

2. Kolokvij MATEMATIKA I

10. 1. 2014

1. (20%) Izračunajte limite:

(a) (10%) $\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$

(b) (5%) $\lim_{x \rightarrow 0} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$

(c) (5%) $\lim_{x \rightarrow \infty} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$

2. (30%) Dana je funkcija $f(x) = \frac{\ln^2 x}{x}$.

(a) (10%) Poiščite **DNPAE** : tj. poiščite **D**efinicijsko območje, **N**ičle, **P**ole, **A**simptote in **E**kstreme funkcije.

(b) (10%) Poiščite intervale naraščanja in padanja funkcije.

(c) (10%) Narišite graf $y = f(x)$.

3. (25%) Izračunajte nedoločeni integral

$$\int \frac{\sin^3 x}{\cos^4 x} dx$$

4. (25%) Dana je krivulja $y = \frac{1}{1+x^2}$.

(a) (10%) Zapišite enačbo tangente na krivuljo v točki $x = 1$.

(b) (15%) Izračunajte ploščino med krivuljo, tangento in ordinatno osjo.

Rešitve

1. naloga

$$\text{a) } \lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right) = \lim_{x \rightarrow 1} \frac{x-1-\ln x}{\ln x \cdot (x-1)} = \lim_{x \rightarrow 1} \frac{1-\frac{1}{x}}{\frac{1}{x}(x-1)+\ln x} =$$
$$\lim_{x \rightarrow 1} \frac{x-1}{(x-1)+x \ln x} = \lim_{x \rightarrow 1} \frac{1}{1+\ln x+x\frac{1}{x}} = \boxed{\frac{1}{2}}$$

$$\text{b) } \lim_{x \rightarrow 0} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right) = \boxed{1}$$

$$\text{c) } \lim_{x \rightarrow \infty} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right) = \boxed{0}$$

2. naloga

a)

Na asimptoto sklepamo iz limite

$$\lim_{x \rightarrow \infty} \frac{\ln^2 x}{x} = \lim_{x \rightarrow \infty} \frac{2 \ln x \cdot \frac{1}{x}}{1} = 2 \lim_{x \rightarrow \infty} \frac{\ln x}{x} = 2 \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = 0$$

Stacionarne točke so ničle odvoda

$$f'(x) = \frac{2 \ln x \cdot \frac{1}{x} \cdot x - \ln^2 x \cdot 1}{x^2} = \frac{\ln x(2 - \ln x)}{x^2}$$

$$\ln x = 0 \quad \rightarrow \quad x_1 = 1$$

$$\ln x = 2 \quad \rightarrow \quad x_2 = e^2$$

Naravo ekstremov odkrijemo iz intervalov naraščanja in padanja funkcije.

$D : (0, \infty)$
$N : x = 1$
$P : x = 0$
$A : y = 0$
$E : x = 1 \text{ min}$
$x = e^2 \text{ max}$

b)

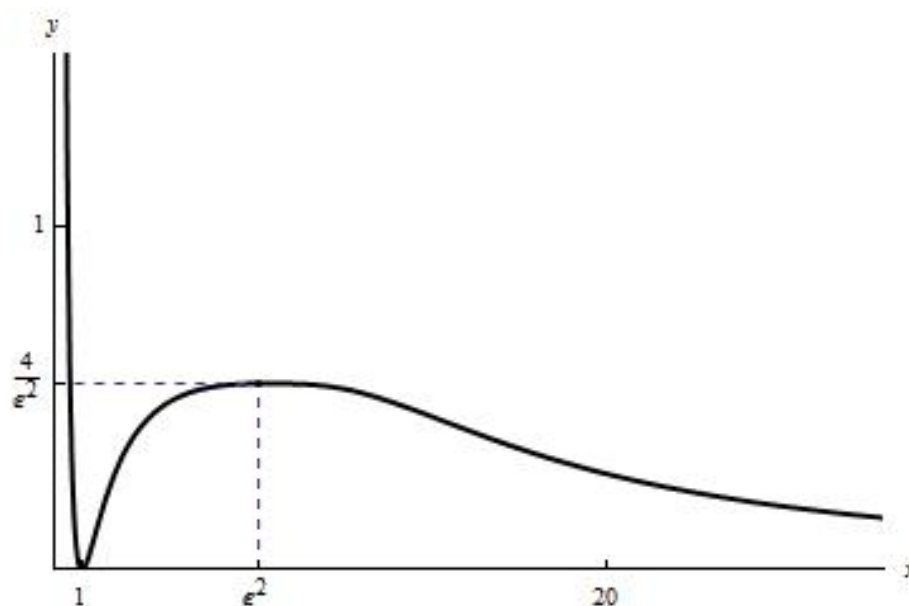
$0 < x < 1 \rightarrow \ln x < 0 \rightarrow f'(x) < 0 \rightarrow f(x)$ pada

