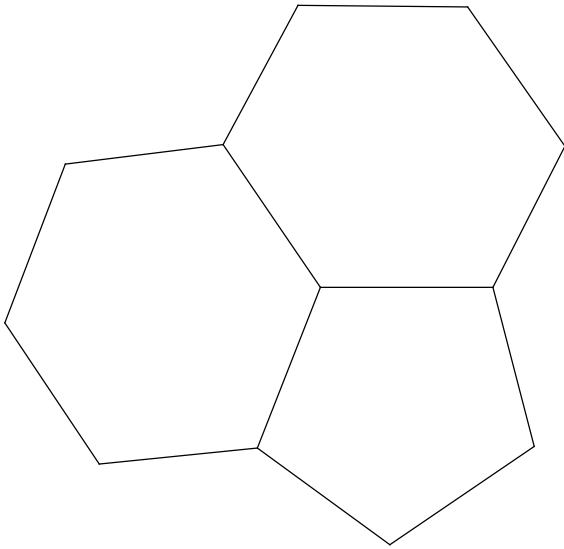




25.

truncated icosahedron

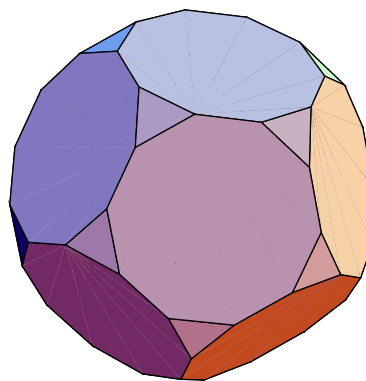
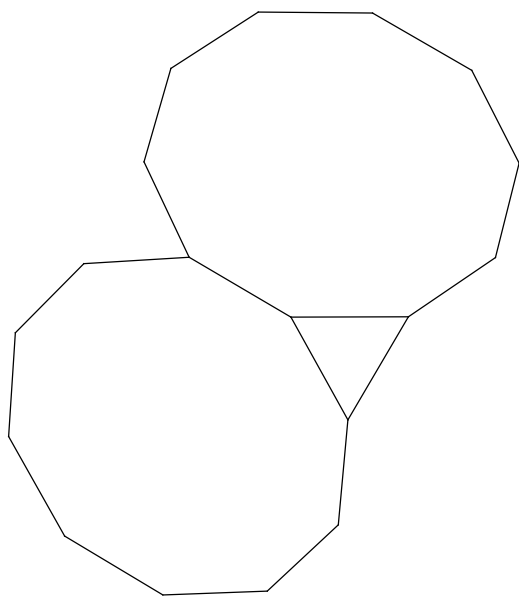
{6, 6, 5}

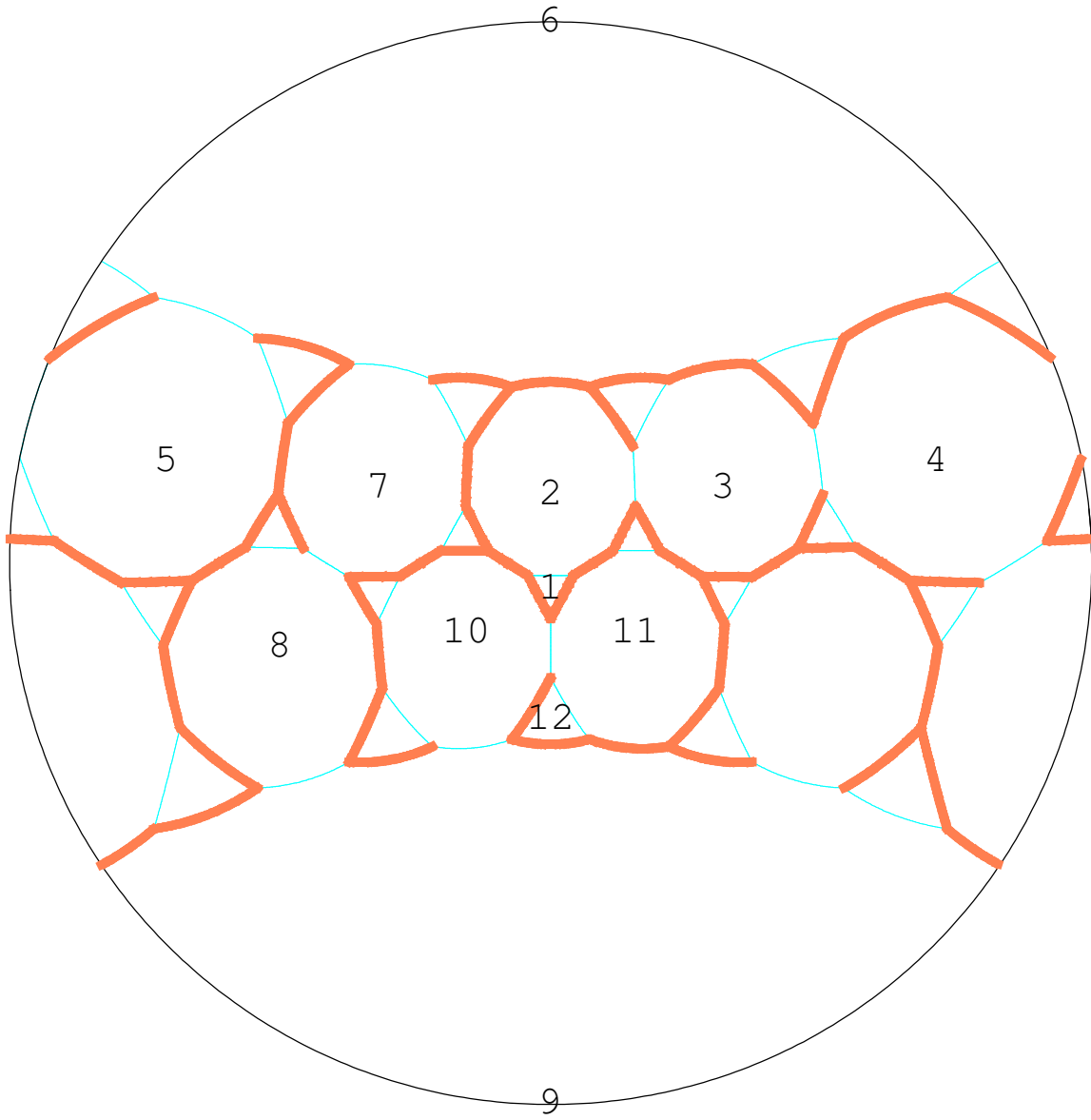


26.

truncated dodecahedron

{10, 10, 3}

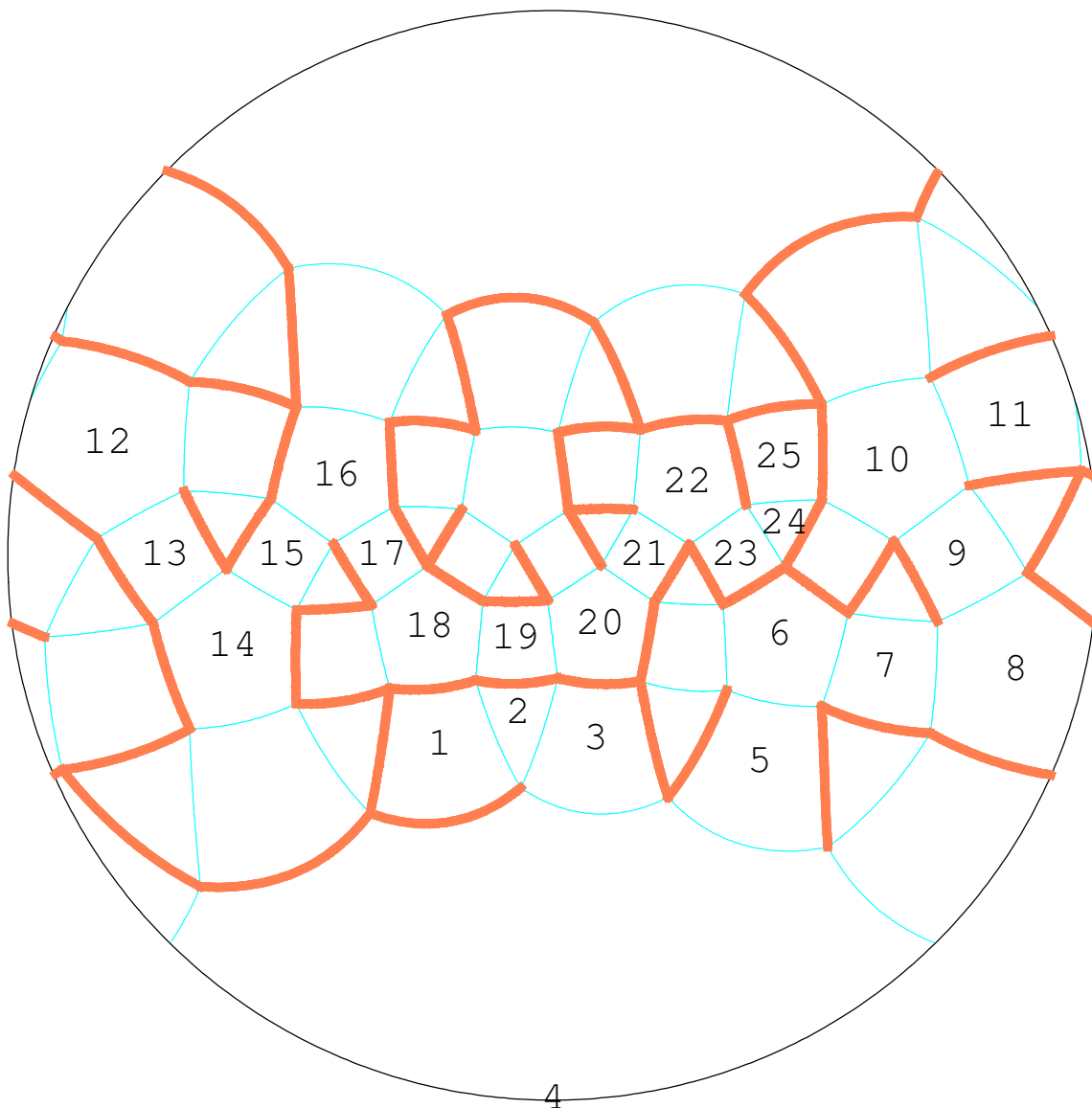
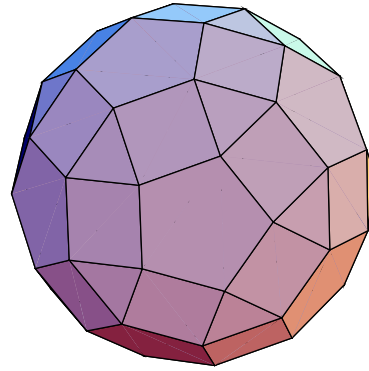
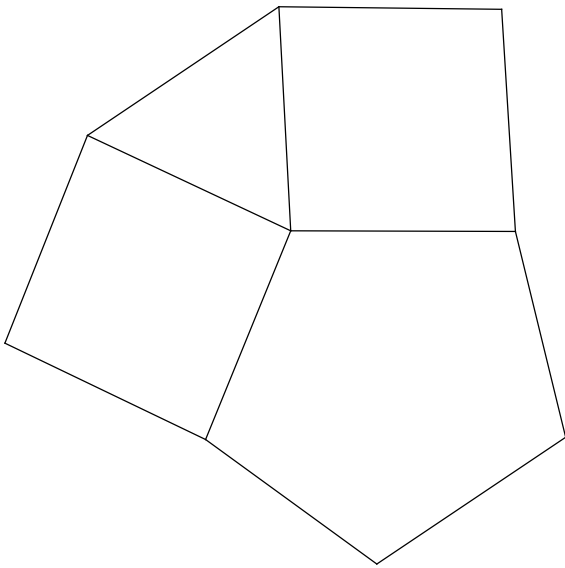




