Izpit Matematika IV

20.6.2011

Rešitve

1. naloga

$F(s) = \frac{s+1}{s^2(s^2+9)} = \frac{As+B}{s^2} + \frac{Cs+D}{s^2+9}$
$s+1 = (As+B)(s^2+9) + (Cs+D)s^2$
$s+1 = s^3(A+C) + s^2(B+D) + 9As + 9B$
$B = \frac{1}{9}\ ,\ A = \frac{1}{9}\ ,\ C = -\frac{1}{9}\ ,\ D = -\frac{1}{9}$

$$\boxed{f(t) = \frac{1}{9}\Big(1 + t - \cos 3t - \frac{1}{3}\sin 3t\Big)}$$

2. naloga

$y' = u'\sqrt{x} + u\frac{1}{2}x^{-\frac{1}{2}}$
$y'' = u''\sqrt{x} + 2u'\frac{1}{2}x^{-\frac{1}{2}} - u\frac{1}{4}x^{-\frac{3}{2}}$
$x^2\sqrt{x}u'' + x\sqrt{x}u' - \frac{1}{4}u\sqrt{x} + x^2\sqrt{x}u + \frac{1}{4}u\sqrt{x} = 0 \qquad / : \sqrt{x}$
$x^2u'' + xu' + (x^2 - 0)u = 0$
$u = A\mathcal{J}_0(x) + B\mathcal{Y}_0(x)$

$$\boxed{y = \sqrt{x}\big(A\mathcal{J}_0(x) + B\mathcal{Y}_0(x)\big)}$$

3. naloga

$u_y = v$
$xv'_x = v + y$
$\frac{dv}{v+y} = \frac{dx}{x}$
$\ln(v+y) = \ln x + \ln C$
$v + y = Cx$

$u_y = C(y)x - y$
$x = 1 \quad \rightarrow \quad C(y) - y = y$
$C(y) = 2y$
$u_y = 2xy - y$

$u = \int (2xy - y)dy = xy^2 - y^2/2 + C(x)$
$y = 1 \quad \rightarrow \quad x - \frac{1}{2} + C(x) = x$
$C(x) = \frac{1}{2}$

$\boxed{u(x,y) = xy^2 + \frac{1-y^2}{2}}$

4. naloga

$2y - 2x - (-2y')' = 0$
$y'' + y = x$
$\lambda^2 + 1 = 0$
$\lambda_{1,2} = \pm i$
$y_h = A\cos x + B\sin x$

$y_p = Cx + D$
$Cx + D = x$
$y_p = x$

$y = A\cos x + B\sin x + x$
$x = 0 \quad \rightarrow \quad A = 2$
$x = \frac{\pi}{2} \quad \rightarrow \quad B + \frac{\pi}{2} = \frac{\pi}{2} \quad \rightarrow \quad B = 0$

$\boxed{y = 2\cos x + x}$

5. naloga

$E(X) = \frac{1}{36}\big(1 \cdot 1 + 2 \cdot 3 + 3 \cdot 5 + 4 \cdot 7 + 5 \cdot 9 + 6 \cdot 11\big) =$

$\frac{1}{36}\big(1 + 6 + 15 + 28 + 45 + 66\big) = \frac{161}{36}$

$E(X^2) = \frac{1}{36}\big(1 \cdot 1 + 4 \cdot 3 + 9 \cdot 5 + 16 \cdot 7 + 25 \cdot 9 + 36 \cdot 11\big) = \frac{791}{36}$

$D(X) = E(X^2) - E^2(X) = \frac{791}{36} - \frac{161^2}{36^2} = \boxed{\frac{2555}{1296}}$