Izpit Matematika IV

26.junij 2006

Rešitve

1. naloga

$$F(s) = \frac{A}{s+1} + \frac{B}{s-2} + \frac{C}{s-3} = \frac{A(s-2)(s-3) + B(s+1)(s-3) + C(s+1)(s-2)}{(s+1)(s-2)(s-3)}$$

$$A(s-2)(s-3) + B(s+1)(s-3) + C(s+1)(s-2) = 2s^2 - 4$$

$$s = -1 \quad \rightarrow \quad 12A = -2$$
$$s = 2 \quad \rightarrow \quad -3B = 4$$
$$s = 3 \quad \rightarrow \quad 4C = 14$$

$$\boxed{f(t) = -\frac{1}{6}e^{-t} - \frac{4}{3}e^{2t} + \frac{7}{2}e^{3t}}$$

2. naloga

$r^2 + \lambda = 0$
$r_{1,2} = \pm\sqrt{-\lambda}$

a) primer $\lambda = -k^2 < 0$
$y = A\,\mathrm{ch}(kx) + B\,\mathrm{sh}(kx)$
$x = 0 \quad \rightarrow \quad A = 0$
$x = 1 \quad \rightarrow \quad Bk\,\mathrm{ch}(k) = 0 \quad \rightarrow \quad B = 0$
trivialna rešitev

b) primer $\lambda = 0$
$y = Ax + B$
$x = 0 \quad \rightarrow \quad B = 0$
$x = 1 \quad \rightarrow \quad A = 0$
trivialna rešitev

c) primer $\lambda = k^2 > 0$
$y = A\cos(kx) + B\sin(kx)$
$x = 0 \quad \rightarrow \quad A = 0$
$x = 1 \quad \rightarrow \quad Bk\cos(k) = 0 \quad \rightarrow \quad k = (n + \frac{1}{2})\pi\ ,\ n \in Z$

$$\boxed{\lambda = \left[(n + \frac{1}{2})\pi\right]^2\ ,\ n \in Z}$$

3. naloga

Označimo $\mathcal{L}\big[u(x,t)\big] = U(x,s)$
Problem za neznanko $u(x,t)$ se transfomira v problem za neznanko $U(x,s)$:

$$x\frac{\partial U}{\partial x} + sU - 0 = \frac{x}{s^2} \quad , \quad U(0,s) = 0$$

To rešimo kot navadno dif. enačbo in na s gledamo kot konstanto.
Je linearna dif.enačba 1.reda; najprej homogena in nato variacija konstante.

$$x\frac{dU}{dx} = -sU$$
$$\int \frac{dU}{U} = \int -\frac{s}{x}dx$$
$$\ln U = -s\ln x + \ln C$$
$$U_h = \frac{C}{x^s}$$

$$U_p = \frac{C(x)}{x^s}$$
$$x\frac{C'}{x^s} + x\frac{C(-s)}{x^{s+1}} + s\frac{C}{x^s} = \frac{x}{s^2}$$
$$C' = \frac{x^s}{s^2}$$
$$C = \frac{x^{s+1}}{(s+1)s^2}$$
$$U_p = \frac{x}{(s+1)s^2}$$
$$U(x,s) = U_h + U_p = \frac{C}{x^s} + \frac{x}{(s+1)s^2}$$
$$x = 0 \quad \rightarrow \quad C = 0$$
$$u(x,t) = \mathcal{L}^{-1}\Big[U(x,s)\Big] = \mathcal{L}^{-1}\Big[\frac{x}{(s+1)s^2}\Big]$$

$$\operatorname*{Res}_{s=-1} = \lim_{s\to-1}\frac{xe^{st}}{s^2} = xe^{-t}$$
$$\operatorname*{Res}_{s=0} = \lim_{s\to 0}\Big[\frac{xe^{st}}{s+1}\Big]' = \lim_{s\to 0}\frac{xe^{st}t(s+1) - xe^{st}}{(s+1)^2} = xt - x$$

$$\boxed{u(x,t) = x(t - 1 + e^{-t})}$$

4. naloga

$$0 - (1 + x^2 y')' = 0$$
$$1 + x^2 y' = A = 1 - 2B$$
$$y' = -\frac{B}{x^2}$$

$$\boxed{y = \frac{B}{x} + C}$$

5. naloga

$$\boxed{P = \left(\frac{92}{100}\right)^3}$$