Rešitve kolokvija MATEMATIKE IV

5.4.2007

skupina A

1. naloga.

$$
\begin{aligned}
x'' + 4x' + 13x &= 0 \\
s^2 X - 3s + 4(sX - 3) + 13X &= 0 \\
(s^2 + 4s + 13)X &= 3s + 12 \\
X &= \frac{3(s+2)+6}{(s+2)^2+9} \\
x(t) &= e^{-2t}(3\cos 3t + 2\sin 3t)
\end{aligned}
$$

2. naloga.

$$
y' = u + xu' 2x = u + 2x^2 u'
$$

$$
y'' = u' 2x + 4xu' + 2x^2 u'' 2x = 6xu' + 4x^3 u''
$$

$$
\begin{aligned}
6x^3 u' + 4x^5 u'' - xu - 2x^3 u' + (4x^3 - 3)xu &= 0 \\
4x^4 u'' + 4x^2 u' + (4x^4 - 4)u &= 0 \\
z^2 u'' + zu' + (z^2 - 1)u &= 0 \\
u &= \mathcal{J}_1(z) \\
y &= x\mathcal{J}_1(x^2)
\end{aligned}
$$

3. naloga.

$$
\int_0^p (qx - 1)x\, dx = q\frac{p^3}{3} - \frac{p^2}{2} \longrightarrow \qquad q\frac{p}{3} = \frac{1}{2}
$$

$$
\int_0^p (15\sqrt{x} - q)x\, dx = 15\frac{2\sqrt{p^5}}{5} - q\frac{p^2}{2} \longrightarrow \quad 6\sqrt{p} = \frac{q}{2}
$$

$$
\text{zmnoži} \quad \text{oboje}
$$

$$
\begin{aligned}
2\sqrt{p^3} &= \tfrac{1}{4} \\
p &= \tfrac{1}{4} \\
q &= 6
\end{aligned}
$$

skupina B

1. naloga.

$$
\begin{aligned}
x'' + 6x' + 13x &= 0 \\
s^2 X - 2s + 6(sX - 2) + 13X &= 0 \\
(s^2 + 6s + 13)X &= 2s + 12 \\
X &= \frac{2(s+3)+6}{(s+3)^2+4} \\
x(t) &= e^{-3t}(2\cos 2t + 3\sin 2t)
\end{aligned}
$$

2. naloga.

$$
y' = u + xu'\left(-\frac{1}{x^2}\right) = u - \frac{u'}{x}
$$

$$
y'' = u'\left(-\frac{1}{x^2}\right) - \frac{u''\left(-\frac{1}{x^2}\right)x - u'}{x^2} = \frac{u''}{x^3}
$$

$$
\begin{aligned}
\frac{u''}{x^2} - u + \frac{u'}{x} + \frac{u}{x^2} &= 0 \\
z^2 u'' + z u' + (z^2 - 1)u &= 0 \\
u &= \mathcal{J}_1(z) \\
y &= x\mathcal{J}_1\left(\tfrac{1}{x}\right)
\end{aligned}
$$

3. naloga.

$$
\int_0^b (x - a)x\, dx = \frac{b^3}{3} - a\frac{b^2}{2}
$$

$$
\int_0^b \left(\frac{4}{\sqrt{x}} - a\right)x\, dx = 4\frac{2\sqrt{b^3}}{3} - a\frac{b^2}{2}
$$

izenači   oboje

$$
\begin{aligned}
\frac{b^3}{3} &= \tfrac{8}{3}\sqrt{b^3} \\
\sqrt{b^3} &= 8 \\
b &= 4 \\
a &= \tfrac{8}{3}
\end{aligned}
$$