1. Kolokvij iz MATEMATIKE IV

2.4.2009

skupina A

1. (40%) Z uporabo *Laplace*-ove transformacije poišči rešitev $y(t)$ diferencialne enačbe

$$\begin{aligned} y'' - \tfrac{5}{2}y' + y &= 1 \\ y(0) &= 0 \\ y'(0) &= 4 \end{aligned}$$

2. (30%) Rešuj diferencialno enačbo

$$\begin{aligned} 2y' - xy' - y &= 0 \\ y(0) &= 1 \end{aligned}$$

z nastavkom $y = \sum_{n=0}^{\infty} C_n x^n$!

(a) (20%) Poišči rekurzijsko formulo za koeficiente

(b) (10%) Izračunaj vsoto vrste

3. (30%) Poišči splošno rešitev diferencialne enačbe

$$x^2y'' + xy' + 4(x^4 - 2)y = 0$$

z vpeljavo neodvisne spremenljivke $x^2 = z$!

skupina B

1. (40%) Z uporabo *Laplace*-ove transformacije poišči rešitev $y(t)$ diferencialne enačbe

$$\begin{aligned} 2y'' - 3y' + y &= 1 \\ y(0) &= 0 \\ y'(0) &= 2 \end{aligned}$$

2. (30%) Rešuj diferencialno enačbo

$$\begin{aligned} xy' + 2y' + y &= 0 \\ y(0) &= 1 \end{aligned}$$

   z nastavkom $y = \sum_{n=0}^{\infty} C_n x^n$!

   (a) (20%) Poišči rekurzijsko formulo za koeficiente

   (b) (10%) Izračunaj vsoto vrste

3. (30%) Poišči splošno rešitev diferencialne enačbe

$$x^2 y'' + xy' + 9(x^6 - 3)y = 0$$

   z vpeljavo neodvisne spremenljivke $x^3 = z$!

Rešitve - skupina A

1. naloga.

$$\begin{aligned}
y'' - \tfrac{5}{2}y' + y &= 1 \\
s^2Y - 4 - \tfrac{5}{2}sY + Y &= \tfrac{1}{s} \\
(s^2 - \tfrac{5}{2}s + 1)Y &= 4 + \tfrac{1}{s} \\
Y &= \tfrac{4s+1}{s(s-2)(s-\frac{1}{2})} \\
Y &= \tfrac{A}{s} + \tfrac{B}{s-2} + \tfrac{C}{s-\frac{1}{2}} \\
4s + 1 &= A(s-2)(s-\tfrac{1}{2}) + Bs(s-\tfrac{1}{2}) + Cs(s-2)
\end{aligned}$$

$$\begin{aligned}
s = 2 &\rightarrow 9 = 3B \\
s = 0 &\rightarrow 1 = A \\
s = \tfrac{1}{2} &\rightarrow 3 = -\tfrac{3}{4}C
\end{aligned}$$

$$\begin{aligned}
Y &= \tfrac{1}{s} + \tfrac{3}{(s-2)} - \tfrac{4}{s-\frac{1}{2}} \\
y(t) &= 1 + 3e^{2t} - 4e^{t/2}
\end{aligned}$$

2. naloga.

$$y = \sum_{n=0}^{\infty} C_n x^n$$

$$y' = \sum_{n=1}^{\infty} C_n n x^{n-1}$$

$$2\sum_{n=1}^{\infty} C_n n x^{n-1} - \sum_{n=1}^{\infty} C_n n x^n - \sum_{n=0}^{\infty} C_n x^n = 0$$

$$2\sum_{n=0}^{\infty} C_{n+1}(n+1) x^n - \sum_{n=1}^{\infty} C_n n x^n - \sum_{n=0}^{\infty} C_n x^n = 0$$

$$2\sum_{n=0}^{\infty} C_{n+1}(n+1) x^n - \sum_{n=0}^{\infty} C_n n x^n - \sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=0}^{\infty} [2C_{n+1}(n+1) - C_n n - C_n] x^n = 0$$

$$2C_{n+1}(n+1) - C_n n - C_n = 0 \qquad , \quad n = 0, 1, 2, \ldots$$

(a)

$$C_0 = 1$$

$$C_{n+1} = C_n/2 \quad , \quad n = 0, 1, 2, \ldots$$

(b)

$$y = 1 + \tfrac{x}{2} + (\tfrac{x}{2})^2 + (\tfrac{x}{2})^3 + \cdots$$

$$y = \tfrac{1}{1-\frac{x}{2}} = \tfrac{2}{2-x}$$

3. naloga.

