Unistable Polyhedral Surface with 15 Faces

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Abstract. The Mathematica Demonstration called Unistable Polyhedron Explorer has been developed. Using it we show that Guy's unistable polyhedron with 19 faces is also unistable as polyhedral surface, that there is a unistable polyhedral surface with 15 faces, and there is a unistable skeleton with 13 faces.

Any regular solid is stable on any face. But there exist polyhedra with only two stable faces [1, p.274, 4, 5]. A uniform-density polyhedral solid is unistable (also called monostable) if it is stable on exactly one face. In 1968 R. Guy constructed unistable polyhedron with 19 faces [1, p. 275, 2, 3]. This polyhedron is very interesting both for teachers and students, but a physical model of it is not easy to make. Using the demonstration we observed that the polyhedron was unistable also as surface. Now we can construct a paper model from a net which is also given by program.

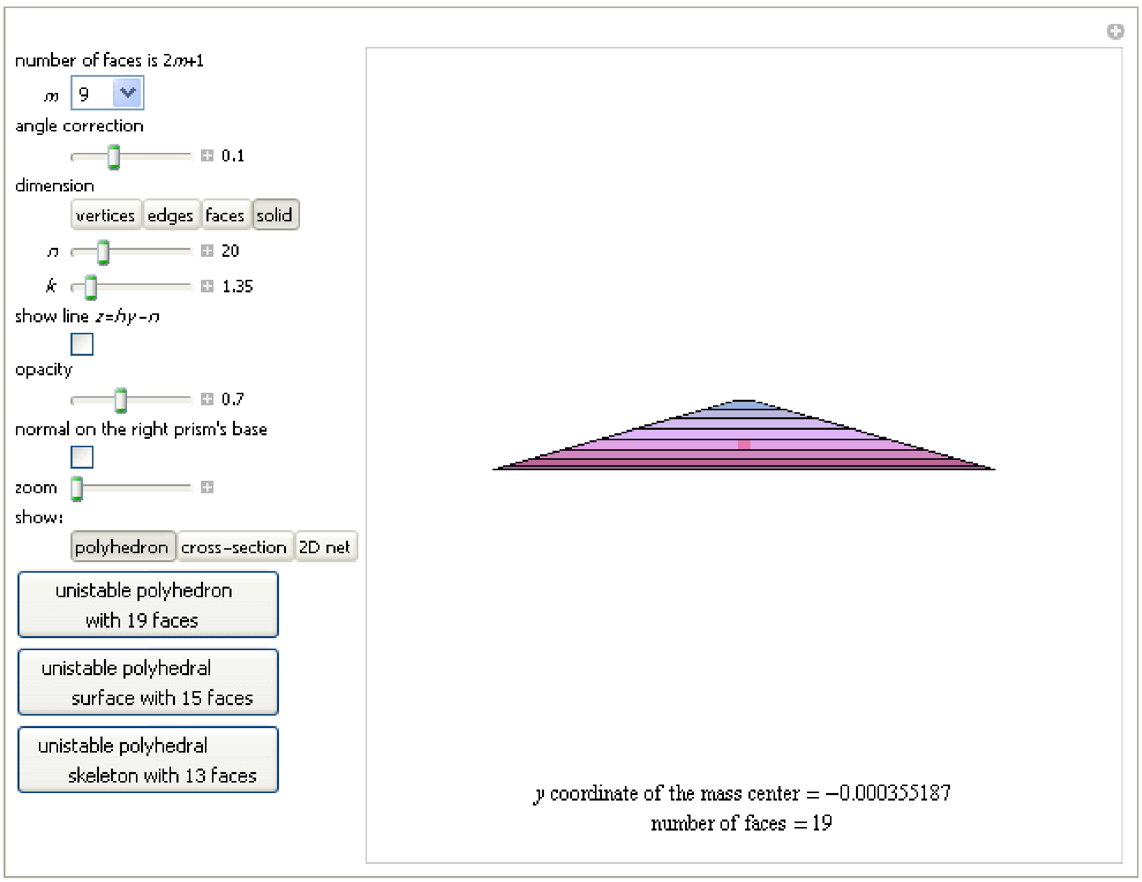

Figure 1. Unistable polyhedron with 19 faces.

