1. naloga

Formula za volumen prizme je mešani produkt vektorjev na sosednjih robovih. Zato najprej poiščemo oglišča piramide, ki so presečišča treh sosednjih stranskih ploskev. Rešimo štiri sisteme enačb, v vsakem izpustimo eno stransko ploskev:

$V(2,2,2)$ je vrh piramide in je rešitev sistema enačb $x = y$, $x = z$, $x + y + z = 6$.
$A(0,0,0)$ je presek ravnin $z = 0$, $x = y$, $x = z$
$B(3,3,0)$ je presek ravnin $z = 0$, $x = y$, $x + y + z = 6$
$C(0,6,0)$ je presek ravnin $z = 0$, $x = z$, $x + y + z = 6$

$$V = \tfrac{1}{6}|(\vec{AB}, \vec{AC}, \vec{AV})| = \tfrac{1}{6}\begin{vmatrix} 3 & 3 & 0 \\ 0 & 6 & 0 \\ 2 & 2 & 2 \end{vmatrix} = 6$$

2. naloga

Konvergenčni radij $R = \lim\limits_{n\to\infty} \frac{2n+3}{2n+5} = 1$, v krajiščih je vrsta divergentna. Konvergenčno območje je interval $(-1,1)$.

$$3 + 5x^2 + 7x^4 + 9x^6 + \cdots =$$
$$\frac{1}{x^2}\Big(3x^2 + 5x^4 + 7x^6 + 9x^8 + \cdots\Big) =$$
$$\frac{1}{x^2}\Big(x^3 + x^5 + x^7 + x^9 + \cdots\Big)' =$$
$$\frac{1}{x^2}\Big(\frac{x^3}{1-x^2}\Big)' =$$

$$\frac{3-x^2}{(1-x^2)^2}$$

3. naloga

$z_x = 4x + 2xy = 2x(2 + y)$

$z_y = 8y + x^2 + 3y^2$

$z_x = 2x(2 + y) = 0 \quad \longrightarrow \quad x_1 = 0\ ,\ y_2 = -2$

$x_1 = 0 \quad \longrightarrow \quad z_y = y(8 + 3y) = 0 \quad \longrightarrow \quad y_{1,1} = 0\ ,\ y_{1,2} = -\frac{8}{3}$

$y_2 = -2 \quad \longrightarrow \quad z_y = x^2 - 4 = 0 \quad \longrightarrow \quad x_{2,1} = 2\ ,\ x_{2,2} = -2$

Stacionarne točke $T_1(0,0)\ ,\ T_2(0,-\frac{8}{3})\ ,\ T_3(2,-2)\ ,\ T_4(-2,-2)$

$D(x,y) = z_{xx}z_{yy} - z_{xy}^2 = (4 + 2y)(8 + 6y) - (2x)^2$

$D(T_1) = 4 * 8 - 0 > 0 \quad \longrightarrow \quad$ minimum

$D(T_2) = (-\frac{4}{3})(-8) - 0 > 0 \quad \longrightarrow \quad$ maximum

$D(T_3) = -16 < 0 \quad \longrightarrow \quad$ ni ekstrema

$D(T_4) = -16 < 0 \quad \longrightarrow \quad$ ni ekstrema

4. naloga

Homogena enačba $xy' - y = 0$

$x\dfrac{dy}{dx} = y$

$\displaystyle\int \frac{dy}{y} = \int \frac{dx}{x}$

$\log y = \log x + \log C$

$y_h = Cx$

Partikularna rešitev $y_p = C(x)x$

$x(C'x + C) - Cx = \dfrac{x^2}{1+x^2}$

$C' = \dfrac{1}{1+x^2}$

$C = \operatorname{arctg} x$

$y_p = x \operatorname{arctg} x$

Splošna rešitev $y = y_h + y_p = Cx + x \operatorname{arctg} x$

$y(1) = \dfrac{\pi}{4} \quad \longrightarrow \quad C + \dfrac{\pi}{4} = \dfrac{\pi}{4} \quad \longrightarrow \quad C = 0$

$y = x \operatorname{arctg} x$

5. naloga

V *Euler*jevo enačbo se vpelje neodvisno spremenljivko $x = e^t$

$y' = \frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \dot{y}\frac{1}{x}$

$y'' = \left(\dot{y}\frac{1}{x}\right)' = (\dot{y})'\frac{1}{x} + \dot{y}\left(\frac{1}{x}\right)' = \ddot{y}\frac{1}{x^2} - \dot{y}\frac{1}{x^2}$

$\ddot{y} - \dot{y} - 2y = 3e^{2t}$

$r^2 - r - 2 = 0$

$r_1 = -1\ ,\ r_2 = 2$

$y_h = Ae^{-t} + Be^{2t}$

$y_p = Ce^{2t}t$

$\dot{y}_p = Ce^{2t}(2t+1)$

$\ddot{y}_p = Ce^{2t}(4t+4)$

$Ce^{2t}(4t + 4 - 2t - 1 - 2t) = 3e^{2t} \quad \longrightarrow \quad C = 1$

$y = y_h + y_p = Ae^{-t} + Be^{2t} + e^{2t}t$

$y = \frac{A}{x} + Bx^2 + x^2 \log x$

$y' = -\frac{A}{x^2} + 2Bx + 2x \log x + x$

$y(1) = 1 \quad \longrightarrow \quad A + B = 1$

$y'(1) = 0 \quad \longrightarrow \quad -A + 2B + 1 = 0$

$B = 0\ ,\ A = 1$

$y = \frac{1}{x} + x^2 \log x$