Izpit Matematika IV

18.september 2006

Rešitve

1. naloga

$Y(s) = \frac{1}{s^2} + 2\frac{s}{s^2+1}Y(s)$

$\big[s^2(s^2+1) - 2s^3\big]Y(s) = s^2+1$

$Y(s) = \dfrac{s^2+1}{s^2(s-1)^2}$

$\operatorname*{Res}_{s=0} = \lim_{s\to 0}\Big[\dfrac{(s^2+1)e^{st}}{(s-1)^2}\Big]' = \lim_{s\to 0}\dfrac{[2se^{st} + (s^2+1)e^{st}t](s-1)^2 - (s^2+1)e^{st}2(s-1)}{(s-1)^4} = t+2$

$\operatorname*{Res}_{s=1} = \lim_{s\to 1}\Big[\dfrac{(s^2+1)e^{st}}{s^2}\Big]' = \lim_{s\to 1}\Big[\big(1+\dfrac{1}{s^2}\big)e^{st}\Big]' = \lim_{s\to 1}\Big[\dfrac{-2}{s^3}e^{st} + \big(1+\dfrac{1}{s^2}\big)e^{st}t\Big] = -2e^t + 2te^t$

$$\boxed{y(t) = (t+2) + 2(t-1)e^t}$$

2. naloga

$\sum_{n=1}^{\infty} C_n n x^{n-1} = \sum_{n=0}^{\infty} C_n x^n + x^2$

$\sum_{n=0}^{\infty} C_{n+1}(n+1)x^n = \sum_{n=0}^{\infty} C_n x^n + x^2$

$C_3 3 = C_2 + 1$

Rekurzijska formula $C_{n+1} = \frac{C_n}{n+1}$ velja za vse $n = 0, 1, 3, 4, \ldots$ razen za $n = 2$

$y(0) = -2 \quad \to \quad C_0 = -2$

$C_1 = \frac{C_0}{1} = -2$

$C_2 = \frac{C_1}{2} = -1$

$C_3 3 = C_2 + 1 \quad \to \quad C_3 = 0$

$C_4 = C_5 = \cdots = 0$

$$\boxed{y = -(2 + 2x + x^2)}$$

3. naloga

$u = F(r)G(\theta)$

$F''G + \frac{1}{r}F'G + \frac{1}{r^2}FG'' = 0$

$r^2\frac{F''}{F} + r\frac{F'}{F} = -\frac{G''}{G} = \lambda^2$

$G'' + \lambda^2 G = 0$

$G = A\cos(\lambda\theta) + B\sin(\lambda\theta)$

$G(\theta + 2\pi) = G(\theta) = \quad \rightarrow \quad \lambda = n \ , \ n = 0, 1, 2, \ldots$

$G_n(\theta) = A_n\cos(n\theta) + B_n\sin(n\theta)$

$G_0(\theta) = A_0$

$r^2F'' + rF' - n^2F = 0$

To je *Eulerjeva* dif. enačba, ki se jo reši z nastavkom $F = r^\alpha$

$r^2\alpha(\alpha-1)r^{\alpha-2} + r\alpha r^{\alpha-1} - n^2r^\alpha = 0$

$\alpha(\alpha-1) + \alpha - n^2 = 0$

$\alpha^2 = n^2$

$\alpha_{1,2} = \pm n$

$F_n(r) = C_nr^n + D_nr^{-n}$

$F_0(r) = C_0 + D_0\ln r$

$$u(r,\theta) = (c_0 + d_0\ln r) + \sum_{n=1}^{\infty}[A_n\cos(n\theta) + B_n\sin(n\theta)][C_nr^n + D_nr^{-n}]$$

$r = 0 \quad \rightarrow \quad d_0 = D_n = 0$

lahko še izberemo $C_n = 1$

$$r = 1 \quad \rightarrow \quad c_0 + \sum_{n=1}^{\infty}[A_n\cos(n\theta) + B_n\sin(n\theta)] = \sin^3\theta = \tfrac{3}{4}\sin\theta - \tfrac{1}{4}\sin(3\theta)$$

$c_0 = 0$, $A_n = 0$, vsi $B_n = 0$ razen $B_1 = \frac{3}{4}$, $B_3 = -\frac{1}{4}$

$$\boxed{u(r,\theta) = \frac{3}{4}r\sin\theta - \frac{1}{4}r^3\sin(3\theta)}$$

4. naloga

$2y - 2\operatorname{ch} x - (-2y')' = 0$
$y'' + y = \operatorname{ch} x$
$\lambda^2 + 1 = 0$
$\lambda_{1,2} = \pm i$
$y_h = A\cos x + B\sin x$

$y_p = C\operatorname{ch} x$
$y_p'' = C\operatorname{ch} x$
$\operatorname{ch} x + \operatorname{ch} x = \operatorname{ch} x$
$A = \frac{1}{2}$

$$\boxed{y = A\cos x + B\sin x + \frac{1}{2}\operatorname{ch} x}$$

5. naloga

$$P = P(ZZZ) + P(\check{C}\check{C}\check{C}) = \tfrac{4}{10} \cdot \tfrac{3}{9} \cdot \tfrac{2}{8} + \tfrac{6}{10} \cdot \tfrac{5}{9} \cdot \tfrac{4}{8} = \boxed{\frac{1}{5}}$$