27.

rhombicosidodecahedron

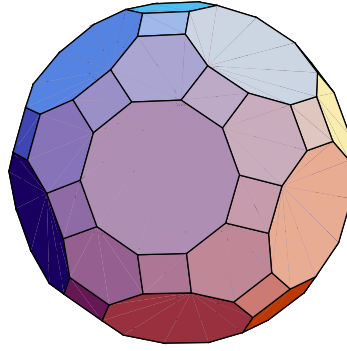
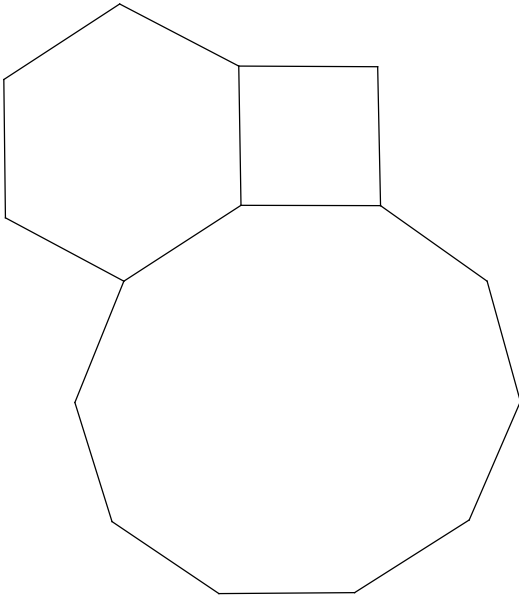
{4, 3, 4, 5}

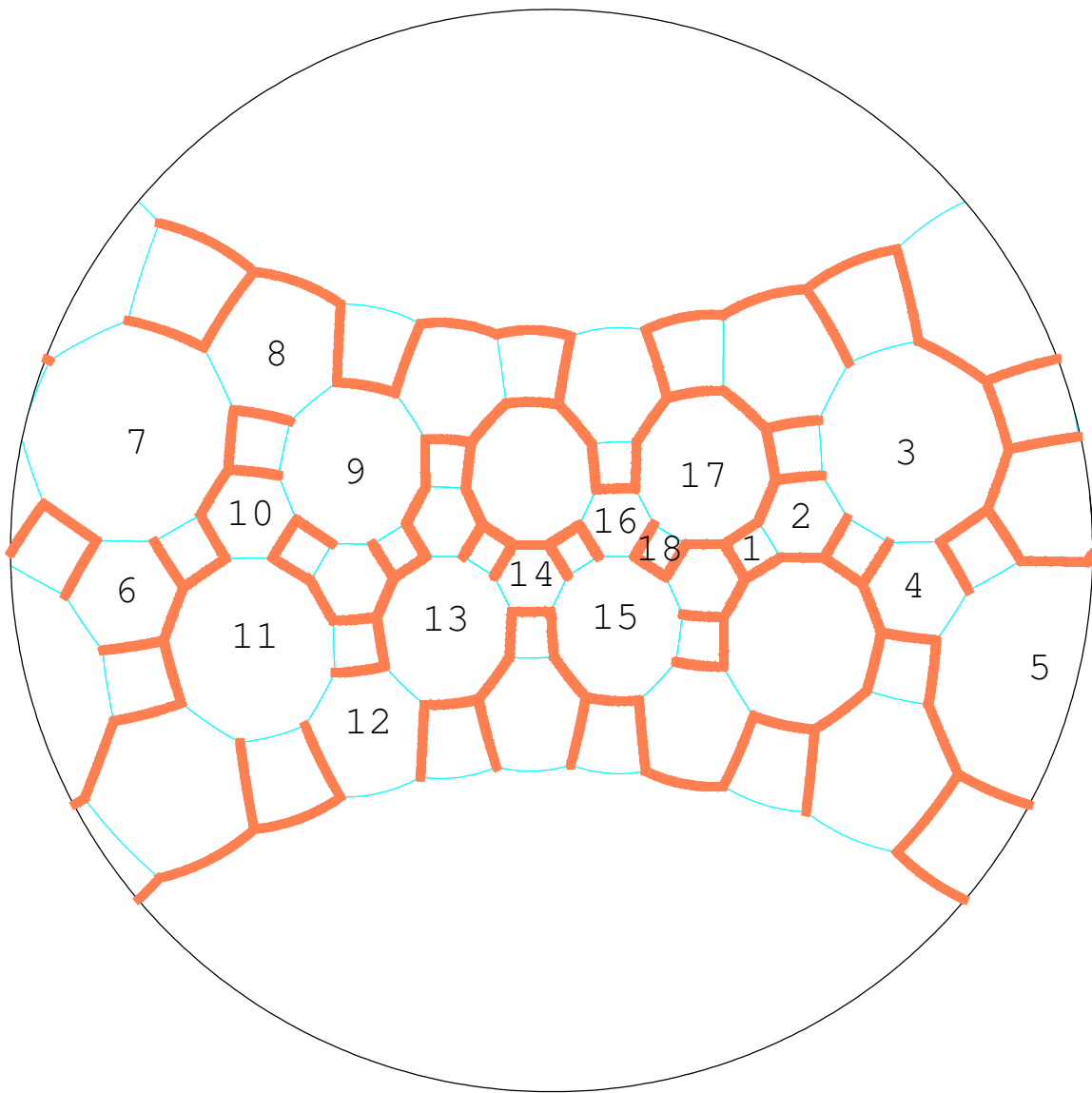


28.

truncated icosidodecahedron

{4, 6, 10}

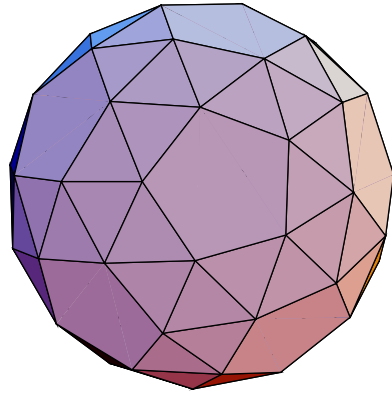
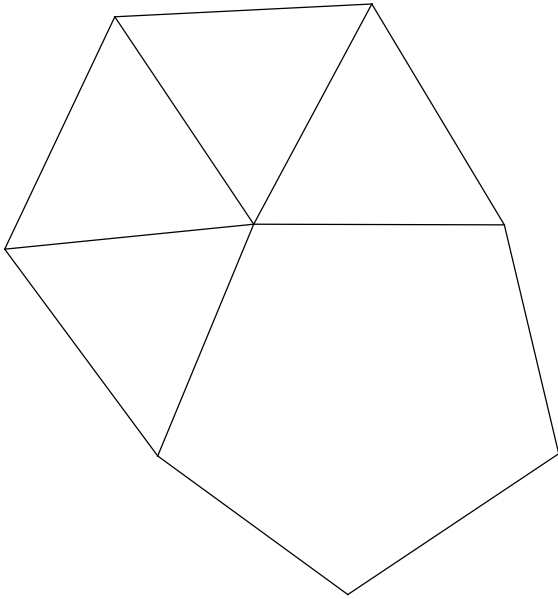


