# 1. Kolokvij MATEMATIKA IV

## 12.4.2013

**1.** (30%) Z *Laplacovo* transformacijo poiščite rešitev $y(t)$ diferencialne enačbe

$$\begin{aligned} y''' - 2y'' + 5y' &= 25 \\ y(0) &= 0 \\ y'(0) &= 3 \\ y''(0) &= 6 \end{aligned}$$

**2.** (30%) Poiščite *Laplacovo* transformacijo funkcije

$$y(t) = \begin{cases} 0 & , \text{ če } \; t < 0 \\ \text{2t+1} & , \text{ če } \; 0 \le t < 1 \\ 3 & , \text{ če } \; 1 \le t < 2 \\ 1 & , \text{ če } \; 2 \le t \end{cases}$$

**3.** (40%) Z nastavkom $y = \sum_{n=0}^{\infty} C_n x^n$ rešite diferencialno enačbo

$$\begin{aligned} xy'' + 2y' + 4xy &= 0 \\ y(0) &= 2 \\ y'(0) &= 0 \end{aligned}$$

**(a)** (30%) Zapišite rekurzijsko formulo za koeficiente $C_n$ !

**(b)** (10%) Zapišite rešitev do vključno potence $x^5$ !

**(c)** (+10%) Izrazite rešitev z elementarnimi funkcijami !

## Rešitve

### 1. naloga

$$(s^3Y - 3s - 6) - 2(s^2Y - 3) + 5sY = \frac{25}{s}$$

$$(s^3 - 2s^2 + 5s)Y = 3s + \frac{25}{s}$$

$$Y = \frac{3s^2 + 25}{s^2(s^2 - 2s + 5)} = \frac{A}{s^2} + \frac{B}{s} + \frac{Cs + D}{s^2 - 2s + 5}$$

$$3s^2 + 25 = A(s^2 - 2s + 5) + Bs(s^2 - 2s + 5) + (Cs + D)s^2$$

$$k \quad \to \quad 25 = 5A \quad \to \quad A = 5$$

$$ks \quad \to \quad 0 = -2A + 5B \quad \to \quad B = 2$$

$$ks^2 \quad \to \quad 3 = A - 2B + D \quad \to \quad D = 2$$

$$ks^3 \quad \to \quad 0 = B + C \quad \to \quad C = -2$$

$$Y = \frac{5}{s^2} + \frac{2}{s} + \frac{-2s + 2}{s^2 - 2s + 5} = \frac{5}{s^2} + \frac{2}{s} - 2\frac{(s - 1)}{(s - 1)^2 + 4}$$

$$\boxed{x(t) = 5t + 2 - 2e^t \cos 2t}$$

### 2. naloga

$$y(t) = (2t + 1)[u_0(t) - u_1(t)] + 3[u_1(t) - u_2(t)] + u_2(t) =$$
$$(2t + 1)u_0(t) - 2(t - 1)u_1(t) - 2u_2(t)$$

$$\boxed{Y(s) = \frac{2}{s^2} + \frac{1}{s} - \frac{2}{s^2}e^{-s} - \frac{2}{s}e^{-2s}}$$

**3. naloga**

**a)**

$$y = \sum_{n=0}^{\infty} C_n x^n$$

$$y' = \sum_{n=1}^{\infty} C_n n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} C_n n(n-1) x^{n-2}$$

$$\sum_{n=2}^{\infty} C_n n(n-1) x^{n-1} + 2\sum_{n=1}^{\infty} C_n n x^{n-1} + 4\sum_{n=0}^{\infty} C_n x^{n+1} = 0$$

V tretji vrsti zamenjamo indeks $n+1 \to n-1$ :

$$\sum_{n=2}^{\infty} C_n n(n-1) x^{n-1} + 2\sum_{n=1}^{\infty} C_n n x^{n-1} + 4\sum_{n=2}^{\infty} C_{n-2} x^{n-1} = 0$$

$$2C_1 + \sum_{n=2}^{\infty} \Big[C_n n(n-1) + 2C_n n + 4C_{n-2}\Big] x^{n-1} = 0$$

$$\boxed{C_1 = 0 \quad , \quad C_n = \frac{-4}{n(n+1)} C_{n-2} \quad , \quad n = 2, 3, \ldots}$$

**b)**

$$C_0 = 2 \; , \; C_1 = C_3 = C_5 = 0 \; , \; C_2 = \frac{-4}{2 \cdot 3} C_0 = -\frac{4}{3} \; , \; C_4 = \frac{-4}{4 \cdot 5} C_2 = \frac{4}{15}$$

$$\boxed{y = 2 - \frac{4}{3}x^2 + \frac{4}{15}x^4 - \cdots}$$

**c)**

$$C_0 = y'(0) = 2$$

$$C_2 = \frac{-4}{2 \cdot 3} C_0 = -\frac{2^3}{3!}$$

$$C_4 = \frac{-4}{4 \cdot 5} C_2 = \frac{2^5}{5!}$$

$$C_6 = \frac{-4}{6 \cdot 7} C_4 = -\frac{2^7}{7!}$$

$\cdots$

$$C_{2n} = (-1)^n \frac{2^{2n+1}}{(2n+1)!}$$

$$y = \sum_{n=0}^{\infty} C_{2n} x^{2n} = \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n+1}}{(2n+1)!} x^{2n} = \frac{1}{x} \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!} =$$

$$\boxed{\frac{\sin 2x}{x}}$$