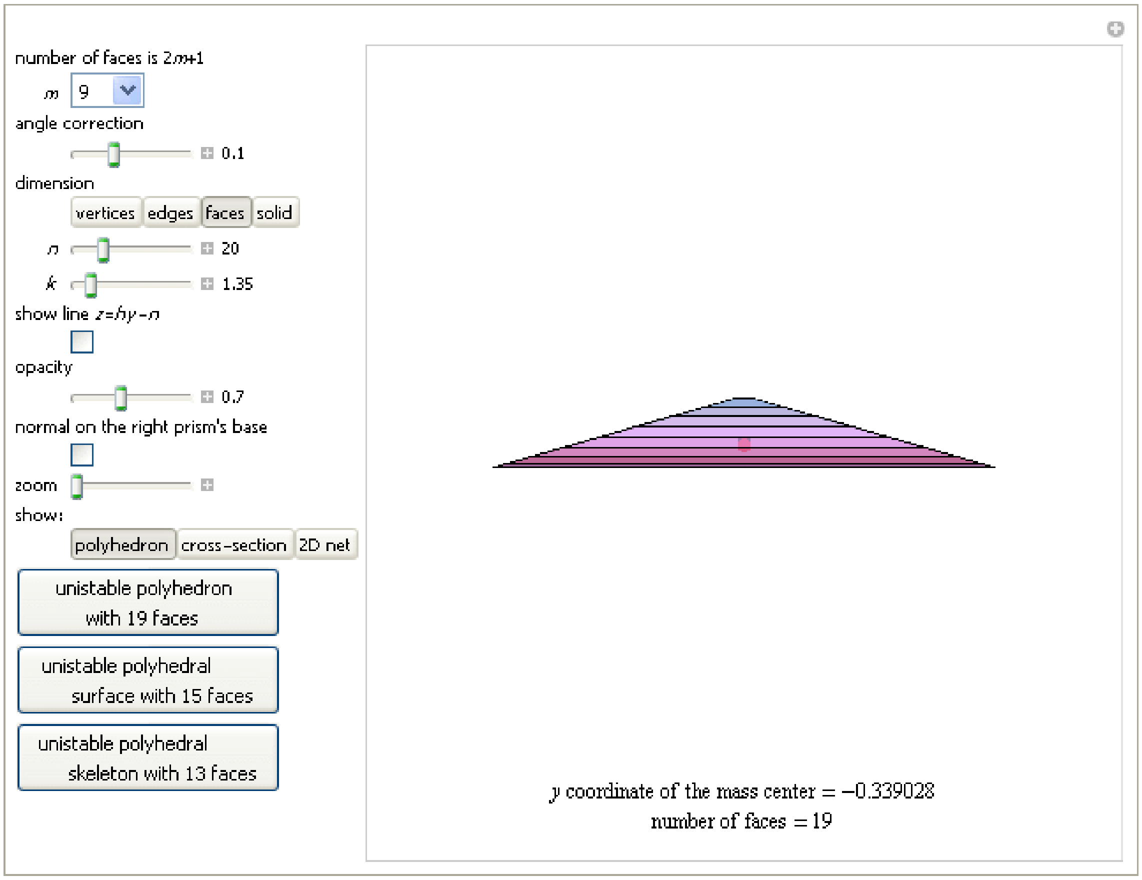

Figure 2. Unistable polyhedral surface with 19 faces.

To construct a unistable polyhedron the normal to any face (with one exception) must not intersect the face. Guy's polyhedron is a kind of prism, bases are not parallel but are truncated symmetricaly as shown on the figure.

