1. Kolokvij MATEMATIKA IV

17.4.2015

1. (40%) Z *Laplacovo* transformacijo poiščite rešitev $y(t)$ diferencialne enačbe

$$\begin{aligned} y''' + y' &= 1 \\ y(0) &= 1 \\ y'(0) &= 2 \\ y''(0) &= 0 \end{aligned}$$

2. (30%) Z nastavkom $y = \sum\limits_{n=0}^{\infty} C_n x^n$ rešite diferencialno enačbo

$$xy'' - xy' - y = 0$$

(a) (20%) Zapišite rekurzijsko formulo za koeficiente C_n .

(b) (10%) Izrazite rešitev z elementarnimi funkcijami .

3. (30%) Poiščite vsaj eno rešitev diferencialne enačbe

$$(x^4 - x^2)y'' + (4x - 2x^3)y' - (6 + 10x^2)y = 0$$

Navodilo: Vpeljite neznano funkcijo $y = x^2 z$.

Rešitve

1. naloga

$$s^3Y - s^2 - 2s + sY - 1 = \frac{1}{s}$$

$$(s^3 + s)Y = \frac{1}{s} + (s^2 + 1) + 2s$$

$$Y = \frac{1}{s^2(s^2+1)} + \frac{(s^2+1)}{s(s^2+1)} + \frac{2s}{s(s^2+1)} = \frac{1}{s^2} - \frac{1}{s^2+1} + \frac{1}{s} + \frac{2}{s^2+1}$$

$$\boxed{y(t) = t + 1 + \sin t}$$

2. naloga

a)

$$y = \sum_{n=0}^{\infty} C_n x^n$$

$$y = \sum_{n=1}^{\infty} C_n n x^{n-1}$$

$$y = \sum_{n=2}^{\infty} C_n n(n-1) x^{n-2}$$

$$\sum_{n=2}^{\infty} C_n n(n-1) x^{n-1} - \sum_{n=1}^{\infty} C_n n x^n - \sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=1}^{\infty} C_{n+1}(n+1) n x^n - \sum_{n=1}^{\infty} C_n n x^n - \sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=1}^{\infty} [C_{n+1}(n+1)n - C_n n - C_n] x^n - C_0 = 0$$

$$C_0 = 0 \quad , \quad C_{n+1}(n+1)n = C_n(n+1)$$

$$\boxed{C_0 = 0 \quad , \quad C_{n+1} = \frac{C_n}{n} \, , \, n = 1, 2, \ldots}$$

b)

$$C_1\ ,\ C_2 = C_1\ ,\ C_3 = \frac{1}{2}\,C_1\ ,\ C_4 = \frac{1}{2\cdot 3}\,C_1\ ,\ \ldots C_n = \frac{1}{(n-1)!}\,C_1$$

$$y = C_1 x + C_1 x^2 + C_1\frac{x^3}{2!} + \cdots C_1\frac{x^n}{(n-1)!} + \cdots$$

$$\boxed{y = C_1 x e^x}$$

3. naloga

$$y = x^2 z$$
$$y' = 2xz + x^2 z'$$
$$y'' = 2z + 2xz' + 2xz' + x^2 z''$$

$$(x^4 - x^2)(2z + 4xz' + x^2 z'') + (4x - 2x^3)(2xz + x^2 z') - (6 + 10x^2)x^2 z = 0$$
$$(x^4 - x^2)x^2 z'' + (4x^5 - 4x^3 + 4x^3 - 2x^5)z' +$$
$$+(2x^4 - 2x^2 + 8x^2 - 4x^4 - 6x^2 - 10x^4)z = 0$$
$$(x^4 - x^2)x^2 z'' + 2x^5 z' - 12x^4 z = 0$$
$$(x^2 - 1)z'' + 2xz' - 3\cdot 4z = 0$$

Legendrova dif. enačba, $n = 3$

$z = P_3(x)$

$$\boxed{y = x^2 P_3(x)}$$