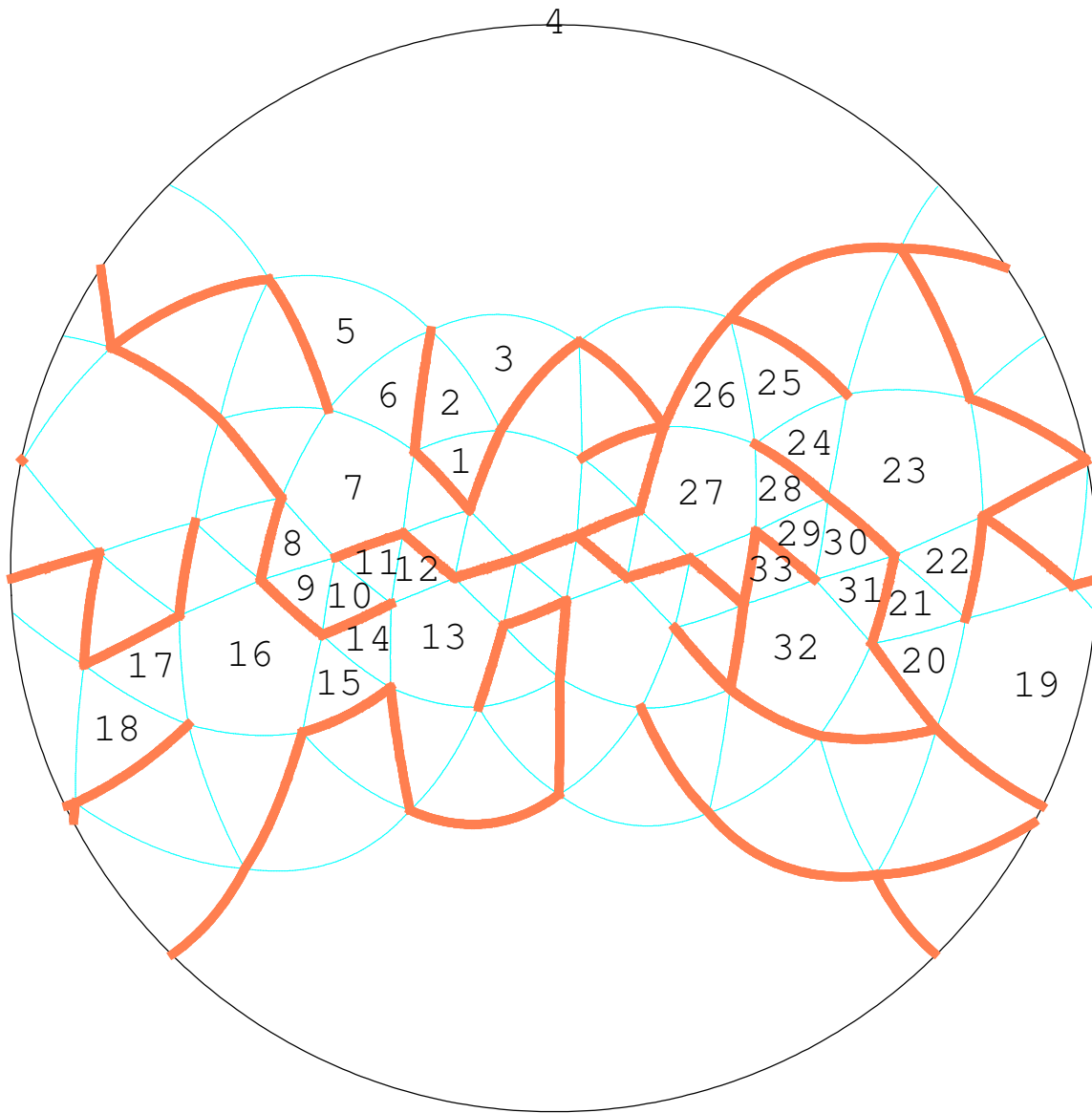


29.

snub dodecahedron

$\{3, 3, 3, 3, 5\}$

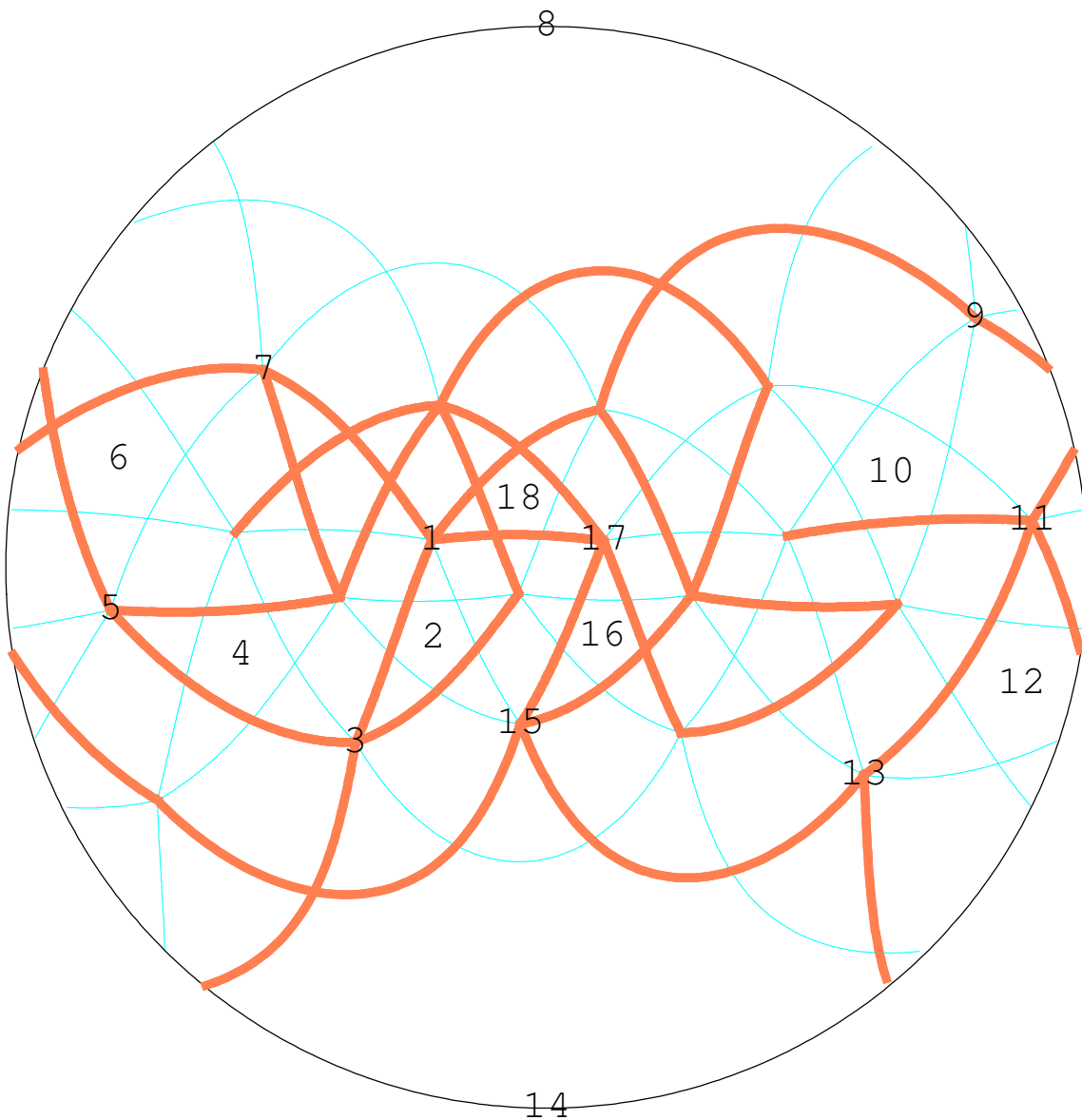
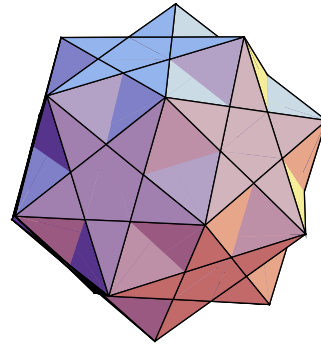
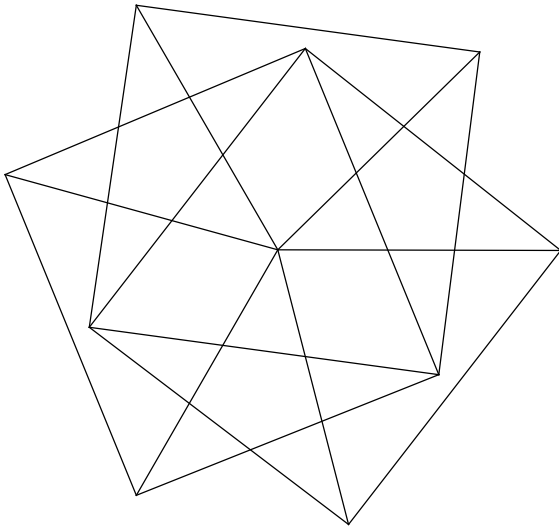




30.

