

1. Kolokvij Matematika III
20.november 2006
Rešitve nalog

1. naloga

a)

Ko sestavljamo parametrično enačbo krivulje, začnemo z $x = t$.

Točka T bo potem določena z vrednostjo parametra $t = 1$.

$$\vec{r}(t) = (t, t^2 + 1, t^2 + 4)$$

$$\dot{\vec{r}} = (1, 2t, 2t)$$

$$\vec{e} = \dot{\vec{r}}(1) = (1, 2, 2)$$

$$t: \quad \vec{r} = (1, 2, 5) + t(1, 2, 2)$$

b)

$$x = 1 + t, \quad y = 2 + 2t, \quad z = 5 + 2t$$

$$(1 + t)^2 - 12(1 + t) - (2 + 2t)^2 + (5 + 2t)^2 - 9 = 0$$

$$t^2 + 2t + 1 = 0$$

$$t = -1$$

$$S(0, 0, 3)$$

c)

$$\vec{v} = (2x - 12, -2y, 2z)$$

$$\vec{n} = \vec{v}(S) = (-12, 0, 6) = 6(-2, 0, 1)$$

$$((x, y, z) - (0, 0, 3)) \cdot (-2, 0, 1) = 0$$

$$-2x + z - 3 = 0$$

$$\cos \alpha = \frac{\vec{e} \cdot \vec{n}}{|\vec{e}| |\vec{n}|} = \frac{(1, 2, 2) \cdot (-2, 0, 1)}{\sqrt{9}\sqrt{5}} = 0$$

$$\alpha = \frac{\pi}{2}$$

2. naloga

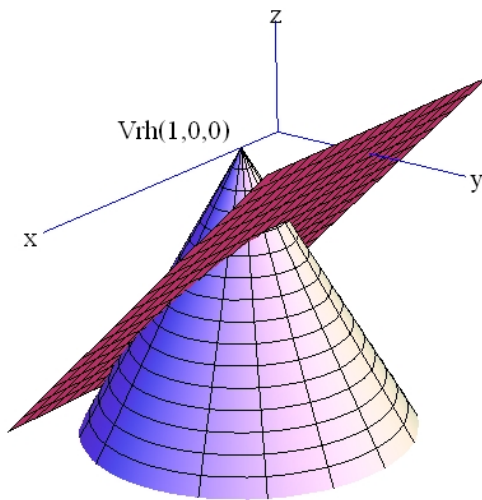
$$\int_0^{a/2} \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} \Big|_0^{a/2} = \arcsin \frac{1}{2} = \frac{\pi}{6}$$

Odvajamo levo in desno stran po a in izračunamo ven iskani intregal:

$$\int_0^{a/2} \frac{1}{2} \frac{2a}{2\sqrt{(a^2 - x^2)^3}} dx + \frac{1}{\sqrt{a^2 - a^2/4}} \frac{1}{2} = 0$$

$$\int_0^{a/2} \frac{dx}{\sqrt{(a^2 - x^2)^3}} = \frac{1}{a} \frac{1}{2} \frac{1}{\sqrt{a^2 - a^2/4}} = \boxed{\frac{1}{a^2\sqrt{3}}}$$

3. naloga



Površina območja je iz dveh delov: plašč stožca in del ravnine znotraj presečne elipse. Obe površini dobimo s formulo $P = \iint \sqrt{1 + p^2 + q^2}$, kjer je integracijsko območje notranjost elipse. Enačbo projekcije elipse na ravnino xy dobimo, če izenačimo enačbi stožca in ravnine:

$$-\sqrt{3(x-1)^2 + 3y^2} = y - 2$$

$$3(x-1)^2 + 3y^2 = y^2 - 4y + 4$$

$$3(x-1)^2 + 2y^2 + 4y + 2 = 4 + 2$$

$$3(x-1)^2 + 2(y+1)^2 = 6$$

$$\frac{(x-1)^2}{2} + \frac{(y+1)^2}{3} = 1$$

Osi elipse sta $a = \sqrt{2}$ in $b = \sqrt{3}$.

$$P_{\text{stožec}} = \int \int_{\text{elipsa}} \sqrt{1 + p^2 + q^2} dx dy =$$

$$\int \int_{\text{elipsa}} \sqrt{1 + \left(-\frac{6(x-1)}{2\sqrt{3(x-1)^2 + 3y^2}}\right)^2 + \left(-\frac{6y}{2\sqrt{3(x-1)^2 + 3y^2}}\right)^2} dx dy =$$

$$\int \int_{\text{elipsa}} \sqrt{4} dx dy = 2 \cdot \text{ploščina elipse} = 2\pi\sqrt{2}\sqrt{3}$$

$$P_{\text{ravnina}} = \int \int_{\text{elipsa}} \sqrt{1 + p^2 + q^2} dx dy =$$

$$\int \int_{\text{elipsa}} \sqrt{1 + 0 + 1} dx dy = \sqrt{2} \cdot \text{ploščina elipse} = \sqrt{2}\pi\sqrt{2}\sqrt{3}$$

$$P = P_{\text{stožec}} + P_{\text{ravnina}} = \boxed{(2 + \sqrt{2})\pi\sqrt{6}}$$