1. Kolokvij iz MATEMATIKE IV

10.4.2008

skupina A

1. (4 točke) Z uporabo *Laplace*-ove transformacije poišči rešitev $y(t)$ diferencialne enačbe

$$\begin{aligned} y''' - 4y'' + 5y' &= 0 \\ y(0) &= 0 \\ y'(0) &= 2 \\ y''(0) &= 3 \end{aligned}$$

2. (3 točke) Reši diferencialno enačbo

$$y'' + \operatorname{ctg} x \; y' + 2y = 0$$

z vpeljavo neodvisne spremenljivke $t = \cos x$!

3. (3 točke) Z metodo separacije spremenljivk reši diferencialno enačbo

$$u_x + y\, u_y + u = 0$$

skupina B

1. (4 točke) Z uporabo *Laplace*-ove transformacije poišči rešitev $y(t)$ diferencialne enačbe

$$\begin{aligned} y''' - 2y'' + 5y' &= 0 \\ y(0) &= 0 \\ y'(0) &= 1 \\ y''(0) &= -3 \end{aligned}$$

2. (3 točke) Reši diferencialno enačbo

$$y'' - \operatorname{tg} x \; y' + 2y = 0$$

z vpeljavo neodvisne spremenljivke $t = \sin x$!

3. (3 točke) Z metodo separacije spremenljivk reši diferencialno enačbo

$$x\, u_x + u_y + u = 0$$

Rešitve - skupina A

1. naloga.

$$\begin{aligned} y''' - 4y'' + 5y' &= 0 \\ s^3Y - 2s - 3 - 4(s^2Y - 2) + 5sY &= 0 \\ (s^3 - 4s^2 + 5s)Y &= 2s - 5 \\ Y &= \tfrac{2s-5}{s(s^2-4s+5)} \\ Y &= \tfrac{A}{s} + \tfrac{Bs+C}{s^2-4s+5} \\ 2s - 5 &= A(s^2 - 4s + 5) + (Bs + C)s \end{aligned}$$

$$\begin{aligned} A + B &= 0 \\ -4A + C &= 2 \\ 5A &= -5 \end{aligned}$$

$$\begin{aligned} A &= -1 \\ B &= 1 \\ C &= -2 \end{aligned}$$

$$\begin{aligned} Y &= \tfrac{-1}{s} + \tfrac{(s-2)}{(s-2)^2+1} \\ x(t) &= e^{2t}\cos t - 1 \end{aligned}$$

2. naloga.

$$y' = \dot{y}(-\sin x)$$

$$y'' = \ddot{y}\sin^2 x - \dot{y}\cos x$$

$$\ddot{y}\sin^2 x - \dot{y}\cos x + \frac{\cos x}{\sin x}\dot{y}(-\sin x) + 2y = 0$$

$$(1-t^2)\ddot{y} - 2t\dot{y} + 2y = 0$$

$$Legendre - \text{ova dif. en. } n = 1$$

$$y = P_1(t) = t$$

$$y = \cos x$$

3. naloga.

$$u(x,y) = F(x)G(y)$$

$$F'(x)G(y) + yF(x)G'(y) + F(x)G(y) = 0$$

$$\frac{F'(x)}{F(x)} + y\frac{G'(y)}{G(y)} = -1$$

$$y\frac{G'(y)}{G(y)} = -\frac{F'(x)}{F(x)} - 1 = \lambda$$

$$y\frac{G'(y)}{G(y)} = \lambda$$

$$\frac{dG}{G} = \lambda\frac{dy}{y}$$

$$\ln G = \lambda \ln y + \ln C$$

$$G(y) = Cy^{\lambda}$$

$$\frac{F'(x)}{F(x)} = -(\lambda + 1)$$

$$\frac{dF}{F} = -(\lambda + 1)dx$$

$$\ln F = -(\lambda + 1)x + \ln D$$

$$F(x) = De^{-(\lambda+1)x}$$

$$u(x,y) = Ay^{\lambda}e^{-(\lambda+1)x}$$

Rešitve - skupina B

1. naloga.

$$\begin{aligned} y''' - 2y'' + 5y' &= 0 \\ s^3Y - s + 3 - 2(s^2Y - 1) + 5sY &= 0 \\ (s^3 - 2s^2 + 5s)Y &= s - 5 \\ Y &= \tfrac{s-5}{s(s^2-2s+5)} \\ Y &= \tfrac{A}{s} + \tfrac{Bs+C}{s^2-2s+5} \\ s - 5 &= A(s^2 - 2s + 5) + (Bs + C)s \end{aligned}$$

$$\begin{aligned} A + B &= 0 \\ -2A + C &= 1 \\ 5A &= -5 \end{aligned}$$

$$\begin{aligned} A &= -1 \\ B &= 1 \\ C &= -1 \end{aligned}$$

$$\begin{aligned} Y &= \tfrac{-1}{s} + \tfrac{(s-1)}{(s-1)^2+4} \\ x(t) &= e^t \cos 2t - 1 \end{aligned}$$

2. naloga.

$$y' = \dot{y}(\cos x)$$

$$y'' = \ddot{y}\cos^2 x - \dot{y}\sin x$$

$$\ddot{y}\cos^2 x - \dot{y}\sin x - \frac{\sin x}{\cos x}\dot{y}(\cos x) + 2y = 0$$

$$(1-t^2)\ddot{y} - 2t\dot{y} + 2y = 0$$

$$Legendre - \text{ova dif. en. } n = 1$$

$$y = P_1(t) = t$$

$$y = \sin x$$

3. naloga.

$$u(x,y) = F(x)G(y)$$

$$xF'(x)G(y) + F(x)G'(y) + F(x)G(y) = 0$$

$$x\frac{F'(x)}{F(x)} + \frac{G'(y)}{G(y)} = -1$$

$$x\frac{F'(x)}{F(x)} = -\frac{G'(y)}{G(y)} - 1 = \lambda$$

$$\frac{F'(x)}{F(x)} = \frac{\lambda}{x}$$

$$\frac{dF}{F} = \frac{\lambda}{x}dx$$

$$\ln F = \lambda \ln x + \ln C$$

$$F(x) = Cx^{\lambda}$$

$$\frac{G'(y)}{G(y)} = -(\lambda + 1)$$

$$\frac{dG}{G} = -(\lambda + 1)dy$$

$$\ln G = -(\lambda + 1)y + \ln D$$

$$G(y) = De^{-(\lambda+1)y}$$

$$u(x,y) = Ax^{\lambda}e^{-(\lambda+1)y}$$