2. Kolokvij MATEMATIKA IV

3.6.2011

Bolonjski študij

1. (30%) Brez uporabe separacije spremenljivk poiščite rešitev $u(x, y)$ parcialne diferencialne enačbe

$$\begin{aligned} y\, u_{xy} &= u_x \\ u_x(x, 1) &= 2x \\ u(1, y) &= -y \end{aligned}$$

2. (40%) Poiščite rešitev $u(x, t)$ parcialne diferencialne enačbe

$$\begin{aligned} u_{xx} &= u_t - u \\ u(0, t) &= 0 \\ u(\pi, t) &= 0 \\ u(x, 0) &= \sin x + \sin 2x \end{aligned}$$

3. (30%) Poiščite ekstremalo funkcionala

$$I(y) = \int_0^{\ln 3} \frac{y + 2}{y'}\, dx$$

$$\begin{aligned} y(0) &= 1 \\ y(\ln 3) &= 7 \end{aligned}$$

2. Kolokvij MATEMATIKA IV

3.6.2011

Pred-Bolonjski študij

1. (30%) Brez uporabe separacije spremenljivk poiščite rešitev $u(x, y)$ parcialne diferencialne enačbe

$$\begin{aligned} y\, u_{xy} &= u_x \\ u_x(x, 1) &= 2x \\ u(1, y) &= -y \end{aligned}$$

2. (40%) Poiščite rešitev $u(x, t)$ parcialne diferencialne enačbe

$$\begin{aligned} u_{xx} &= u_t - u \\ u(0, t) &= 0 \\ u(\pi, t) &= 0 \\ u(x, 0) &= \sin x + \sin 2x \end{aligned}$$

3. Vržemo 3 kovance, slučajna spremenljivka $X =$ število padlih grbov.
   (a) (10%) Podajte verjetnostno funkcijo slučajne spremenljivke X!
   (b) (10%) Po prvem metu odstranimo kovance padle na cifro; ostale še enkrat vržemo, slučajna spremenljivka $Y =$ število padlih grbov. Koliko je $P(Y = 2)$?
   (c) (10%) Podajte verjetnostno funkcijo slučajne spremenljivke Y!

Rešitve

1. naloga

$$u_x = v$$

$$yv_y = v$$

$$y\frac{dv}{dy} = v$$

$$\int \frac{dv}{v} = \int \frac{dy}{y}$$

$$\ln v = \ln y + \ln C$$

$$u_x = v = Cy$$

$$y = 1 \quad \rightarrow \quad C = 2x$$

$$u_x = 2xy$$

$$u = x^2y + C$$

$$x = 1 \quad \rightarrow \quad y + C = -y$$

$$\boxed{u(x, y) = x^2y - 2y}$$

2. naloga

$$u = F(x)G(t)$$
$$F''(x)G(t) = F(x)G'(t) - F(x)G(t)$$
$$\frac{F''(x)}{F(x)} = \frac{G'(t)}{G(t)} - 1 = -\lambda^2$$

$$F''(x) + \lambda^2 F(x) = 0$$
$$k^2 + \lambda^2 = 0$$
$$k_{1,2} = \pm\lambda i$$
$$F(x) = A\cos(\lambda x) + B\sin(\lambda x)$$
$$x = 0 \quad \rightarrow \quad A = 0$$
$$x = \pi \quad \rightarrow \quad B\sin(\lambda\pi) = 0 \quad \rightarrow \quad \lambda = n$$
$$F_n(x) = B_n \sin(nx)$$

$$\frac{G'(t)}{G(t)} = 1 - n^2$$
$$\int \frac{dG}{G} = \int (1 - n^2)dt$$
$$\ln G = (1 - n^2)t + \ln C$$
$$G_n(t) = C_n e^{(1-n^2)t}$$

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx) e^{(1-n^2)t}$$

$$t = 0 \quad \rightarrow \quad \sum_{n=1}^{\infty} A_n \sin(nx) = \sin x + \sin 2x$$
$$A_1 = A_2 = 1\ ,\ A_n = 0$$

$$\boxed{u(x,t) = \sin x + \sin(2x)e^{-3t}}$$

3. naloga

$$\frac{y+2}{y'} - y'\left(-\frac{y+2}{y'^2}\right) = A$$

$$\frac{2(y+2)}{y'} = A$$

$$Ay' = 2(y+2)$$

$$A\int \frac{dy}{y+2} = \int 2dx$$

$$A\ln(y+2) = 2x + B$$

$$x = 0 \quad \rightarrow \quad A\ln 3 = B$$

$$x = \ln 3 \quad \rightarrow \quad A\ln 9 = 2\ln 3 + B$$

$$A(\ln 9 - \ln 3) = 2\ln 3$$

$$A = 2\ ,\ B = 2\ln 3$$

$$y = e^{x+\ln 3} - 2$$

$$\boxed{y = 3e^x - 2}$$

3. naloga

a)

$$X: \begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{pmatrix}$$

b)

Označimo $H_i = (X = i) \quad , \quad A = (Y = 2)$ in uporabimo formulo $P(A) = \sum_{i=0}^{3} P(H_i)P(A/H_i)$

$$P(Y=2) = \frac{1}{8} \cdot 0 + \frac{3}{8} \cdot 0 + \frac{3}{8} \cdot \frac{1}{4} + \frac{1}{8} \cdot \frac{3}{8} = \frac{9}{64}$$

c)

$$P(Y=3) = \frac{1}{8} \cdot \frac{1}{8} = \frac{1}{64}$$

$$P(Y=1) = \frac{1}{8} \cdot 0 + \frac{3}{8} \cdot \frac{1}{2} + \frac{3}{8} \cdot \frac{2}{4} + \frac{1}{8} \cdot \frac{3}{8} = \frac{27}{64}$$

$$P(Y=0) = \frac{1}{8} \cdot 1 + \frac{3}{8} \cdot \frac{1}{2} + \frac{3}{8} \cdot \frac{1}{4} + \frac{1}{8} \cdot \frac{1}{8} = \frac{27}{64}$$

$$Y: \begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{27}{64} & \frac{27}{64} & \frac{9}{64} & \frac{1}{64} \end{pmatrix}$$