small ditrigonal icosidodecahedron

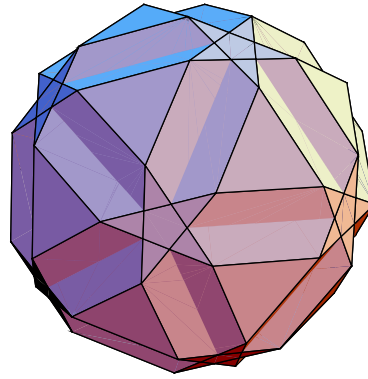
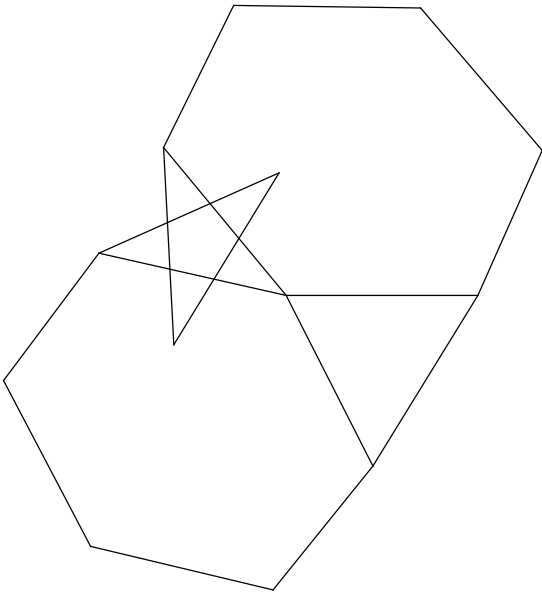
$$\left\{ \frac{5}{2}, 3, \frac{5}{2}, 3, \frac{5}{2}, 3 \right\}$$

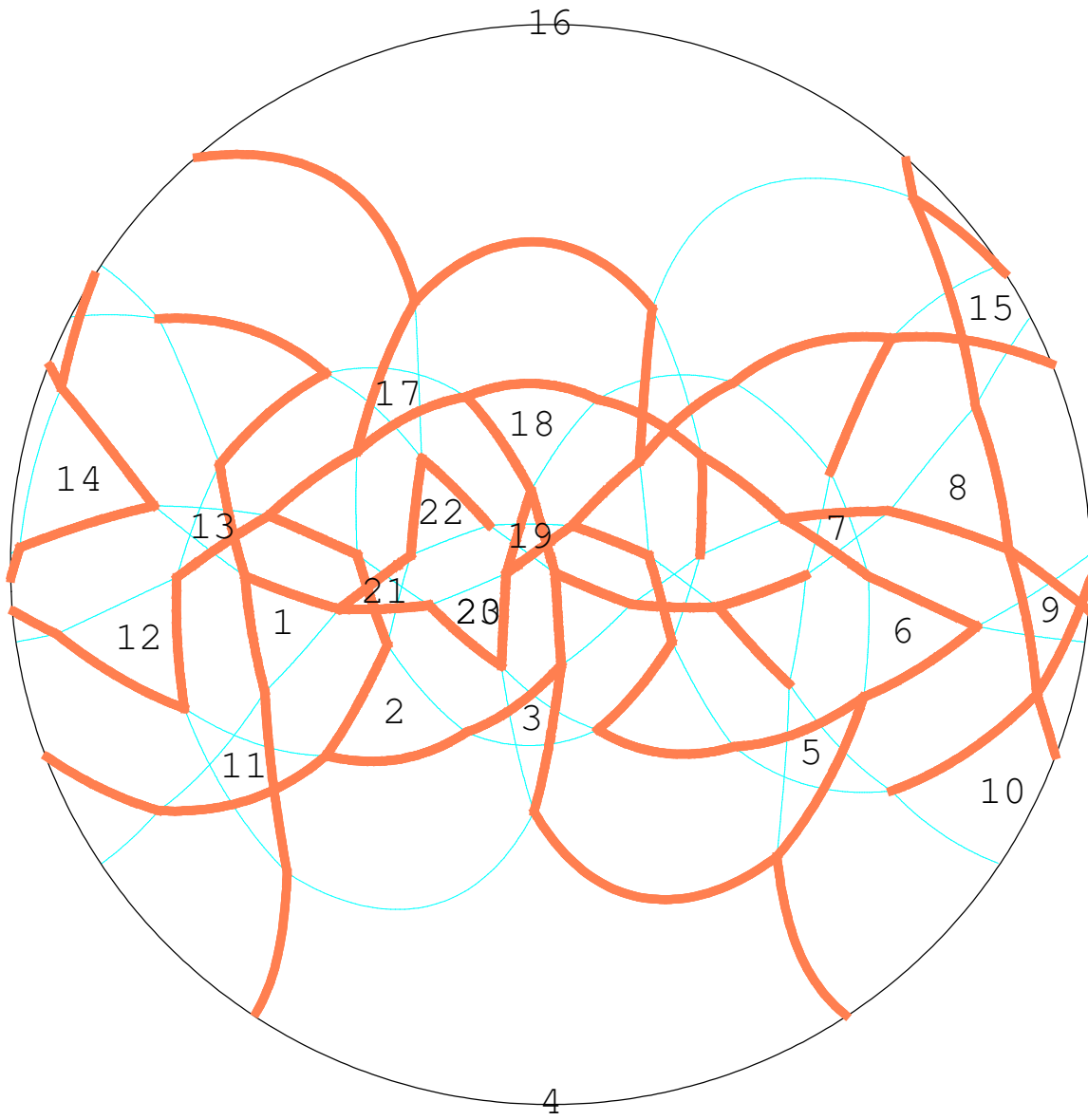


31.

small icosicosidodecahedron

$$\left\{6, \frac{5}{2}, 6, 3\right\}$$

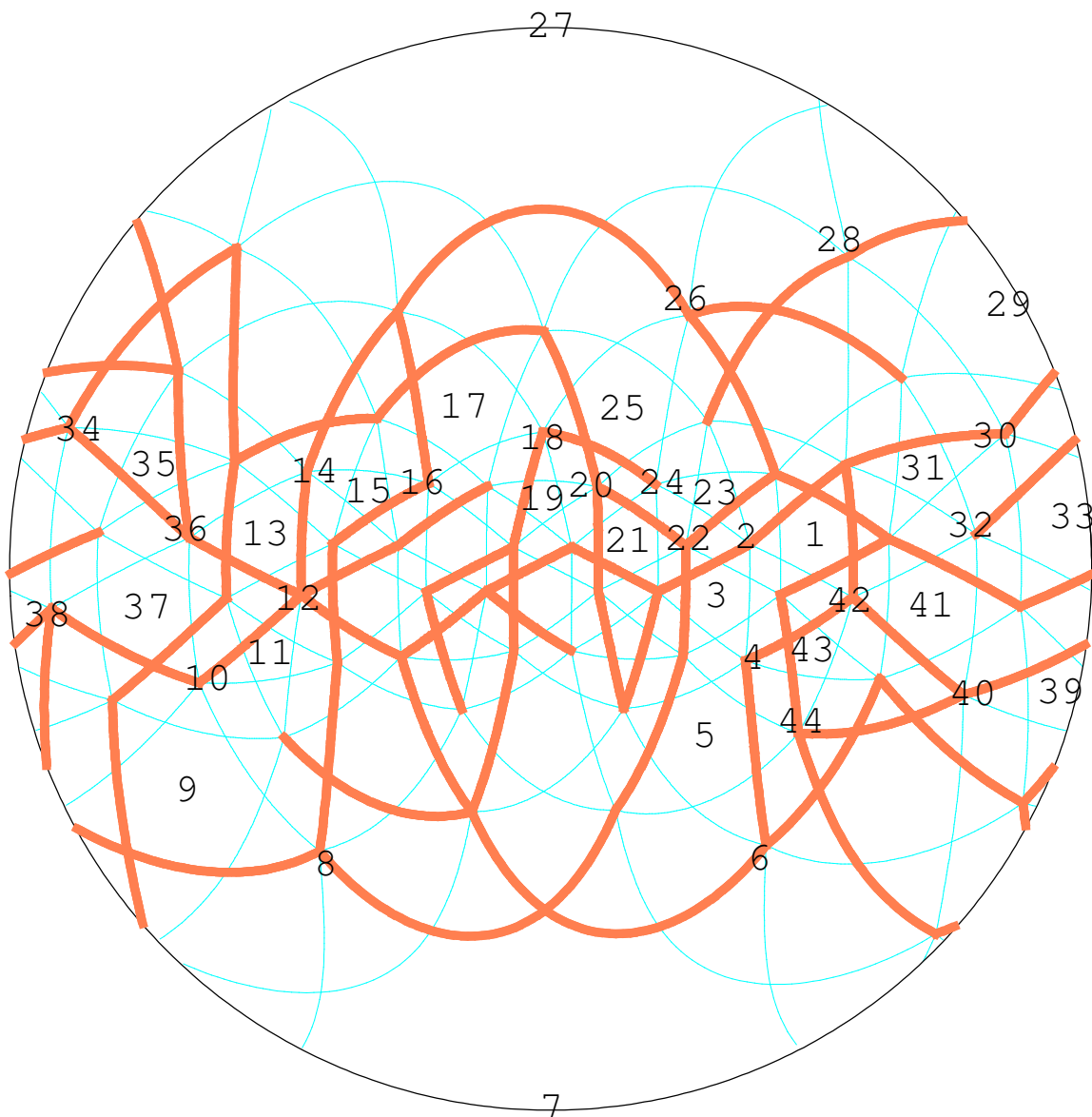
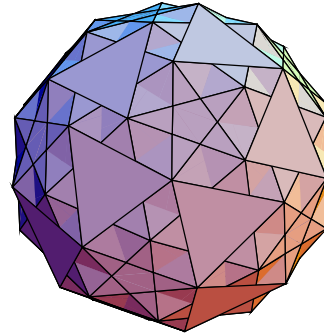
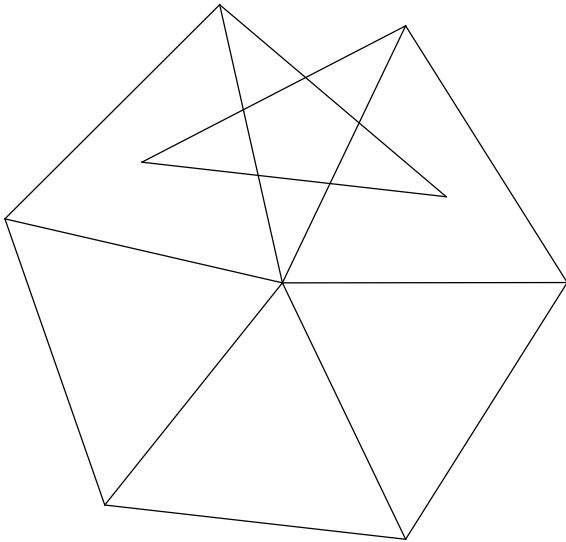




