

# Izpit Matematika IV

16.junij 2006

## Rešitve

### 1. naloga

$$F(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{(s+1)^2}$$

$$As(s+1)^2 + B(s+1)^2 + Cs^2(s+1) + Ds^2 = 1$$

$$s = 0 \rightarrow B = 1$$

$$s = -1 \rightarrow D = 1$$

$$ks \rightarrow A + 2B = 0 \rightarrow A = -2$$

$$ks^3 \rightarrow A + C = 0 \rightarrow C = 2$$

$$f(t) = -2 + t + 2e^{-t} + te^{-t}$$

## 2. naloga

$$y = \sum_{n=0}^{\infty} C_n x^n$$

$$y' = \sum_{n=1}^{\infty} C_n n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} C_n n(n-1) x^{n-2}$$

$$\sum_{n=2}^{\infty} C_n n(n-1) x^{n-1} + 2 \sum_{n=1}^{\infty} C_n n x^{n-1} + \sum_{n=0}^{\infty} C_n x^{n+1} = 0$$

$$\sum_{n=2}^{\infty} C_n n(n-1) x^{n-1} + 2 \sum_{n=1}^{\infty} C_n n x^{n-1} + \sum_{n=2}^{\infty} C_{n-2} x^{n-1} = 0$$

$$2C_1 + \sum_{n=2}^{\infty} [C_n n(n-1) + 2C_n n + C_{n-2}] x^{n-1} = 0$$

$$C_0 \text{ poljuben}, \quad C_1 = 0, \quad C_n = \frac{-1}{n(n+1)} C_{n-2}, \quad n = 2, 3, \dots$$

$$C_1 = C_3 = C_5 = \dots = 0$$

$$C_{2n} = \frac{(-1)^n}{(2n+1)!}$$

$$y = C_0 \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n}$$

$$y = C_0 \frac{\sin x}{x}$$

## 3. naloga

$$u_x = u_v + u_z y$$

$$u_{xy} = u_{vz} x + u_{zz} y x + u_z$$

$$u_y = u_z x$$

$$u_{yy} = u_{zz} x^2$$

$$x^2 u_{vz} + x^2 y u_{zz} + x u_z = x^2 y u_{zz} + x u_z$$

$$u_{vz} = 0$$

$$u_v = f_1(v)$$

$$u = \int f_1(v) dv = f(v) + g(z)$$

$$u(x, y) = f(x) + g(xy)$$

#### 4. naloga

$$2y - 2 \sin x - (-2y)' = 0$$

$$y'' + y = \sin x$$

$$r^2 + 1 = 0$$

$$r_{1,2} = \pm i$$

$$y_h = A \cos x + B \sin x$$

$$y_p = (C \sin x + D \cos x)x$$

$$y'_p = (C \cos x - D \sin x)x + (C \sin x + D \cos x)$$

$$y''_p = (-C \sin x - D \cos x)x + 2(C \cos x - D \sin x)$$

$$2(C \cos x - D \sin x) = \sin x$$

$$C = 0, D = -\frac{1}{2}$$

$$y_p = -\frac{x}{2} \cos x$$

$$y = A \cos x + B \sin x - \frac{x}{2} \cos x$$

#### 5. naloga

$$P(\text{deljivo z 2 ali s 3}) = P(A \cup B) = P(A) + P(B) - P(A \cap B) =$$

$$P(\text{deljivo z 2}) + P(\text{deljivo s 3}) - P(\text{deljivo s 6}) = \frac{25}{50} + \frac{16}{50} - \frac{8}{50} = \boxed{\frac{33}{50}}$$