$1 < x < e^2 \rightarrow \ln x > 0$ in $\ln x < 2 \rightarrow f'(x) > 0 \rightarrow f(x)$ narašča

$x > e^2 \rightarrow \ln x > 2 \rightarrow f'(x) < 0 \rightarrow f(x)$ pada

$(0, 1)$	$f(x)$ pada
$(1, e^2)$	$f(x)$ narašča
(e^2, ∞)	$f(x)$ pada

c)



3. naloga

$$\cos x = u$$

$$-\sin x dx = du$$

$$dx = \frac{du}{-\sin x}$$

$$\int \frac{\sin^3 x}{\cos^4 x} dx = \int \frac{\sin^3 x}{u^4} \frac{du}{-\sin x} = \int -\frac{1 - \cos^2 x}{u^4} du = \int \frac{u^2 - 1}{u^4} du =$$

$$\int (u^{-2} - u^{-4}) du = \frac{u^{-1}}{-1} - \frac{u^{-3}}{-3} + C = \boxed{\frac{1}{3} \frac{1}{\cos^3 x} - \frac{1}{\cos x} + C}$$

4. naloga

a)

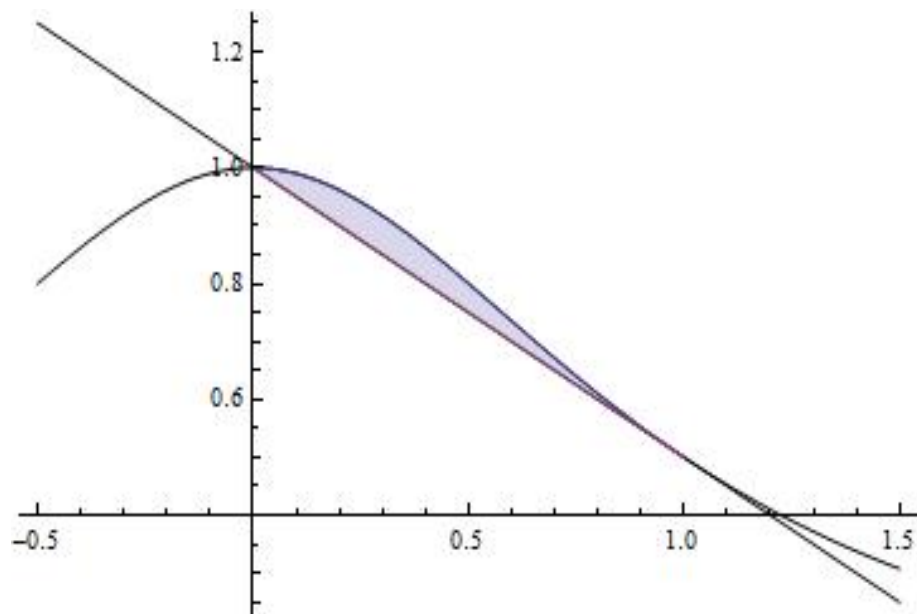
$$y'(x) = \frac{-2x}{(1+x^2)^2}$$

$$k = y'(1) = -\frac{1}{2}$$

$$y - \frac{1}{2} = -\frac{1}{2}(x - 1)$$

$$y = -\frac{1}{2}x + 1$$

b)



$$P = \int_0^1 \left(\frac{1}{1+x^2} + \frac{x}{2} - 1 \right) dx = \arctg x + \frac{x^2}{4} - x \Big|_0^1 = \frac{\pi}{4} + \frac{1}{4} - 1 = \boxed{\frac{\pi - 3}{4}}$$