32.

small snub icosicosidodecahedron

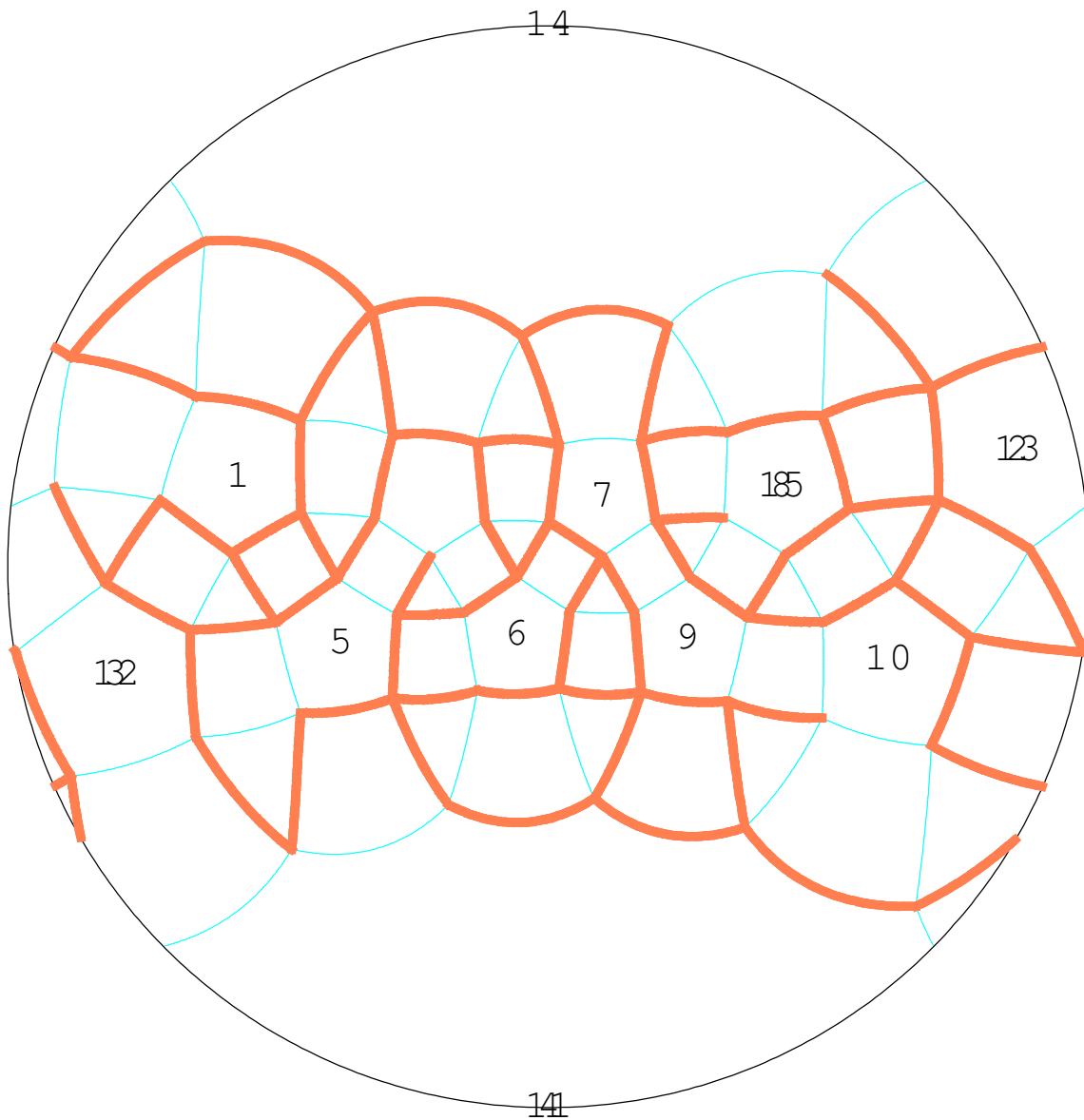
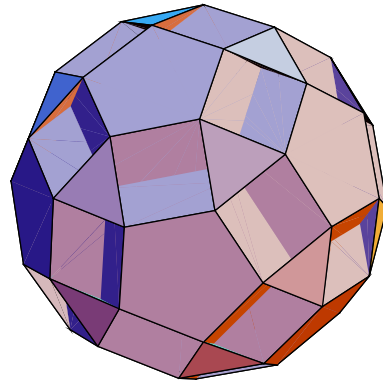
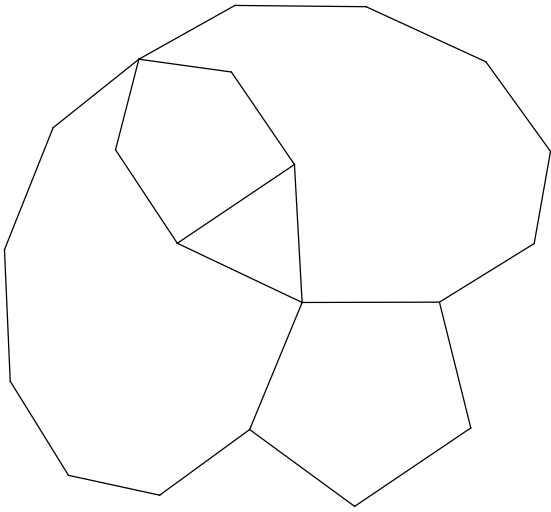
$$\left\{3, \frac{5}{2}, 3, 3, 3, 3\right\}$$



33.

small dodecicosidodecahedron

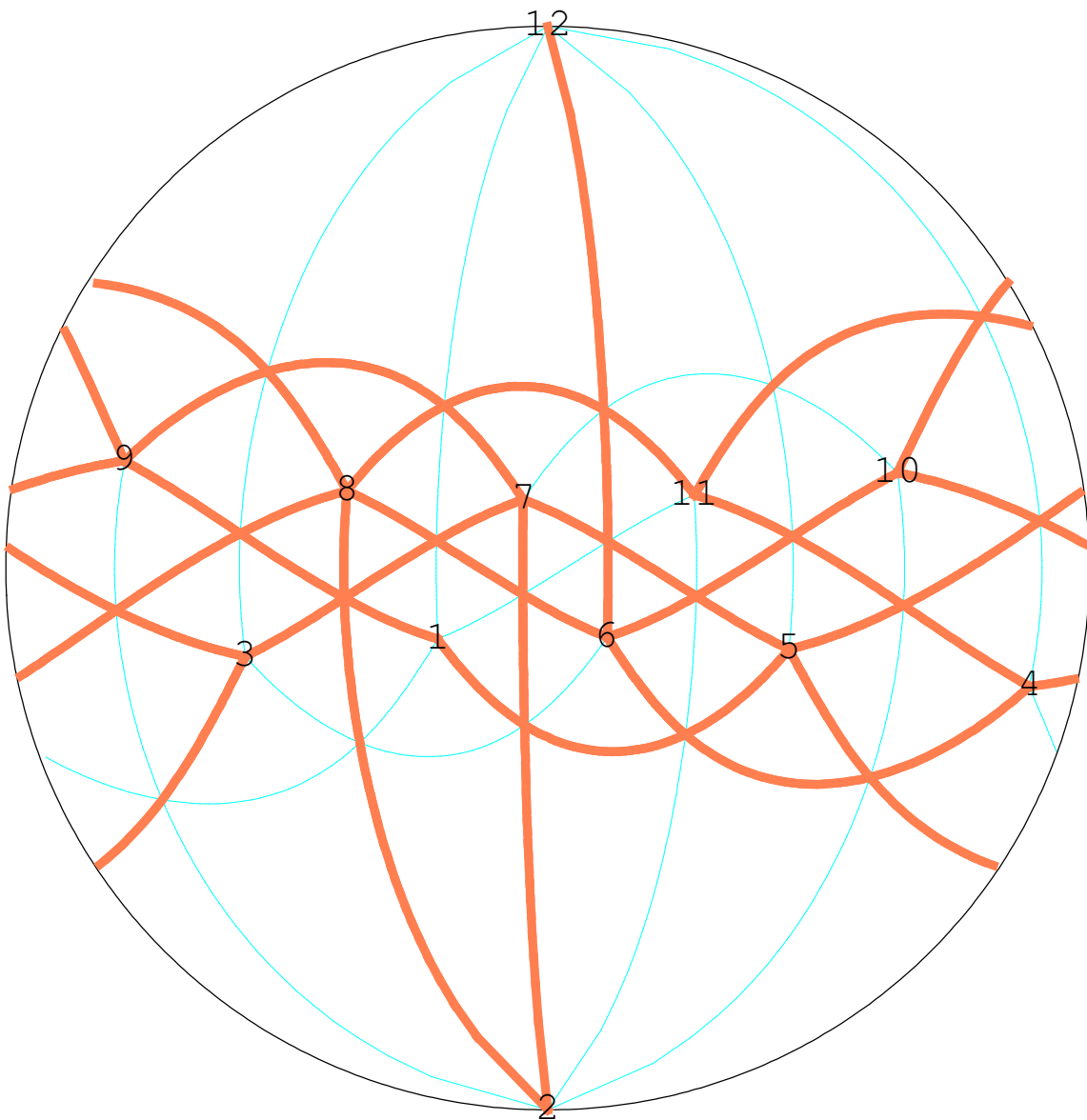
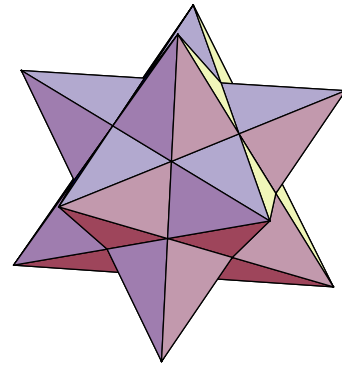
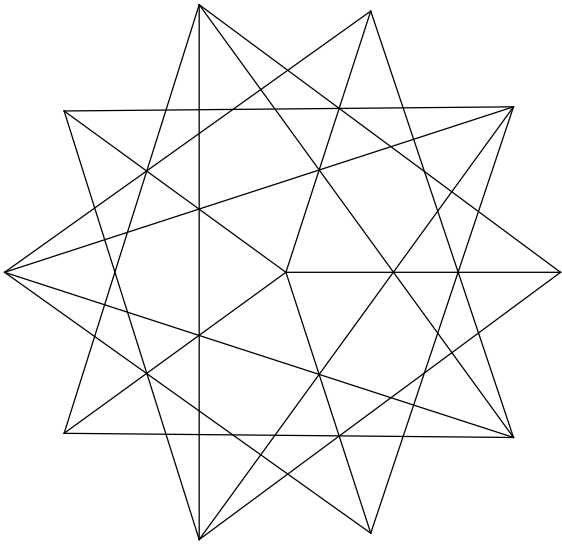
$$\{10, \frac{3}{2}, 10, 5\}$$



