Izpit Matematika IV

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Rešitve

1. naloga

Z uporabo pravila $\mathcal{L}[f(t-a)u_a(t)] = F(s)e^{-as}$ upoštevamo faktor $e^{-\pi s}$ nazadnje.
Prej poiščemo $\mathcal{L}^{-1}$ funkcij $G(s)$ in $H(s)$.

$$G(s) = \frac{s+1}{s^2+s+1} = \frac{(s+\frac{1}{2})+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}} = H(s+\frac{1}{2}) \quad , \quad H(s) = \frac{s+\frac{1}{2}}{s^2+\frac{3}{4}}$$

Poiščemo najprej $\mathcal{L}^{-1}\Big[H(s)\Big]$ in uporabimo pravilo $\mathcal{L}^{-1}\Big[H(s-a)\Big] = e^{-at}h(t)$.

$$h(t) = \mathcal{L}^{-1}\Big[\frac{s+\frac{1}{2}}{s^2+\frac{3}{4}}\Big] = \mathcal{L}^{-1}\Big[\frac{s}{s^2+(\sqrt{3}/2)^2} + \frac{1}{2}\frac{2}{\sqrt{3}}\frac{\sqrt{3}/2}{s^2+(\sqrt{3}/2)^2}\Big] = \cos\Big(\frac{\sqrt{3}}{2}t\Big) + \frac{1}{\sqrt{3}}\sin\Big(\frac{\sqrt{3}}{2}t\Big)$$

$$g(t) = \mathcal{L}^{-1}\Big[H(s+\frac{1}{2})\Big] = e^{-t/2}\Big[\cos\Big(\frac{\sqrt{3}}{2}t\Big) + \frac{1}{\sqrt{3}}\sin\Big(\frac{\sqrt{3}}{2}t\Big)\Big]$$

$$\boxed{f(t) = e^{-(t-\pi)/2}\Big[\cos\Big(\frac{\sqrt{3}}{2}(t-\pi)\Big) + \frac{1}{\sqrt{3}}\sin\Big(\frac{\sqrt{3}}{2}(t-\pi)\Big)\Big]\cdot u_\pi(t)}$$

2. naloga

$$y = \sum_{n=0}^{\infty} C_n x^n$$

$$\sum_{n=2}^{\infty} C_n n(n-1)x^{n-2} + \sum_{n=0}^{\infty} C_n x^{n+1} = 0$$

S ciljem enotne potence x^n v obeh vrstah zamaknemo sumacijska indeksa.

$$\sum_{n=0}^{\infty} C_{n+2}(n+2)(n+1)x^n + \sum_{n=1}^{\infty} C_{n-1}x^n = 0$$

$$2C_2 + \sum_{n=1}^{\infty} \Big[C_{n+2}(n+2)(n+1) + C_{n-1}\Big]x^n = 0$$

$$C_2 = 0 \quad , \quad C_{n+2} = \frac{-1}{(n+2)(n+1)}C_n \ , \ n = 1, 2, \dots$$

$$C_2 = 0 \quad \to \quad C_5 = C_8 = C_{11} = \cdots = 0$$

$$C_1 = y'(0) = 0 \quad \to \quad C_4 = C_7 = C_{10} = \cdots = 0$$

$$C_0 = y(0) = 1 \ , \ C_3 = -\tfrac{1}{6} \ , \ C_6 = \tfrac{1}{180} \ , \ \dots$$

$$\boxed{y = 1 - \frac{1}{6}x^3 + \frac{1}{180}x^6 - \dots}$$

3. naloga

$$u_x = u_v + 3u_z$$
$$u_{xx} = (u_v + 3u_z)_v + 3(u_v + 3u_z)_z = u_{vv} + 6u_{vz} + 9u_{zz}$$
$$u_{xy} = (u_v + 3u_z)_v + (u_v + 3u_z)_z = u_{vv} + 4u_{vz} + 3u_{zz}$$
$$u_y = u_v + u_z$$
$$u_{yy} = (u_v + u_z)_v + (u_v + u_z)_z = u_{vv} + 2u_{vz} + u_{zz}$$

$$u_{vv} + 6u_{vz} + 9u_{zz} - 4(u_{vv} + 4u_{vz} + 3u_{zz}) + 3(u_{vv} + 2u_{vz} + u_{zz}) = 0$$
$$u_{vv} + 6u_{vz} + 9u_{zz} - 4u_{vv} - 16u_{vz} - 12u_{zz} + 3u_{vv} + 6u_{vz} + 3u_{zz} = 0$$
$$-4u_{vz} = 0$$

$$u_v = f_1(v)$$
$$u = \int f_1(v)dv = f(v) + g(z)$$

$$\boxed{u(x,y) = f(x+y) + g(x+3y)}$$

4. naloga

$2y - 2\sin x - (-2y')' = 0$
$y'' + y = \sin x$

$r^2 + 1 = 0$
$r_{1,2} = \pm i$
$y_h = A\cos x + B\sin x$

$y_p = (C\sin x + D\cos x)x$
$y_p' = (C\cos x - D\sin x)x + (C\sin x + D\cos x)$
$y_p'' = (-C\sin x - D\cos x)x + 2(C\cos x - D\sin x)$
$2(C\cos x - D\sin x) = \sin x$
$C = 0\ ,\ D = -\frac{1}{2}$
$y_p = -\frac{x}{2}\cos x$

$$\boxed{y = A\cos x + B\sin x - \frac{x}{2}\cos x}$$

5. naloga

$p = P(\text{ pade stran A }) = \frac{2}{3}$

Poskus ponovimo n=3 krat, najti je treba verjetnost, da se dogodek zgodi k=1 krat.

$$P = \binom{n}{k} p^k (1-p)^{n-k} = \binom{3}{1}\frac{2}{3}\left(\frac{1}{3}\right)^2 = \boxed{\frac{2}{9}}$$