1. Kolokvij MATEMATIKA I

19. 11. 2013

1. (20%) Z uporabo matematične indukcije dokažite, da je $n-ta$ *delna vsota* vrste

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)} \quad \text{enaka} \quad S_n = \frac{n}{2(n+2)} .$$

2. **(a)** (10%) V kompleksni ravnini narišite množico $\{ z \, ; \, |z-(2+i)| = \sqrt{10} \}$.

(b) (20%) Poiščite vsa kompleksna števila z, za katera je

$$|z-(4+3i)| = |z-(2+i)| = \sqrt{10}$$

3. (20%) Izračunajte limito zaporedja

$$\lim_{n\to\infty} \frac{(2n-1)n^n}{(n+1)^{n+1}}$$

4. (30%) Poiščite *definicijsko območje* funkcije $f(x) = \log(x-\sqrt{4x-3})$.

Rešitve

1. naloga

$$a_1 = \frac{1}{2 \cdot 3} = S_1$$

$$S_{n+1} = S_n + a_{n+1} = \frac{n}{2(n+2)} + \frac{1}{(n+2)(n+3)} =$$

$$\frac{n^2+3n+2}{2(n+2)(n+3)} = \frac{(n+1)(n+2)}{2(n+2)(n+3)} =$$

$$\frac{(n+1)}{2((n+1)+2)}$$

2. naloga

a)

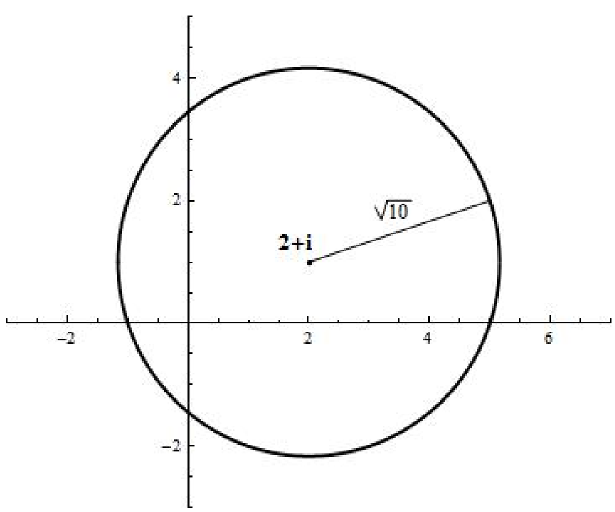
$z = x + iy$

$|x + iy - (2+i)| = \sqrt{10}$

$|(x-2) + i(y-1)| = \sqrt{10}$

$\sqrt{(x-2)^2 + (y-1)^2} = \sqrt{10}$

$(x-2)^2 + (y-1)^2 = 10$



b)

$$|x+iy-(4+3i)| = |x+iy-(2+i)|$$
$$|(x-4)+i(y-3)| = |(x-2)+i(y-1)|$$
$$\sqrt{(x-4)^2+(y-3)^2} = \sqrt{(x-2)^2+(y-1)^2}$$
$$x^2-8x+16+y^2-6y+9 = x^2-4x+4+y^2-2y+1$$
$$-4x-4y+20=0 \quad , \quad y=5-x$$
$y=5-x$ vstavimo v $(x-2)^2+(y-1)^2=10$
$$(x-2)^2+(4-x)^2=10$$
$$x^2-4x+4+16-8x+x^2=10$$
$$x^2-6x+5=0$$
$$(x-1)(x-5)=0$$
$$x_1=1\ ,\ x_2=5$$
$$y_1=4\ ,\ y_2=0$$

$$\boxed{z_1=1+4i\ ,\ z_2=5}$$

3. naloga

$$\lim_{n\to\infty}\frac{(2n-1)n^n}{(n+1)^{n+1}} = \lim_{n\to\infty}\frac{2n-1}{n+1}\cdot\left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty}\frac{2n-1}{n+1}\cdot\frac{1}{\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n} =$$

$$\boxed{\frac{2}{e}}$$

4. naloga

$$4x-3\geq 0 \ \text{ in } \ x-\sqrt{4x-3}>0$$
$$x\geq\frac{3}{4}$$
$$x>\sqrt{4x-3}$$
$$x^2>4x-3$$
$$x^2-4x+3>0$$
$$(x-1)(x-3)>0$$
$$x<1 \ \text{ ali } \ x>3$$

$$\boxed{\mathcal{D}=\left[\frac{3}{4},1\right)\cup\left(3,\infty\right)}$$