34.

small stellated dodecahedron

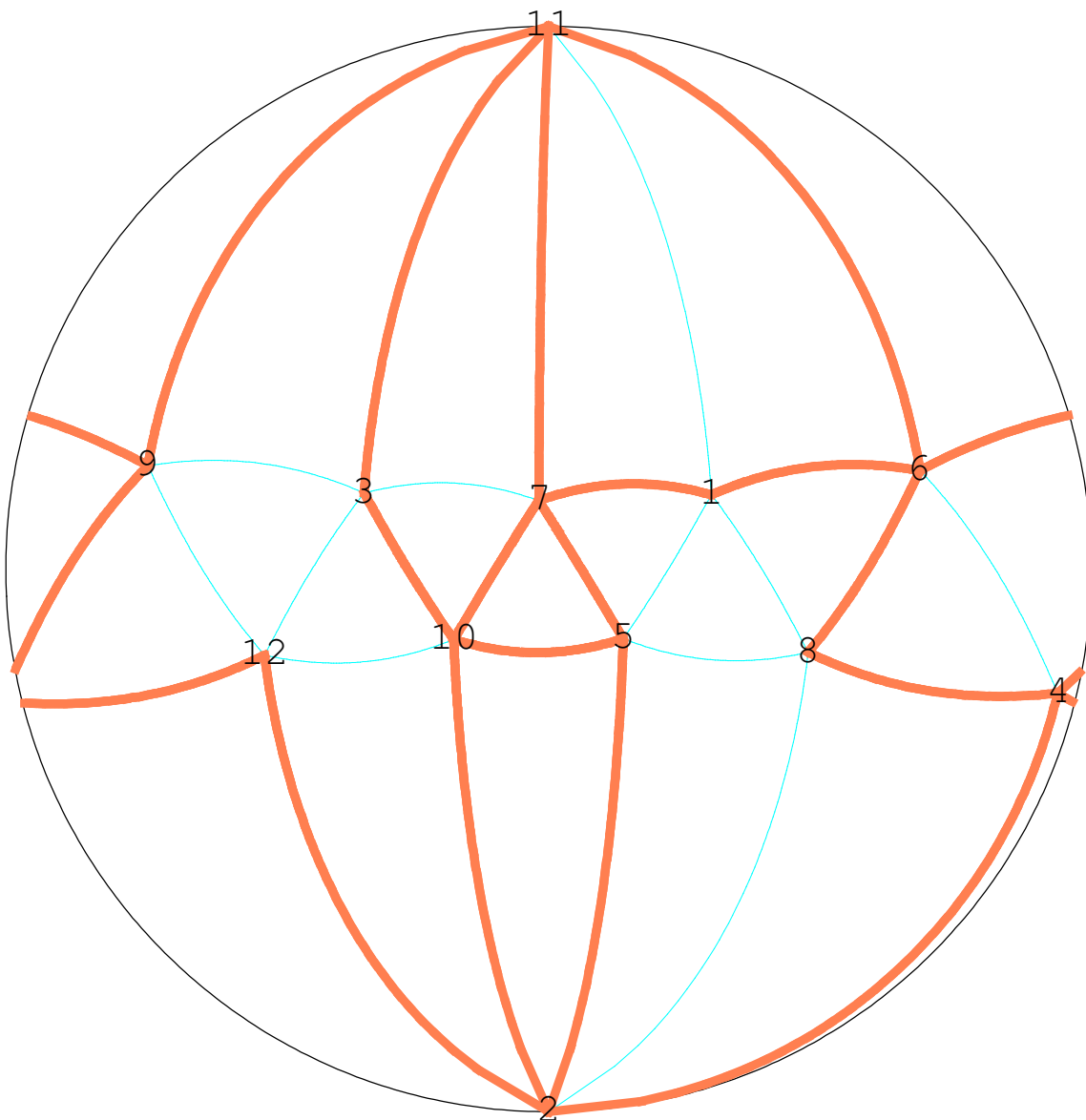
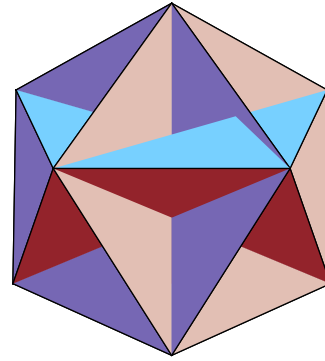
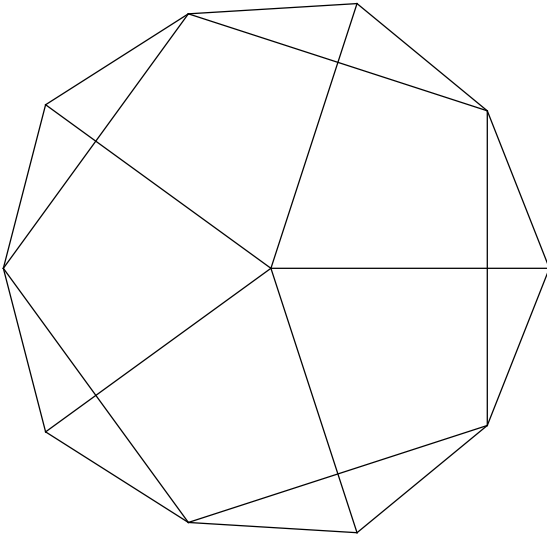
$$\left\{ \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2} \right\}$$



35.

great dodecahedron

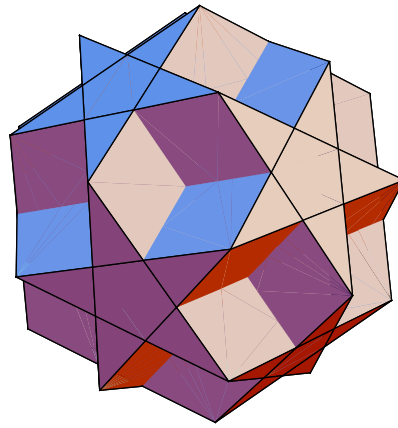
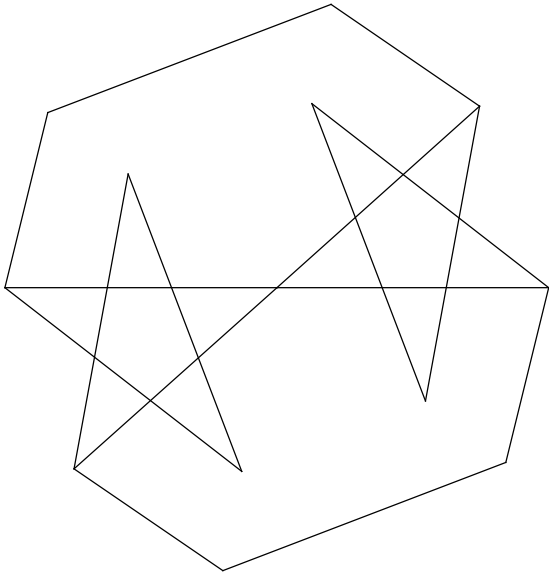
$$\frac{1}{2} \{5, 5, 5, 5, 5\}$$