$$y' = \frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx} = \dot{y}2x$$

$$y'' = (\dot{y}2x)' = \dot{y}'2x + \dot{y}2 = \frac{d\dot{y}}{dz}\frac{dz}{dx}2x + \dot{y}2 = \ddot{y}4x^2 + 2\dot{y} = 4z\ddot{y} + 2\dot{y}$$

$$z(4z\ddot{y} + 2\dot{y}) + 2x^2\dot{y} + 4(z^2 - 2)y = 0$$

$$z^2\ddot{y} + z\dot{y} + (z^2 - 2)y = 0$$

Besselova diff. enačba $\nu = \sqrt{2}$

$$y = A\mathcal{J}_{\sqrt{2}}(z) + B\mathcal{J}_{-\sqrt{2}}(z)$$

$$y = A\mathcal{J}_{\sqrt{2}}(x^2) + B\mathcal{J}_{-\sqrt{2}}(x^2)$$

Rešitve - skupina B

1. naloga.

$$\begin{aligned}
2y'' - 3y' + y &= 1 \\
2(s^2Y - 2) - 3sY + Y &= \tfrac{1}{s} \\
(2s^2 - 3s + 1)Y &= 4 + \tfrac{1}{s} \\
Y &= \tfrac{4s+1}{s(2s-1)(s-1)} \\
Y &= \tfrac{A}{s} + \tfrac{B}{2s-1} + \tfrac{C}{s-1} \\
4s + 1 &= A(2s-1)(s-1) + Bs(s-1) + Cs(2s-1)
\end{aligned}$$

$$\begin{aligned}
s = 1 &\rightarrow 5 = C \\
s = 0 &\rightarrow 1 = A \\
s = \tfrac{1}{2} &\rightarrow 3 = -\tfrac{1}{4}B
\end{aligned}$$

$$\begin{aligned}
Y &= \tfrac{1}{s} - \tfrac{6}{(s-\frac{1}{2})} + \tfrac{5}{s-1} \\
y(t) &= 1 - 6e^{t/2} + 5e^{t}
\end{aligned}$$

2. naloga.

$$y = \sum_{n=0}^{\infty} C_n x^n$$

$$y' = \sum_{n=1}^{\infty} C_n n x^{n-1}$$

$$\sum_{n=1}^{\infty} C_n n x^n + 2\sum_{n=1}^{\infty} C_n n x^{n-1} + \sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=1}^{\infty} C_n n x^n + 2\sum_{n=0}^{\infty} C_{n+1}(n+1) x^n + \sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=0}^{\infty} C_n n x^n + 2\sum_{n=0}^{\infty} C_{n+1}(n+1) x^n + \sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=0}^{\infty} [C_n n + 2C_{n+1}(n+1) + C_n] x^n = 0$$

$$2C_{n+1}(n+1) + C_n n + C_n = 0 \qquad , \quad n = 0, 1, 2, \ldots$$

(a)

$$C_0 = 1$$

$$C_{n+1} = -C_n/2 \quad , \quad n = 0, 1, 2, \ldots$$

(b)

$$y = 1 - \tfrac{x}{2} + \left(\tfrac{x}{2}\right)^2 - \left(\tfrac{x}{2}\right)^3 + \cdots$$

$$y = \tfrac{1}{1+\frac{x}{2}} = \tfrac{2}{2+x}$$

3. naloga.

$$y' = \frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx} = \dot{y}3x^2$$

$$y'' = (\dot{y}3x^2)' = \dot{y}'3x^2 + \dot{y}6x = \frac{d\dot{y}}{dz}\frac{dz}{dx}3x^2 + \dot{y}6x = \ddot{y}9x^4 + 6x\dot{y}$$

$$9x^6\ddot{y} + 6x^3\dot{y} + 3x^3\dot{y} + 9(z^2-3)y = 0$$

$$z^2\ddot{y} + z\dot{y} + (z^2-3)y = 0$$

Besselova diff. enačba $\nu = \sqrt{3}$

$$y = A\mathcal{J}_{\sqrt{3}}(z) + B\mathcal{J}_{-\sqrt{3}}(z)$$

$$y = A\mathcal{J}_{\sqrt{3}}(x^3) + B\mathcal{J}_{-\sqrt{3}}(x^3)$$