1. KOLOKVIJ MATEMATIKA IV

8.4.2011

1. (40%) Z *Laplacovo* transformacijo poišči rešitev $y(t)$ diferencialne enačbe

$$\begin{aligned} y'' - 3y' + 2y &= e^t \\ y(0) &= 1 \\ y'(0) &= -2 \end{aligned}$$

2. (30%) Pošči prve štiri od 0 različne člene v vrsti $y = \sum_{n=0}^{\infty} C_n x^n$, ki je rešitev diferencialne enačbe

$$\begin{aligned} y'' - xy' + y &= 0 \\ y(0) &= -12 \\ y'(0) &= 1 \end{aligned}$$

3. (30%) Z vpeljavo neodvisne spremenljivke $t = \frac{1}{x}$ in funkcije u, kjer je $y = ux$, poišči splošno rešitev diferencialne enačbe

$$y'' - \frac{1}{x}y' + \frac{1}{x^4}y = 0$$

Rešitve

1. naloga

$$(s^2Y - s + 2) - 3(sY - 1) + 2Y = \frac{1}{s-1}$$

$$(s^2 - 3s + 2)Y = s - 5 + \frac{1}{s-1}$$

$$Y = \frac{s^2 - 6s + 6}{(s-1)^2(s-2)} = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s-2}$$

$$s^2 - 6s + 6 = A(s-1)(s-2) + B(s-2) + C(s-1)^2$$

$$\begin{array}{rrl} ks^2: & 1 & = A + C \\ ks: & -6 & = -3A + B - 2C \\ k: & 6 & = 2A - 2B + C \end{array}$$

$$A = 3\ ,\ B = -1\ ,\ C = -2$$

$$Y = \frac{3}{s-1} - \frac{1}{(s-1)^2} - \frac{2}{s-2}$$

$$\boxed{y(t) = 3e^t - te^t - 2e^{2t}}$$

2. naloga

$$y = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + \cdots$$

$$y' = C_1 + 2C_2 x + 3C_3 x^2 + 4C_4 x^3 + \cdots$$

$$y'' = 2C_2 + 6C_3 x + 12C_4 x^2 + \cdots$$

$$y'' - xy' + y =$$

$$\begin{array}{rrrrr} 2C_2 & +6C_3 x & 12C_4 x^2 & +\cdots & - \\ & -C_1 x & -2C_2 x^2 & -\cdots & + \\ +C_0 & +C_1 x & +C_2 x^2 & +\cdots & = \end{array}$$

$$(C_0 + 2C_2) + 6C_3 x + (12C_4 - C_2)x^2 + \cdots$$

$$y(0) = C_0 = -12$$

$$y'(0) = C_1 = 1$$

$$C_2 = -\frac{1}{2} C_0 = 6$$

$$C_3 = 0$$

$$C_4 = \frac{1}{12} C_2 = \frac{1}{2}$$

$$\boxed{y = -12 + x + 6x^2 + \tfrac{1}{2} x^4 + \cdots}$$

3. naloga

Označimo $y' = \frac{dy}{dx}$, $\dot{u} = \frac{du}{dt}$

$y = ux$, $t = \frac{1}{x}$

$$y' = \dot{u}(-\frac{1}{x^2})x + u = -\dot{u}t + u$$

$$y'' = -(\ddot{u}t + \dot{u})(-\frac{1}{x^2}) + \dot{u}(-\frac{1}{x^2}) = \ddot{u}t^3$$

$$\ddot{u}t^3 - t(-\dot{u}t + u) + t^4ux = 0 \quad / : t$$

$$t^2\ddot{u} + t\dot{u} + (t^2 - 1)u = 0$$

Besselova dif.en. $\nu = 1$

$$u = C_1\mathcal{J}_1(t) + C_2\mathcal{Y}_1(t)$$

$$\boxed{y = x\Big(C_1\mathcal{J}_1(\tfrac{1}{x}) + C_2\mathcal{Y}_1(\tfrac{1}{x})\Big)}$$