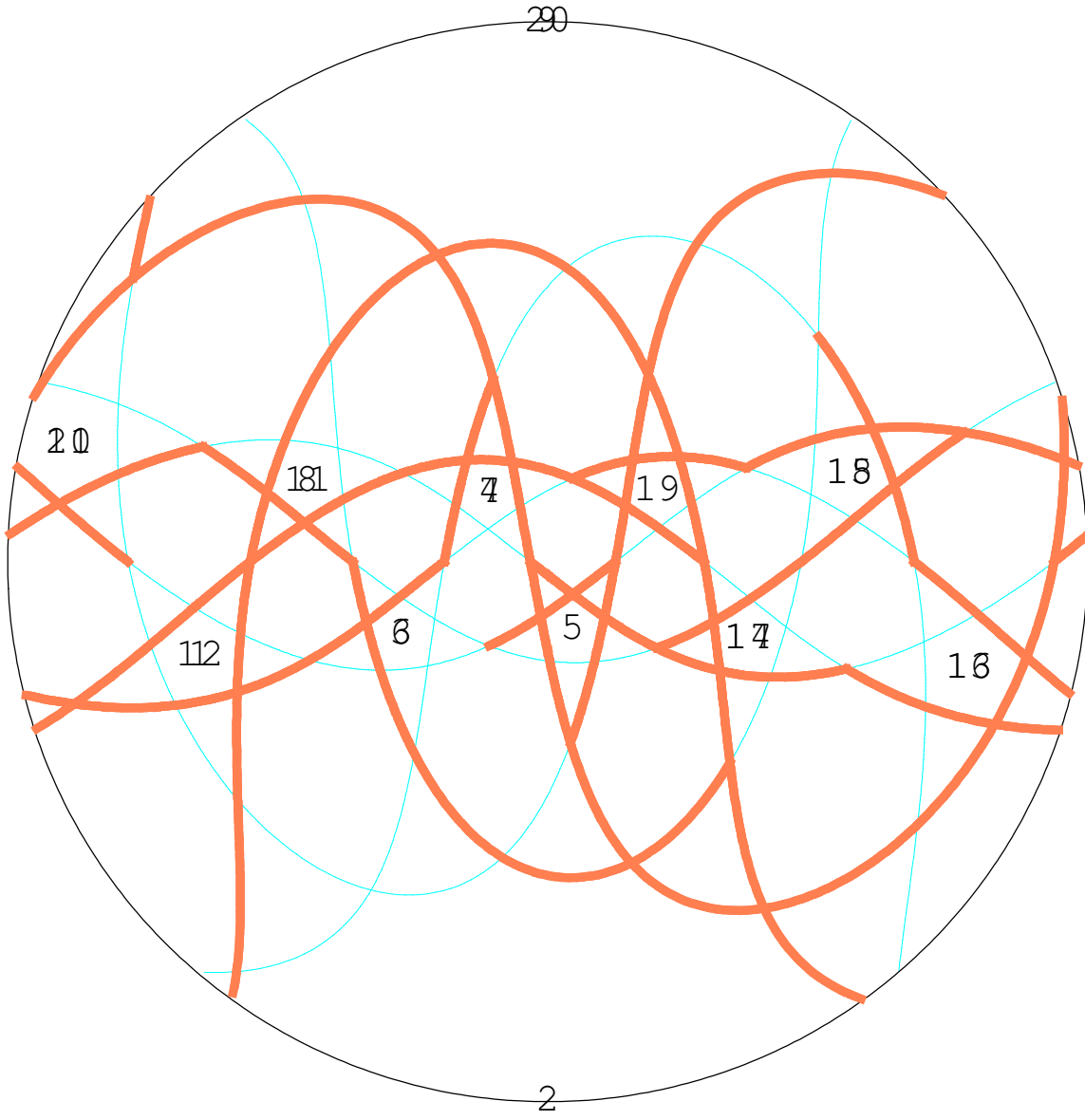


36.

dodecadodecahedron

$$\left\{ \frac{5}{2}, 5, \frac{5}{2}, 5 \right\}$$

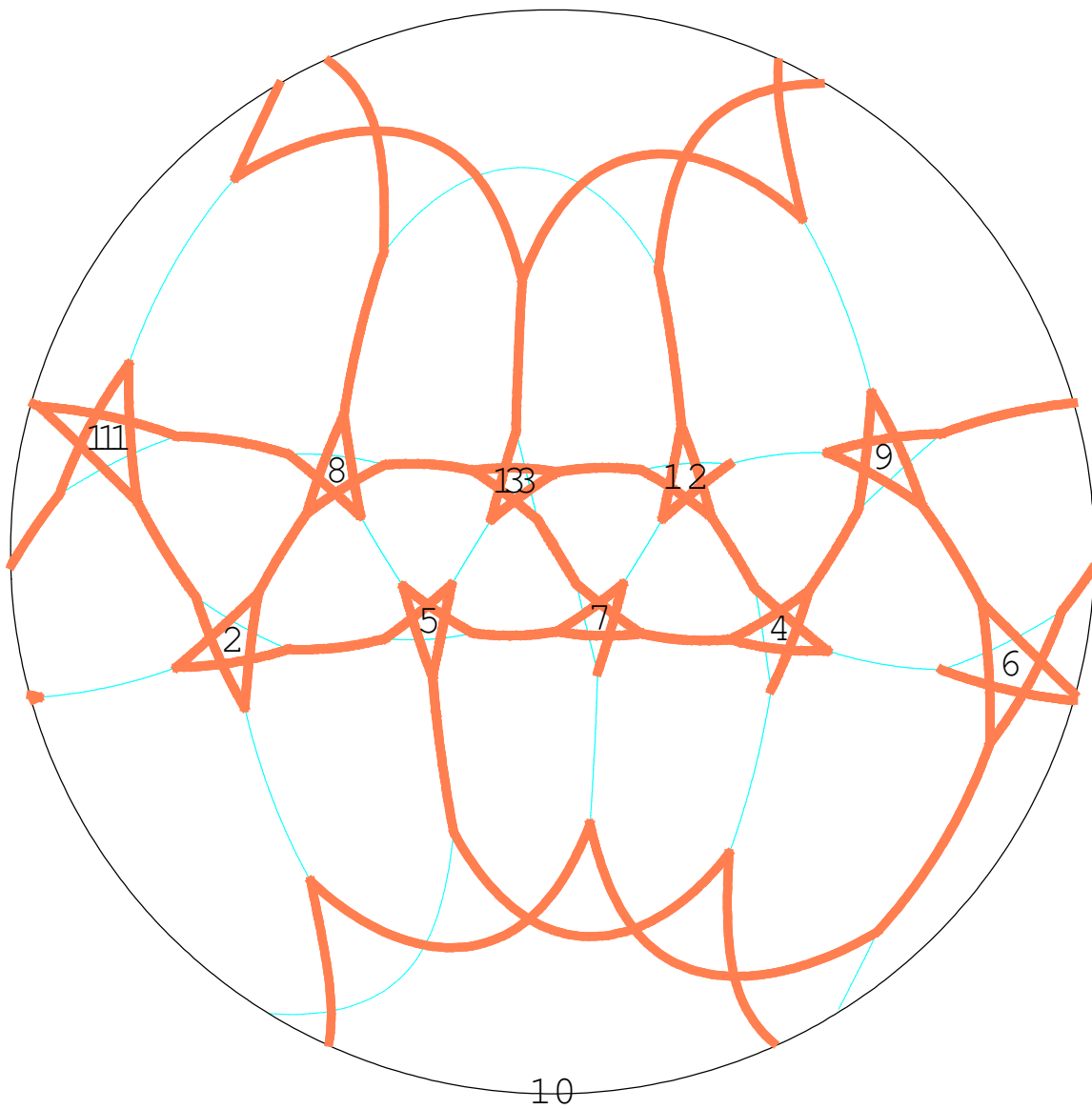
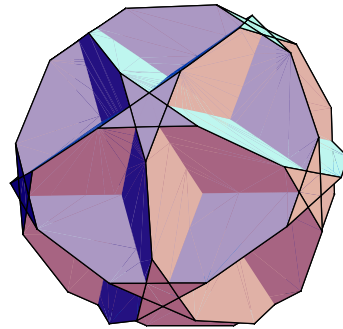
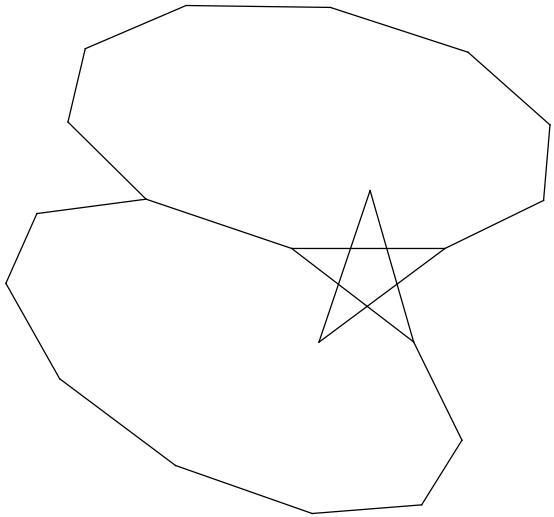




37.

