Izpit Matematika IV

14.junij 2007

Rešitve

1. naloga

$$F(s) = \int\limits_{s}^{\infty} \left(\frac{1}{s+a} - \frac{1}{s+b} \right) ds = \ln \frac{s+a}{s+b} \Bigg|_{s}^{\infty} = \boxed{\frac{s+b}{s+a}}$$

2. naloga

$$y = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + \ldots$$
$$C_0 = y(0) = 0$$
$$C_1 = y'(0) = 1$$

$$y = x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + \ldots$$
$$y'' = 2C_2 + 6C_3 x + 12C_4 x^2 + 20C_5 x^3 + \ldots$$
$$xy'' + y = (2C_2 + 1)x + (6C_3 + C_2)x^2 + (12C_4 + C_3)x^3 + (20C_5 + C_4)x^4 + \ldots$$

$$C_2 = -\frac{1}{2}$$
$$C_3 = -\frac{1}{6}C_2 = \frac{1}{12}$$
$$C_4 = -\frac{1}{12}C_3 = -\frac{1}{144}$$
$$C_5 = -\frac{1}{20}C_4 = \frac{1}{2880}$$

$$\boxed{y = x - \frac{x^2}{2} + \frac{x^3}{12} - \frac{x^4}{144} + \frac{x^5}{2880} - \ldots}$$

3. naloga

$u = F(x)G(y)$

$F''G + FG'' = 0$

$\dfrac{F''}{F} = -\dfrac{G''}{G} = \lambda^2$

$G'' + \lambda^2 G = 0$

$G(y) = A\cos(\lambda y) + B\sin(\lambda y)$

$y = 0 \quad \rightarrow \quad A = 0$

$y = b \quad \rightarrow \quad \sin(\lambda b) = 0 \quad \rightarrow \quad \lambda_n = \dfrac{n\pi}{b}$

$G_n(y) = B_n\sin(\lambda_n y) , \ n = 1, 2, \ldots$

$F'' - \lambda_n^2 F = 0$

$F_n(x) = C_n\,\mathrm{ch}(\lambda_n x) + D_n\,\mathrm{sh}(\lambda_n x)$

$$u(x, y) = \sum_{n=1}^{\infty} \Big[c_n\,\mathrm{ch}(\lambda_n x) + d_n\,\mathrm{sh}(\lambda_n x) \Big] \sin(\lambda_n y)$$

$x = 0 \quad \rightarrow \quad \displaystyle\sum_{n=1}^{\infty} c_n\sin\frac{n\pi y}{b} \quad$ je *Fourier sinusna* vrsta za konstanto 0.

Zato $c_n = 0$

$x = a \quad \rightarrow \quad \displaystyle\sum_{n=1}^{\infty} d_n\,\mathrm{sh}\frac{n\pi a}{b}\sin\frac{n\pi y}{b} \quad$ je *Fourier sinusna* vrsta za funkcijo $A\sin\frac{\pi y}{b}$.

Zato $d_1\,\mathrm{sh}\frac{\pi a}{b} = A$ in ostali $d_n = 0$.

$$\boxed{\,u(x, y) = \frac{A}{\mathrm{sh}\frac{\pi a}{b}}\,\mathrm{sh}\frac{\pi x}{b}\sin\frac{\pi y}{b}\,}$$

4. naloga

$-(1 + x^2 2y')' = 0$

$1 + x^2 2y' = C = 1 - 2A$

$y' = -\dfrac{A}{x^2}$

$$\boxed{\,y = \frac{A}{x} + B\,}$$

5. naloga

Izberemo si označbe:

$H_{00} = (\text{1.zgreši}) \text{ in } (\text{2.zgreši})$

$H_{01} = (\text{1.zgreši}) \text{ in } (\text{2.zadane})$

$H_{10} = (\text{1.zadane}) \text{ in } (\text{2.zgreši})$

$H_{11} = (\text{1.zadane}) \text{ in } (\text{2.zadane})$

$A = (\text{cilj je enkrat zadet})$

$$P(A) = \sum_i P(H_i)P(A/H_i) =$$

$$P(H_{00}) \cdot P(A/H_{00}) + P(H_{01}) \cdot P(A/H_{01}) + P(H_{10}) \cdot P(A/H_{10}) + P(H_{11}) \cdot P(A/H_{11}) =$$

$$0,2 \cdot 0,4 \cdot 0 + 0,2 \cdot 0,6 \cdot 1 + 0,8 \cdot 0,4 \cdot 1 + 0,8 \cdot 0,6 \cdot 0 = 0,44$$

$$P(H_{10}/A) = \frac{P(H_{10})P(A/H_{10})}{P(A)} = \frac{0,8 \cdot 0,4 \cdot 1}{0,44} = \boxed{\frac{8}{11}}$$