1. Kolokvij Matematika III

10. november 2003

Rešitve

1. naloga

$t + 1 = u$

$\cos t = u \cos v$

$\sin t = u \sin v$

Druga in tretja enačba na kvadrat, seštejemo in upoštevamo $u \geq 0$:

$u = 1\ ,\ t = 0\ ,\ v = 0$

$$\boxed{P(1,1,0)}$$

$\dot{\vec{r}}(t) = (1\,,\,-\sin t\,,\,\cos t)$

$\vec{e} = \dot{\vec{r}}(0) = (1,0,1)$

$$\vec{\nu}(u,v) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & \cos v & \sin v \\ 0 & -\sin v & \cos v \end{vmatrix} = (1\,,\,-\cos v\,,\,-\sin v)$$

$\vec{n} = \vec{\nu}(1,0) = (1,-1,0)$

$$\cos\alpha = \frac{\vec{e}\cdot\vec{n}}{|\vec{e}||\vec{n}|} = \frac{(1,0,1)\cdot(1,-1,0)}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$$

$$\boxed{\alpha = \frac{\pi}{3}}$$

2. naloga

$$F(y) = \int_{\frac{1}{y}}^{\infty} \frac{\ln(1+y^2x^2)}{x^2}\,dx$$

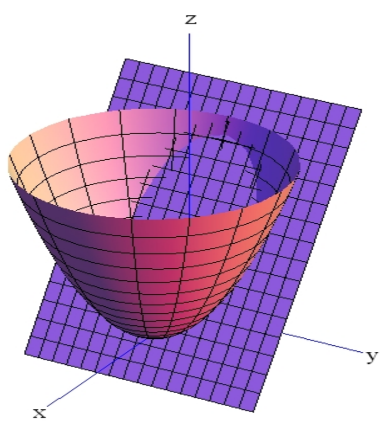
$$F'(y) = \int_{\frac{1}{y}}^{\infty} \frac{2yx^2}{(1+y^2x^2)x^2}\,dx - \frac{\ln(1+1)}{1/y^2}\cdot\frac{-1}{y^2} = 2\,\mathrm{arctg}(yx)\Big|_{1/y}^{\infty} + \ln 2 = 2(\tfrac{\pi}{2}-\tfrac{\pi}{4}) + \ln 2 =$$

$\frac{\pi}{2} + \ln 2$

$$F(y) = \int (\frac{\pi}{2} + \ln 2) dy = \boxed{(\frac{\pi}{2} + \ln 2) y + C}$$

Opomba: pri tej nalogi določitev konstante C presega znanje, ki ga mora študent obvladati. S poglobljeno matematično analizo se da odkriti, da je $C = 0$.

3. naloga



Prostornina je dvojni integral zgornje - spodnje ploskve. Integracijsko območje je določeno s presekom obeh ploskev.

$$(x-1)^2 + y^2 = 2 - 2x$$
$$x^2 - 2x + 1 + y^2 = 2 - 2x$$
$$x^2 + y^2 = 1$$

Ker je integracijsko območje krog, bomo vpeljali polarne koordinate.

$$V = \iint\limits_{x^2+y^2<1} \Big[(2-2x) - (x-1)^2 - y^2\Big] dxdy = \iint\limits_{x^2+y^2<1} (1 - x^2 - y^2) dxdy =$$

$$\int_0^{2\pi} d\varphi \int_0^1 (1 - r^2) r \, dr = 2\pi \Big(\frac{r^2}{2} - \frac{r^4}{4}\Big)\Big|_0^1 = \boxed{\frac{\pi}{2}}$$