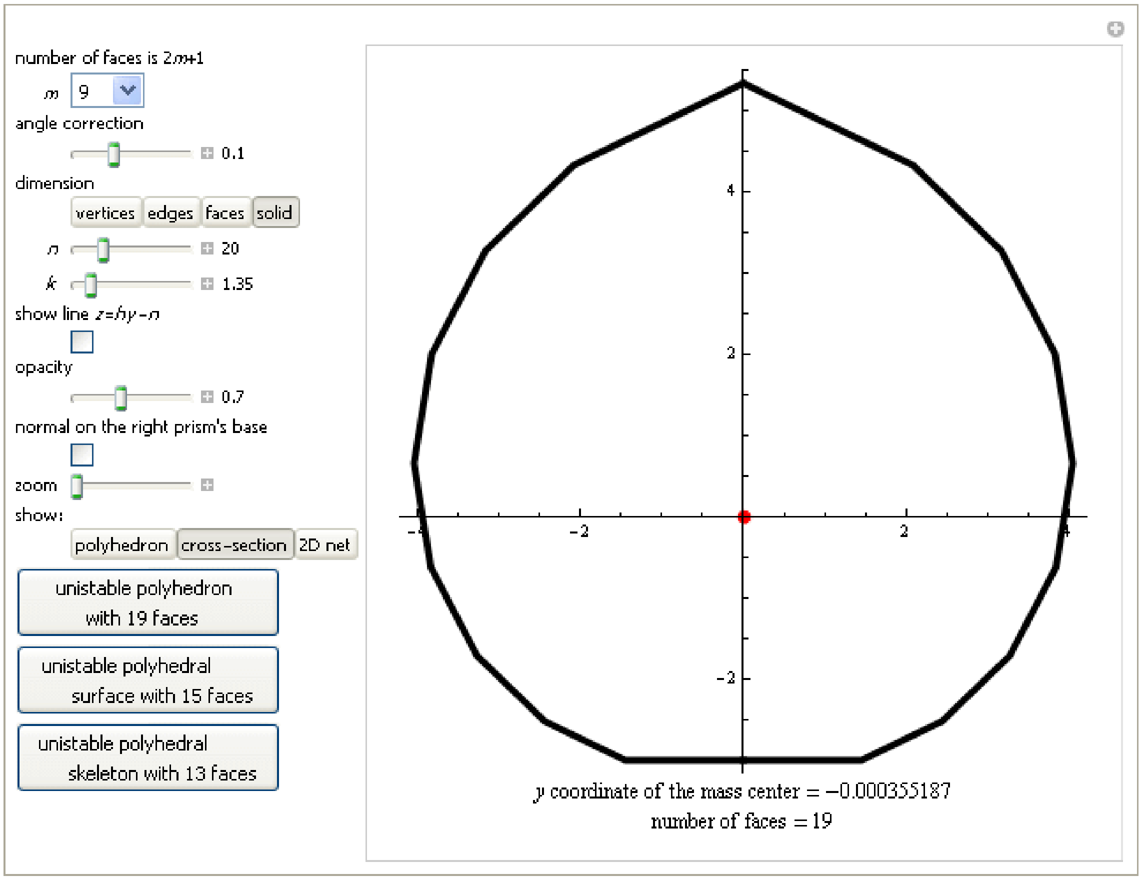

Figure 3. Cross-section of Guy's polyhedron

Each straight line from the origin to lower vertex of a side of the cross-section is perpendicular to the side. Only the lower face is stable if the center of mass is below the origin, and the normal to the right face doesn't intersect it as is shown on Figure 4.

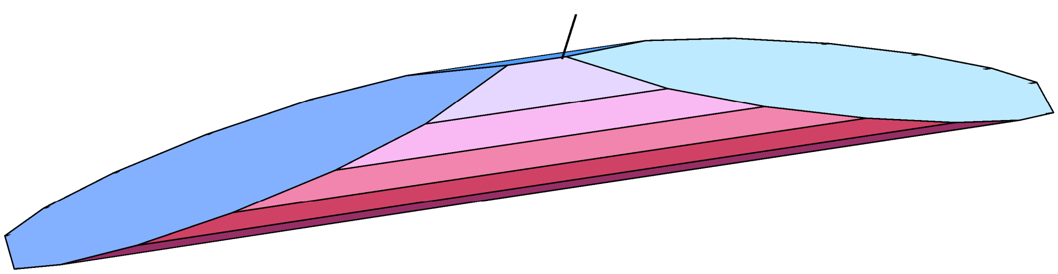

Figure 4. Unistable polyhedral surface with 15 faces

Figure 5. The net of Guy's polyhedron

In the Demonstration we examine also polyhedra as surfaces and skeletons. Example: to find a unistable polyhedral surface with 15 faces, choose m=7, dimension="faces", n=20, k=2. To get a unistable polyhedron, its mass center (red point) must be below coordinate origin (black point). The meaning of parameters is: $z=\pm n$ are z coordinates of intersections of the polyhedron and z axis, 2k is the length of upper edge of the polyhedron. So the slope of the line on the right face of the polyhedron and on y z plane is $h=(n-k)/r$, where r is the distance of the origin from the upper edge.