truncated great dodecahedron

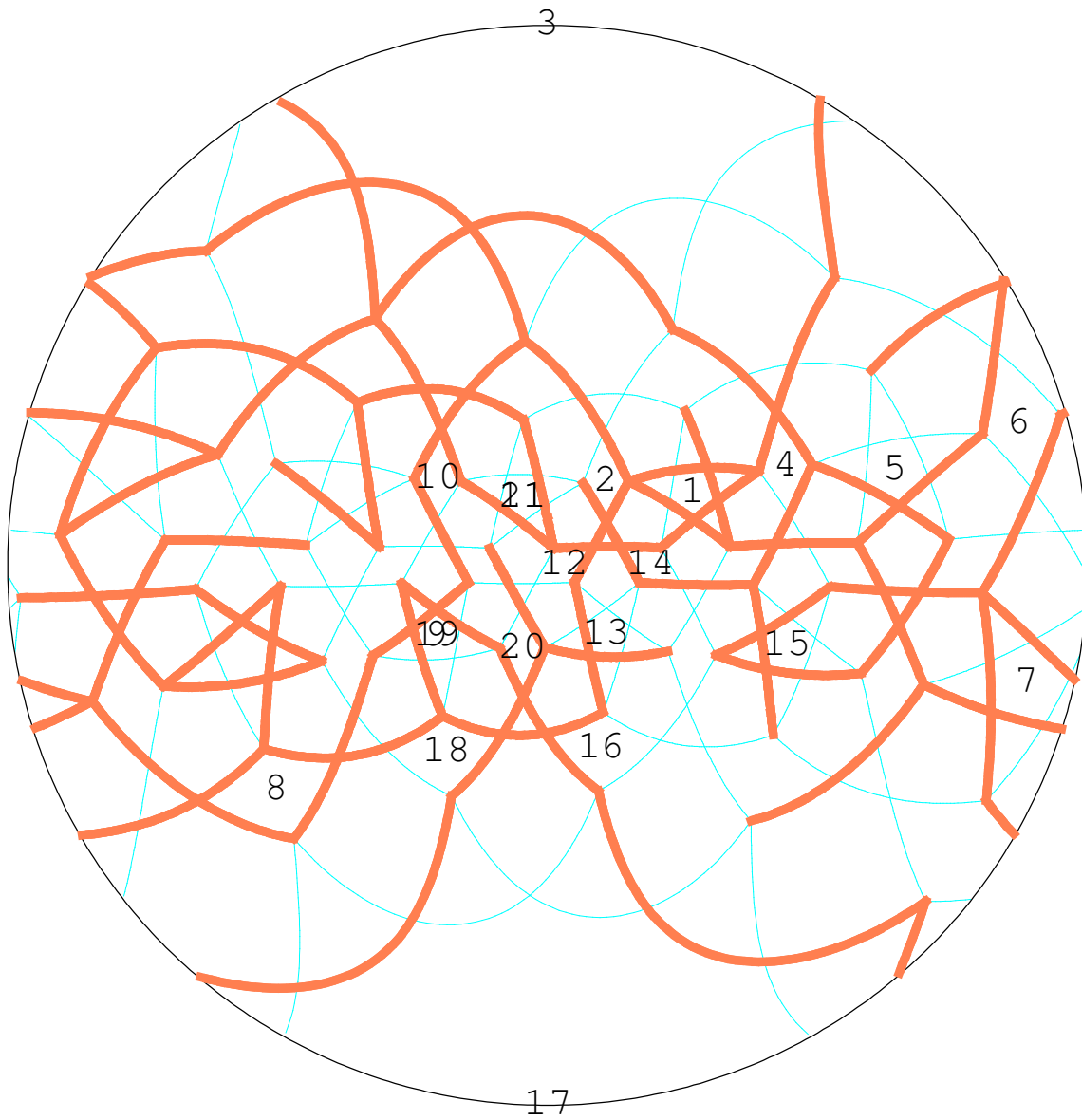
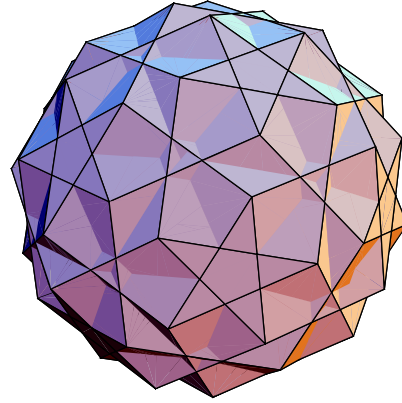
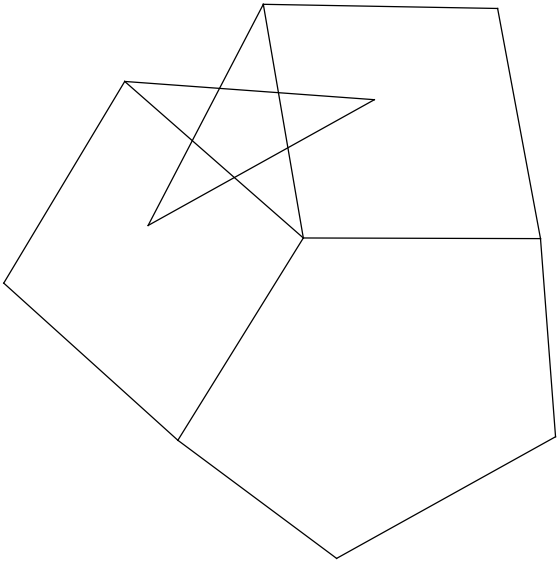
$$\{10, 10, \frac{5}{2}\}$$



38.

rhombidodecadodecahedron

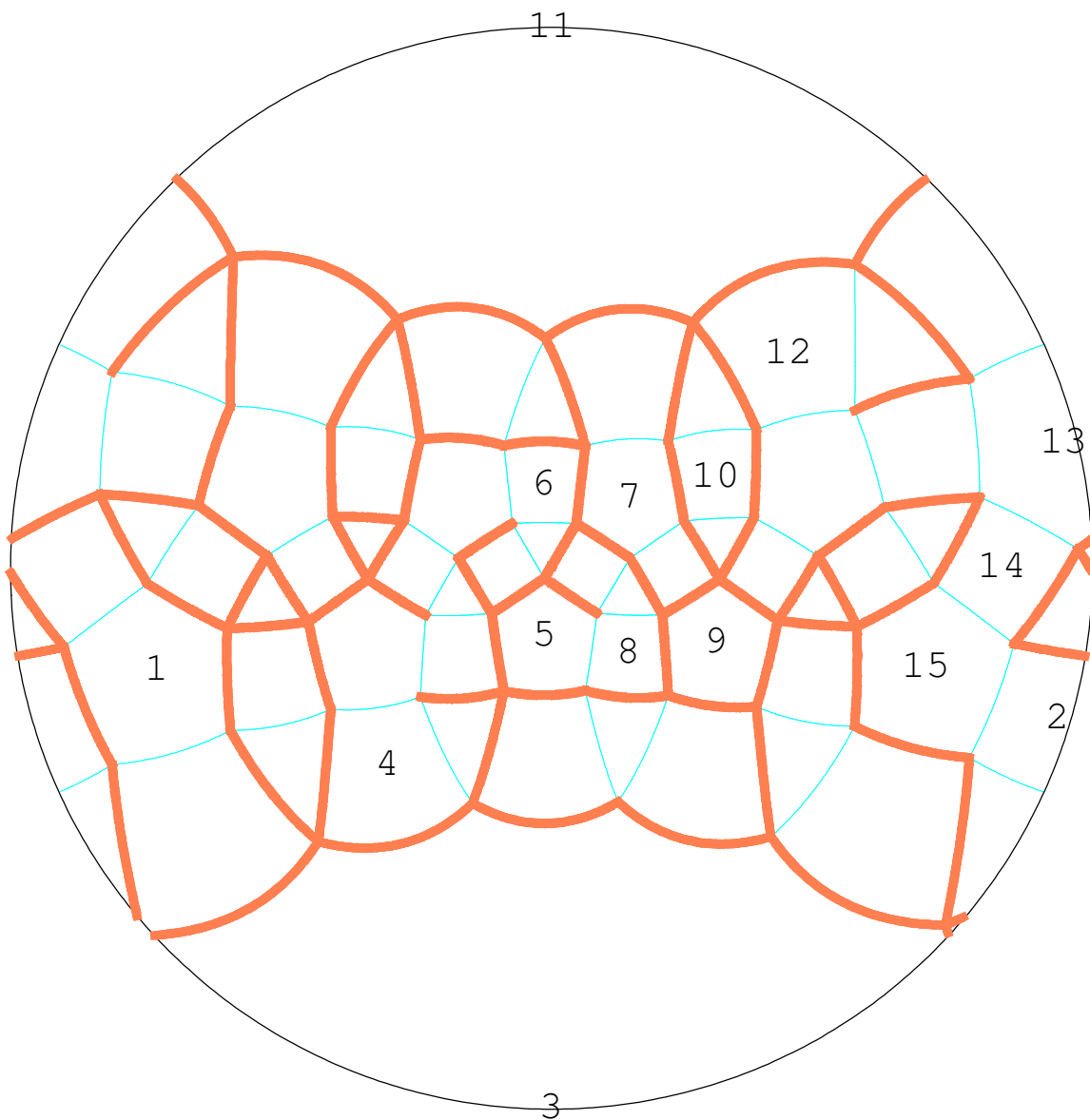
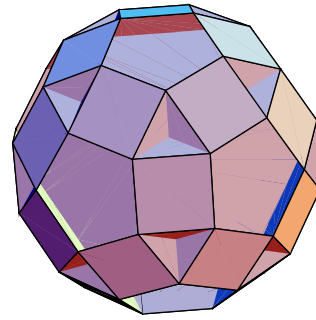
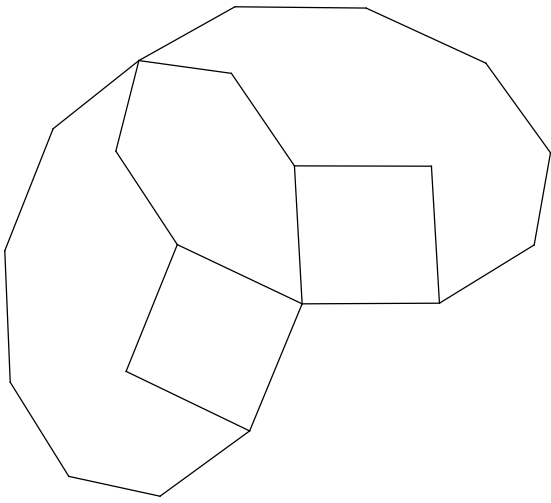
$$\{4, \frac{5}{2}, 4, 5\}$$



39.

small rhombidodecahedron

$$\{10, 4, \frac{10}{9}, \frac{4}{3}\}$$



40.

snub dodecadodecahedron

$$\left\{3, 3, \frac{5}{2}, 3, 5\right\}$$