We can save graphics directly from the Demonstration. Even if we don't have Mathematica, we could use free Mathematica CDF Player [7]

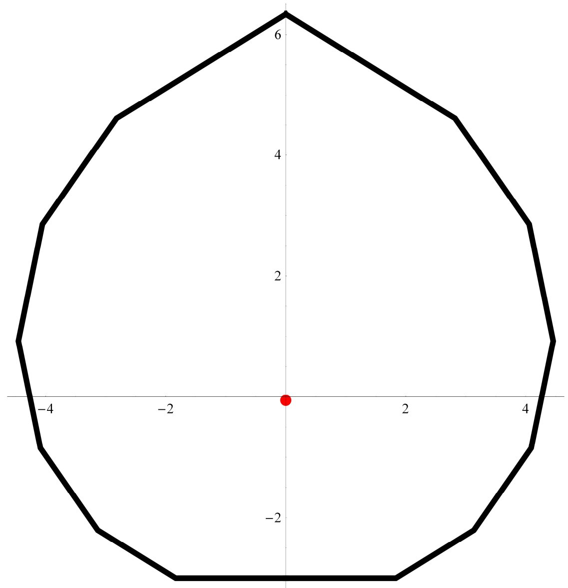

Figure 6. The cross-section of the polyhedral surface with 15 faces

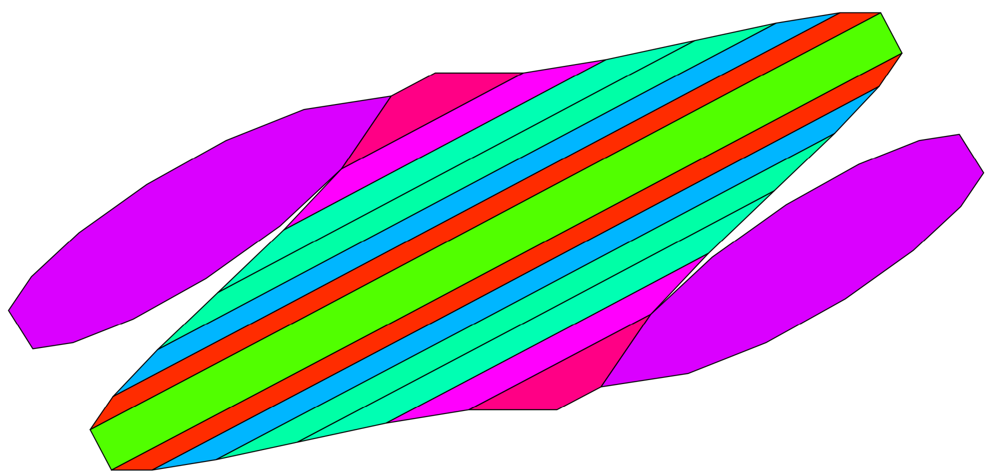

Figure 7. A net of polyhedral surface with 15 faces.

We are convinced that teachers and students will enjoy building unistable polyhedra.

Reference

[1] J. Bryant and C. Sangwin, How Round Is Your Circle?: Where Engineering and Mathematics Meet, Princeton, NJ: Princeton University Press, 2008, pp. 274–276.

[2] Richard K. Guy, A Unistable Polyhedron, University of Calgary Dept. of Mathematics, 1968.

[3] J.H. Conway, M. Goldberg and R.K. Guy, Problem 66-12, SIAM Review 11 (1969), 78–82.

[4] Izidor Hafner, "Heppes's Two-Tip Tetrahedron" http://demonstrations.wolfram.com/HeppessTwoTipTetrahedron/

[5] Izidor Hafner, "Stable and Unstable Faces of Antiprisms" http://demonstrations.wolfram.com/StableAndUnstableFacesOfAntiprisms/

[6] Izidor Hafner, Unistable Polyhedron Explorer, sent to Wolfram Demonstration Project.

[7] Mathematica CDF Player, http://www.wolfram.com/cdf-player/

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